

PHYS 251: Atomic Physics Lab  
Due date (in lab): Sept. 6, 2023 (Wednesday section)  
Sept. 7, 2023 (Thursday section)

## Error Analysis Exercises

### Problem #1: Statistical analysis

Ten different students use a 1 m ruler to measure the length of rope that is about 1.5 meters long and get the following values (in cm):

149.5 149.8 150.1 149.8 150.3 149.6 149.9 150.0 150.4 149.7

- a) Calculate the average length of the rope.
- b) Calculate/estimate the precision (i.e. error) of a single measurement.
- c) Calculate the uncertainty on the average of the measurements of the rope length.

### Problem #2: Error propagation

A student measures the lengths of the four sides of a rectangle and gets the following results (in cm):

$300.5 \pm 0.5$     $153.2 \pm 0.3$     $299.9 \pm 0.5$     $152.6 \pm 0.3$

- a) Are the measurements consistent with each other? Explain.
- b) Calculate the average lengths of the long and short sides of the rectangle and the associated errors on those averages.
- c) Calculate the perimeter of the rectangle and the error on it.
- d) Calculate the area of the rectangle and the error on it.
- e) Calculate length of diagonal on rectangle and the error on it.
- f) Calculate the angle in degrees that the diagonal makes with the long side of the rectangle and the error on it.

### Problem #3: Unweighted vs weighted average with error bars

Five different students measure  $\pi$  (i.e. 3.1415926...) by various methods and get the following results:

$3.15 \pm 0.03$     $3.12 \pm 0.01$     $3.16 \pm 0.2$     $3.20 \pm 0.05$     $3.14 \pm 0.04$

- a) What is the average value of these measurements, and what is the error on this average.
- b) A sixth student finds the following value for  $\pi$  with a much larger error bar:  $2.7 \pm 0.7$ . Calculate the average of the six measurements and the error on this average.
- c) The sixth value is throwing off the average, even though it has a larger error bar. A more accurate way to calculate the average in this case is to use a weighted average that gives more weight to measurements with smaller error bars:

$$\langle x \rangle_{weighted} = \frac{\sum(x_i/\sigma_i^2)}{\sum(1/\sigma_i^2)} \quad \text{with } \sigma_{weighted} = \sqrt{\frac{1}{\sum(1/\sigma_i^2)}}$$

Calculate the weighted average of the six measurements of  $\pi$  and the associated error on this weighted average, and then compare with the result from (b).