Optical Interferometry

Experiment objectives: Assemble and align Michelson interferometer, check its calibration by using a laser of known wavelength, and then determine the refractive index of the air.

Introduction

Optical interferometers are the instruments that use interference of two or more superimposed light beams. These are one of the most common optical tools, and are used for precision measurements, surface diagnostics, astrophysics, seismology, quantum information, etc. There are many configurations of optical interferometers, and in this lab you will become familiar with probably the most famous one - Michelson interferometer.

The **Michelson interferometer**, shown in Fig. 1, is based on the interference of two light fields: the initial beam is split into two arms on a beam splitter, and then these resulting beams are retro-reflected and recombined on the same beamsplitter again. The difference in optical paths in the two arms leads to a changing relative phase of two beams, so when overlapped the two light fields will interfere constructively or destructively. Such an interferometer was first used by Michelson and Morley [1] in 1887 to determine that electromagnetic waves propagate in vacuum, giving the first strong evidence against the theory of a *luminiferous aether* (a fictitious medium for light wave propagation) and providing insight into the true nature of electromagnetic radiation. Michelson interferometers are widely used in many areas of physics and engineering, including the recent discovery of gravitational waves at the LIGO facility (see materials at page 8 for more information).

Figure 1 shows the traditional setting for a Michelson interferometer. A beam splitter (a glass plate which is partially silver-coated on the front surface and angled at 45 degrees) splits the laser beam into two parts of equal amplitude. One beam (that was initially transmitted by the beam splitter) travels to a movable mirror M_1 and back again. One-half of this amplitude is then reflected from the partially-silvered surface and directed at 90 degrees toward the observer (you will use a viewing screen). At the same time the second beam (reflected by the beam splitter) travels at 90 degrees toward an adjustable mirror M_2 and back. Since this beam travels through the glass beamsplitter plate 3 times, its optical path length is longer than the first beam which only passes through the beam splitter one time. To compensate for that, the first beam passes twice through a clear glass plate called the compensator plate, that has the same thickness. At the beam splitter one-
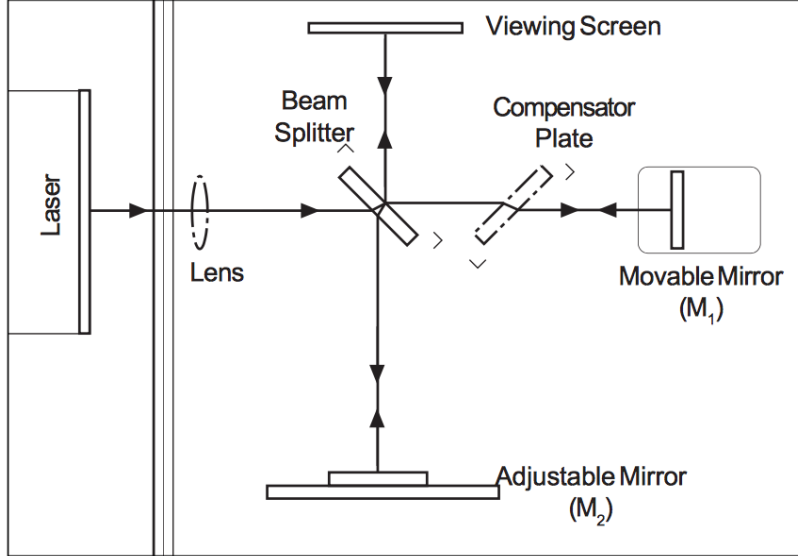
Figure 1: A Michelson interferometer setup.

half of this light is transmitted to an observer, overlapping with the beam reflected by M_2, and the total amplitude of the light at the screen is a combination of amplitude of the two beams. Assuming that the initial optical field is described by $E_0 \cos(kz - \omega t)$, where E_0 is its amplitude, $k = 2\pi\lambda$ is the wave vector, and λ and ω are correspondingly the wavelength and the frequency of the laser . After splitting and recombining at the beam splitter, the total field at the viewing screen beam splitter is a combination of the electric fields from the two arms of the interferometer :

$$
\begin{aligned}
E_{total} &= \frac{1}{2}E_0 \cos(kz - \omega t + kL_1) + \frac{1}{2}E_0 \cos(kz - \omega t + kL_2) \\
&= E_0 \cos(k\Delta L/2) \cos(kz - \omega t + k(L_1 + L_2)/2),
\end{aligned} \tag{1}
$$

where $L_{1,2}$ are the round-trip optical paths for each interferometer arm, and $\Delta L = L_1 - L_2$ is the optical path difference for the two beams. You may notice that the final expression describes a laser fields with the amplitude $E_0 \cos(k\Delta L/2)$. As a result, the intensity we are going to see on the screen will be $I_0 \cos^2(k\Delta L/2)$, where I_0 is the total intensity of the initial laser beam.

