PHYS 402: Electricity & Magnetism II
Due date: Thursday, September 19, 2013

Problem set #2

Griffiths 3rd Ed. [4th Ed.] problems
7.32 [7.35], 7.36 [7.39], 7.43 [7.45], 7.49 [7.52], 7.50 [7.53], 7.58 [7.62]

Problem: AC skin effect

In this exercise, you will show that the AC current in a wire lies primarily on the skin of the conductor.

Consider a long copper wire with a circular cross-section of radius *a*. For convenience, the wire lies along the z-axis.

a. In the limit of an infinitely long wire, use Maxwell's equations and the vector form of Ohm's law to derive the following Helmholtz-type equation for the current density in the wire:

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right] J(r) \cong i\omega\mu\sigma\, J(r)$$

where *J(r)* is the current density, ω is the AC frequency of the current, μ is the magnetic susceptibility, and σ is the conductivity.

b. The above differential equation is difficult to solve exactly (it involves Bessel functions), so you will find an approximate solution. Solve the above differential equation in the limit of a wire with diameter much larger than the skin depth, and show that near the skin of the wire that the current remains confined to the edge of the conductor, with a characteristic length scale of $\delta = \sqrt{\frac{2\rho}{\omega\mu}}$, where ρ is the resistivity.

Explain the physics of the skin effect.

c. The expression you derived in part b gives the general form of the current distribution. The conductivity of the copper does not change. Starting with your approximate solution for the current distribution, make an estimate of the effective cross-sectional area of the wire.

d. Calculate the resistance of the copper wire at DC for a wire diameter of 1 mm and a length of 1 m. The resistivity of copper is 1.7×10^{-8} Ω·m. At what frequency (in Hz) does the effective resistance of the wire double? Does using stranded wire suppress this effect?