PHYS 402: Electricity & Magnetism II
Due date: Friday, December 6, 2013

Problem set #10

0. Multipole expansion
Read Griffiths (3rd and 4th) sections 3.4, 5.4.3, and 11.1.4.

1. 4-vector transformations
For a frame S' moving relative to your rest frame S with a velocity *v* in the *x* direction, use the definition of the 4-velocity and the appropriate Lorentz transformation to derive Griffiths equations 12.45 (3rd and 4th Editions) that describe how the classical 3-velocity transforms.

2. Tensor inner ("scalar") product
Compute the scalar $F^{\mu\nu}F_{\mu\nu}$.

3. 4-tensor transformations
For a frame S' moving relative to your rest frame S with a velocity *v* in the *x* direction, derive the electric and magnetic field transformation equations given by Griffiths (3rd Ed.) equations 12.108 [12.109, 4th Ed.] starting from the definition of $F^{\mu\nu}$ and the appropriate Lorentz transformations.

4. Electromagnetic wave under a Lorentz Transformation
Consider an electromagnetic plane wave polarized in the x-direction travelling in the positive z-direction in an inertial reference frame S.

a) A scientist is in a rocket travelling with a relativistic velocity *v* in the minus z-direction. Using $F^{\mu\nu}$ and the Lorentz transformation, calculate (all) the components of the electric and magnetic field that the scientist measures.

b) Calculate intensity of the plane in the S frame and in the rocket's frame. Comment on the results.

c) So far we have not considered what happens to the frequency and wavelength of the light (i.e. the Doppler shift). Using the quantum mechanical relations $\vec{p} = \hbar\vec{k}$ and $E = \hbar\omega$ for a photon and the expression for the 4-momentum P^{μ}, show that the 4-wavevector K^{μ} is given by

$$K^{\mu} = \begin{pmatrix} \omega / c \\ k_x \\ k_y \\ k_z \end{pmatrix}$$

, where *k* is the wavevector and ω is the frequency of the plane wave.

Using K^{μ} and the appropriate Lorentz transformation, derive expressions for the wavevector and frequency of the plane wave in the frame of the rocket. Use these results in combination with your results from (a) to write down an expression for the plane wave in the frame of the rocket. *Note:* $\hbar$ is Lorentz invariant.

d) Calculate the photon flux (i.e. photons per second) over a given area (x-y plane or x'-y' plane) in the frame S and the frame of the rocket. Comment on the results.