Mirror M_1 is mounted on a precision traveling platform, and thus we can adjust its position (by turning the micrometer screw) with great precision to make the distances traversed on both arms exactly the same. Because the thickness of the compensator plate and the beamsplitter are the same, both wavefronts pass through the same amount of glass and air, so the path length of the light beams in both interferometer arms is exactly the same. Therefore, the two fields will arrive in phase to the observer, their amplitudes will add up constructively, and we should see all the original laser beam to go toward the viewing screen. However, if now you turn the micrometer to offset the length of one arm by a quarter of the wavelength, $\Delta L = \lambda/2$, the two beams arriving to the beam splitter will be out of phase
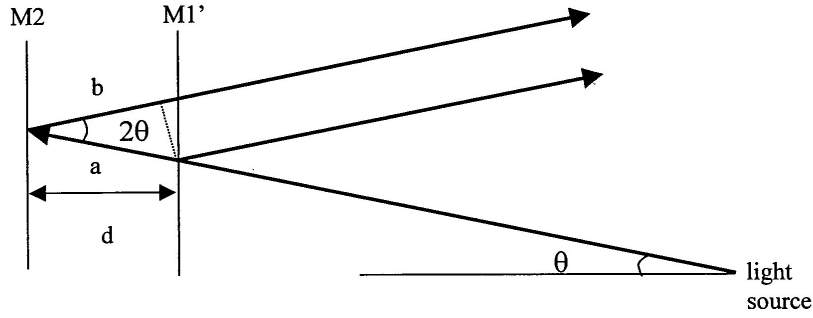
Figure 2: Explanation the interference pattern. Notice that to simplify the figure we have "unfold" the interferometer by neglecting the reflections on the beamsplitter.

$kL_1 - kL_2 = \pm\pi$, resulting in the total cancellation (destructive interference).

It is easy to see that constructive interference happens when the difference between path lengths in the two interferometer arms is equal to the integer number of wavelengths $\Delta L = m\lambda$, and destructive interference corresponds to a half-integer number of wavelengths $\Delta L = (m + 1/2)\lambda$ (here m is an integer number). Since the wavelength of light is small $(600 - 700$ nm for a red laser), Michelson interferometers are able to measure distance variation with very good precision.

To simplify the alignment of a Michelson interferometer, it is convenient to work with diverging optical beams [2]. In this case when the interferometer is aligned properly, the interference pattern looks like a set of concentric bright and dark circles, since the different portions of the diverging beam travel at slightly different angles, and therefore acquire different phases, as illustrated in Figure 2. Suppose that the actual path length difference between two arms is d. Then the path length difference for two off-axis rays arriving at the observer is $\Delta L = a + b$ where $a = d/\cos\theta$ and $b = a\cos 2\theta$:

$$\Delta L = \frac{d}{\cos\theta} + \frac{d}{\cos\theta}\cos 2\theta \qquad (2)$$

Recalling that $\cos 2\theta = 2(\cos\theta)^2 - 1$, we obtain $\Delta L = 2d\cos\theta$. The two rays interfere constructively for any angle θ_c for which $\Delta L = 2d\cos\theta = m\lambda$ (m is an integer number); at the same time, two beams traveling at the angle θ_d interfere destructively when $\Delta L = 2d\cos\theta = (m+1/2)\lambda$. Because of the symmetry about the normal direction to the mirrors, this means that interference (bright and dark fringes) appears in a circular shape. If the fringes are not circular, the mirrors are not parallel, and additional alignment of the interferometer is required.

