

Problem set #8

Griffiths 3rd Ed. [4th Ed.] problems

10.18 [10.20], 10.22 [10.25], 10.23 [10.27], 11.3 [11.3], 11.4 [11.4], 11.6 [11.6]

Lagrangian and Hamiltonian of a non-relativistic charged particle

While the electromagnetic force on a charged particle cannot be derived exclusively from a scalar potential (as required for standard Lagrangian mechanics), in this problem you will show that there exists a Lagrangian functional that produces the correct equations of motion for a charged particle in an arbitrary electromagnetic field.

a) Write down the Lorentz force law for charged particle in an electric field $\vec{E}(\vec{r}, t)$ and magnetic field $\vec{B}(\vec{r}, t)$. Write down the Lorentz force law in terms of the electric potential $V(\vec{r}, t)$ and the vector potential $\vec{A}(\vec{r}, t)$.

b) Show that the Lagrangian functional L below produces the Lorentz force law for a particle of charge q and mass m when it is used in conjunction with the Lagrange equation of motion (i.e. the equations that follow from the principle of least action):

$$L = \frac{1}{2} m \dot{\vec{r}}^2 + q \dot{\vec{r}} \cdot \vec{A}(\vec{r}, t) - qV(\vec{r}, t)$$

c) Derive an expression for the canonical momentum \vec{p} , and show that the Hamiltonian H for the charged particle can be written as

$$H = \frac{1}{2m} (\vec{p} - q\vec{A}(\vec{r}, t))^2 + qV(\vec{r}, t).$$