DSP Research Project: DSP Lock-In Amplifier

I. DSP device requirements

You will design and construct a basic DSP lock-in amplifier with the base specifications listed below. Any improvements on these specifications are welcome, but not necessary.

Reference Signal

Reference signal output: sinusoid, 1 Hz to 100 kHz.

Reference signal stability (0° C to 50° C): 100 ppm or better.

Reference amplitude resolution: equivalent to 8-bit or better.

Output Characteristics

Output with range ± 10 V.

Phase control: 0 to 360° with a resolution of at least 3.6°. You must be able to shift the phase by exactly 90°.

Integration (Δt): 1 ms, 10 ms, 100 ms, 1 s, and 10 s.

2nd harmonic output: -20 dB (i.e. 100 times smaller than the main output) or less.

Output amplitude resolution: equivalent to 8-bit or better.

Input

Input impedance: 1 MΩ.

Sensitivity: 1 mV/V, 10 mV/V, 100 mV/V, 1 V/V.

Input range: ± 10 V.

Dynamic resolution: equivalent to 8-bit or better.

II. Theory of Operation

A lock-in amplifier is a phase-sensitive active bandpass filter. It has one input, an output reference signal, and the lock-in output.

Reference Signal

The reference signal is a very precise reference sinewave, $R(t) = R_0 sin(\omega t)$, with zero DC offset and very stable frequency ω which is sent to an experiment to modulate some quantity (voltage, frequency, current, intensity, magnetic field, etc …).

Lock-In Input

A small signal, S(t), which is affected by the modulated quantity and which is generally below the noise level is sent to the input of the lock-in amplifier.

Lock-In Output

The lock-in amplifier performs the following operation:

$$X(t) = \frac{A}{\Delta t}\int_{t-\Delta t}^{t} R(t + t_{phase})S(t)dt \qquad (1)$$

Where the reference signal, *R(t)*, has been delayed by a time $t_\phi = \phi/\omega$ and A is the amplification. *X(t)* is sent to the lock-in output.

How does a lock-in amplifier work?

A lock-in amplifier is used to search for a small signal in a large noise background. The lock-in technique makes a phase sensitive extraction of the modulation induced by the reference signal.

If *S(t)* contains the modulation induced by the reference signal *R(t)*, then we can write it as a background signal, S_B, with no amplitude at frequency ω, plus the following modulation:

$$S(t) = S_B + S_1 \sin(\omega t + \varphi_1) + S_2 \sin(2\omega t + \varphi_2) + \ldots \qquad (2)$$

where the S_2, S_3, … terms correspond to non-linearities in the transfer of the reference signal modulation to the signal.

We can now rewrite the integral for X(t) as

$$X(t) = \frac{A}{\Delta t}\int_{t-\Delta t}^{t} R_0 \sin(\omega t + \phi)\left[S_B + S_1 \sin(\omega t + \varphi_1) + S_2 \sin(2\omega t + \varphi_2) + \ldots\right]dt \qquad (3)$$

All the above terms average to zero for $\Delta t >> 1/\omega$ except for the "R_0S_1" term. Therefore, we are left with the following expression for *X(t)*, where we have assumed that any variations in S_n are on time scale longer than Δt:

$$X(t) = \frac{AR_0S_1}{\Delta t}\int_{t-\Delta t}^{t} \sin(\omega t + \phi)\sin(\omega t + \varphi_1)dt \qquad (4)$$

Upon completing the integral, we obtain

$$X(t) = AR_0S_1\frac{\cos(\phi - \varphi_1)}{2} \qquad (5)$$

where we have neglected terms that vanish for $\Delta t >> 1/\omega$. This last equation shows that the output does not have any time-dependence at frequency ω. The bandwidth of the lock-in output is $1/\Delta t$, so that one must careful to make sure that any variations in S_1 occur on a time scale longer than Δt.

III. Why would anybody want a lock-in amplifier?

A lock-in amplifier is used for detecting variations in a signal which are much smaller than the background noise.

For example, if you wanted to detect the presence of atoms which scatter light when illuminated by a laser at a particular wavelength, then you would shine the laser on the atoms. If you have plenty of atoms then your fluorescence signal will be very large, and you should have no problems detecting the scattered light. However, if the atom number is small, then you might have a hard time detecting the scattered light, and thus might not be able to detect the presence of the atoms. You could use a lock-in amplifier to modulate the wavelength of the laser light. When the laser light is on resonance, the atoms scatter light, and when the laser light is off resonance is the atoms do not scatter any light. However, the background stray scattering of laser light (i.e. laser light scattered off glass, metal surfaces, etc …) should not change with wavelength. The lock-in amplifier will pick-out the modulation in fluorescence scattering correlated with the modulation in laser wavelength (as long as the phase difference, ϕ-φ, is different from $\pm\pi/2$). The lock-in output signal will be proportional to the number of atoms present.