When the path length difference is varied, by using the micrometer to move one of the mirrors a distance d along the horizontal axis of Figure 2, the fringes appear to "move". As the micrometer is turned, the condition for constructive and destructive interference is alternately satisfied at any given angle. If we fix our eyes on one particular spot and count how many bright fringes pass that spot as we move mirror M_1 by a known distance, we

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can determine the wavelength of light in the media using the condition for constructive interference

$$2d\cos\theta = m\lambda. \tag{3}$$

It is usually convenient to concentrate on the center of the fringe bull's eye at $\theta = 0$. Equation 3 for constructive interference then reduces to $2d = m\lambda$. If X_1 is the initial position of the mirror M_1 (as measured on the micrometer) and X_2 is the final position after a number of fringes Δm has been counted, we have $2d = 2(X_2 - X_1) = \lambda\Delta m$. Then the laser wavelength λ is given by:

$$\lambda = 2(X_2 - X_1)/\Delta m. \tag{4}$$

Laser Safety

Never look directly at the laser beam! Align the laser so that it is not at eye level. Even a weak laser beam can be dangerous for your eyes.

Alignment of Michelson interferometer

Equipment needed: Pasco precision interferometry kit, a laser.

Set up the interferometer as shown in Figure 1 using components from the PASCO interferometry kit. The mirrors $M_{1,2}$ are, correspondingly, a movable and an adjustable mirror from the kit.

1. Begin by installing the movable mirror, adjustable mirror, component holder with viewing screen (magnetically attaches), and the lens holder. Do not install the lens yet. Make sure screws are firmly tightened as we don't want components to move around.

2. Level the table by adjusting its legs and/or adjust the laser height, location and direction to get the beam spot at the center of the movable mirror. It is a good idea to tape your laser down at this point, or make sure it's securely mounted in the optical breadboard (if your station has one).

3. Tweak the laser direction using the two knobs on its back so that the beam reflects back into its aperture.

4. Install the beam splitter, orient it to center the new beam spot on the adjustable mirror.

5. Tweak the adjustable mirror using the two knobs on its back. You want to make the two sets of dots visible on the viewing screen come into alignment. When they are properly aligned you should see interference fringes appear in the dot.

6. It is possible to see interference without the compensator (ask yourself why). But, it can potentially make the pattern better and more distinct. Try inserting it and aligning to improve the pattern.

7. Install the lens to blow up the pattern.

8. Further tweak the adjustable mirror to bring the bulls eye on screen. You will find it's very sensitive and a little tweaking goes a long way.

The interferometer is very sensitive to small displacement of the mirror - but it also makes it very sensitive to any mirror alignments and vibrations. Be patient, make small changes, think about what you are doing, and get some help from the instructor and TA. It helps to start the alignment procedure with the micrometer and the movable mirror near the center of their ranges. Also, the alignment knobs on the laser and adjustable mirror should be at the middle of their ranges, so that the plane of each is perpendicular to the table.

A word of caution: sometimes dust on a mirror or imperfections on optical surfaces may produce similar intensity patterns. True interference disappears if you block one arm of the interferometer. Try it!

Note: before starting the measurements, make sure you understand how to read the micrometer properly!

Figure 3: Micrometer readings. The coarse division equals to 100 μm, and smallest division on the rotary dial is 1 μm (same as 1 micron). The final measurements is the sum of two. So, from left to right, the figures above show 211 μm, 345 μm, and 166 μm.

Wavelength measurements using Michelson interferometer

Record the initial reading on the micrometer. Focus on the central fringe and begin turning the micrometer. You will see that the fringes move. For example, the central spot will change from bright to dark to bright again, that is counted as one fringe. A good method: pick out a reference line on the screen and then softly count fringes as they pass the point. Count a total of about $\Delta m = 50$ fringes and record the new reading on the micrometer.

For each trial, approximately 50 fringes should be accurately counted and tabulated with the initial X_1 and final X_2 micrometer settings. Do this at least five times (e.g., 5×50

fringes). Consider moving the mirror both forward and backward. Make sure that the difference $X_2 - X_1$ is consistent between all the measurements. Calculate the average value and standard deviation of the micrometer readings $< X_2 - X_1 >$. What value do you obtain for the laser wavelength, and what are your uncertainties on that measurement? Does this match what you expect for a red laser?

Experimental tips

1. Make sure that all optical elements are bolted and cannot move if you bump them! It is good idea to tape the laser to the table.

2. Avoid touching the face of the front-surface mirrors, the beam splitter, and any other optical elements!

3. The person turning the micrometer should also do the counting of fringes. It can be easier to count them in bunches of 5 or 10 (*i.e.* 100 fringes = 10 bunches of 10 fringes).

4. Use a reference point or line and count fringes as they pass.

5. Before the initial position X_1 is read make sure that the micrometer has engaged the drive screw. Chose a direction in which you will move the micrometer handle and do a few turns in that direction prior counting, i.e. take a way the slack on a thread. If you decide to reverse the direction, remember there will be slack which you have to take off before micrometer engages. You will notice that fringes do not change until the slack is removed.

6. Avoid hitting the table which can cause a sudden jump in the number of fringes.

Measurement of the index of refraction of air

Up to now, we neglected the fact that air is different from vacuum, and the value for air's index of refraction n_{air} is very close to unity, but slightly larger [3]: $n_{air} \simeq 1.0003$. Thus, the values of the laser wavelength in air (λ) and in vacuum (λ_0) are slightly different $\lambda = \lambda_0/n_{air}$. Amazingly, a Michelson interferometer is precise enough to be able to make an accurate measurement of this quantity, by taking advantage of the fact that n_{air} changes linearly with the air pressure.

To do the measurement, place a cylindrical gas cell which can be evacuated in the path of light heading to mirror M_1 and correct alignment of the Michelson interferometer, if necessary. Make sure that the gas cell is initially at the atmospheric pressure.

Now pump out the cell by using a hand pump at your station and count the number of fringe transitions Δm that occur. When you are done, record Δm and the final reading of the vacuum gauge p. **Note**: most vacuum gauges display the difference between measured and atmospheric pressure. If absolute pressure is needed, it should be found by subtracting the gauge reading from the atmospheric pressure ($p_0 = 76$ cm Hg). For example, if the gauge
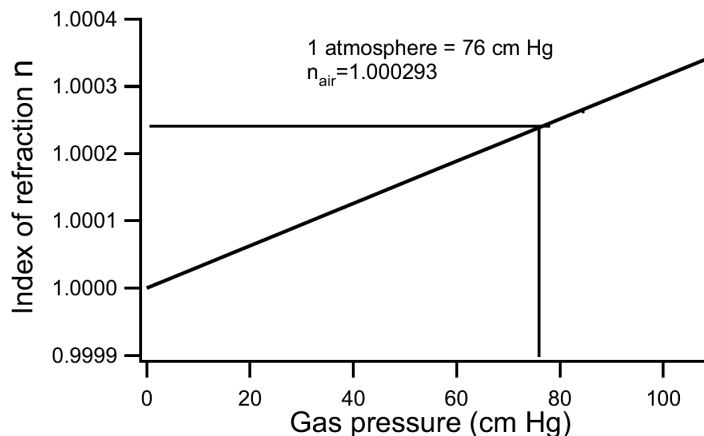
Figure 4: Index of refraction as a function of air gas pressure

reads $\Delta p = 23$ cm Hg, the absolute pressure is $p = p_0 - \Delta p = 53$ cm Hg. Alternatively, you can pump out the air first, and then admit air is slowly to the cell by cracking the valve, while counting the number of fringes that move past a selected fixed point.

The shifting fringes indicate a change in relative optical phase difference for the two arms, which arises from the difference in refractive indices of the gas cell at low and atmospheric pressures Δn. According to Eq.(4), this difference is

$$\Delta n = \Delta m \frac{\lambda_0}{2d_{cell}} \tag{5}$$

where $d_{cell} = 3.0$ cm is the length of the gas cell.

Since the change in the refractive index Δn is linearly depends on the air pressure $\Delta p = p_0 - p_{fin}$, it is now easy to find out the proportionality coefficient $\Delta n/\Delta p$ and calculate the value of the refractive index at the atmospheric pressure n_{air}.

To improve the accuracy of your measurements, calculate the value of the refractive index for several values of air pressure, using Eq.(5), and plot them on a graph, similar to Fig. 4. You won't be able to pump out the cell completely, but it is safe to assume that at $p = 0$ the refractive index is exactly one. Make sure you have enough data points to ensure reliable linear fit.

From the slope of your fit, find the proportionality coefficient $\Delta n/\Delta p$, and use it to calculate the and n_{air} at the atmospheric pressure. Don't forget to include the error bars for all the data points on the graph, and use the fit uncertainty to estimate the uncertainty of the final refractive index. Compare your measurements to the following accepted experimental values: Index of Refraction of Air(STP) = 1.000293

Observation of Gravitational Waves (LIGO)

A Michelson interferometer helps to test the theory of general relativity! Gravity waves, predicted by the theory of relativity, are ripples in the fabric of space and time produced by violent events in the distant universe, such as the collision of two black holes. Gravitational waves are emitted by accelerating masses much as electromagnetic waves are produced by accelerating charges, and often travel to Earth. Until recently the only indirect evidence for these waves was the observation of the rotation of a binary pulsar (for which the 1993 Nobel Prize was awarded).

Hanford, Washington   **Livingston, Louisiana**

Figure 5: For more details see http://www.ligo.caltech.edu/

The Laser Interferometry Gravitational-wave Observatory (LIGO) Michelson interferometer built with the goal of directly detecting gravitational wave. Two mirrors hang 2.5 mi apart, forming one "arm" of the interferometer, and two more mirrors make a second arm perpendicular to the first. Laser light enters the arms through a beam splitter located at the corner of the L, dividing the light between the arms. The light is allowed to bounce between the mirrors repeatedly before it returns to the beam splitter. If the two arms have identical lengths, then interference between the light beams returning to the beam splitter will direct all of the light back toward the laser. But if there is any difference between the lengths of the two arms, some light will travel to where it can be recorded by a photodetector.

The space-time ripples cause the distance measured by a light beam to change as the gravitational wave passes by. These changes are minute: just 10^{-16} centimeters, or one-hundred-millionth the diameter of a hydrogen atom over the 2.5 mile length of the arm. Yet, they are enough to change the amount of light falling on the photodetector, which produces a signal defining how the light falling on the photodetector changes over time. LlGO requires at least two widely separated detectors, operated in unison, to rule out false signals and confirm that a gravitational wave has passed through the earth. Three interferometers were built for LlGO – two near Richland, Washington, and the other near Baton Rouge, Louisiana, shown in Fig. 5.
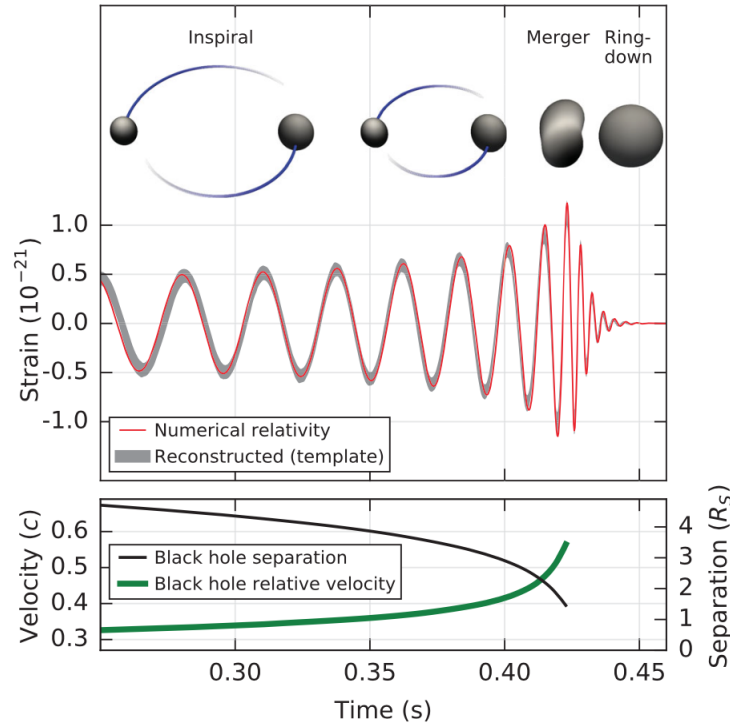
Figure 6: Data from LIGO's two sites from the first detection of gravitational waves caused by the collision of two black holes (from Physical Review Letters **116**, 061102).

In 2015, LIGO observed the first direct detection of gravitational waves from the collision of two black holes. Figure 6 shows data from the black hole merger event where the 'strain' reflects the observed change in distance in the interferometer arm due to the passing gravitation wave, which oscillates as a function of time. For a more information you can browse the LIGO webpage and view a simulation of a black hole merger.

Bibliography

[1] A. A. Michelson,E. W. Morley, "On the Relative Motion of the Earth and the Luminiferous Ether," *American Journal of Science* **34**(203), 333–345 (18870.

[2] A. C. Melissinos and J. Napolitano, *Experiments in Modern Physics*, (Academic Press, New York, 2003).

[3] *Refractive index of air*, available at https://emtoolbox.nist.gov/Wavelength/Documentation.asp.