Tuesday, September 5, 2017

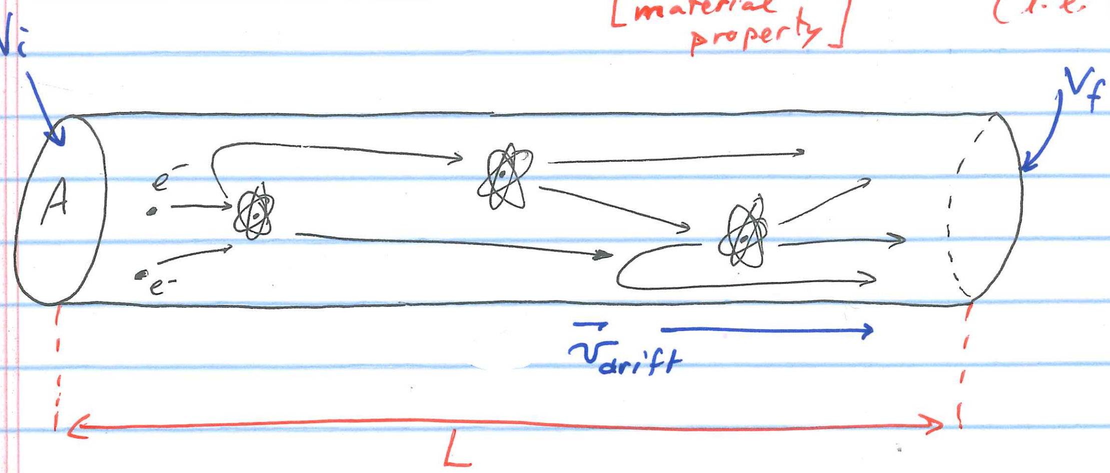
Ohm's Law: (empirical law)

$$\vec{J} = \sigma \vec{f}$$

current density

conductivity (constant) [material property]

force on charges
↳ force per unit charge (i.e. $\vec{E}$)



Ohm's law is counterintuitive, since a force should produce an acceleration, but defects/impurities limit average charge velocities to a drift velocity ($\lesssim$ m/s)

⚠ QM predicts Bloch oscillations, no net acceleration for a defect free lattice

counterintuitive

resistivity $= \rho = \frac{1}{\sigma}$

examples

conductor: $\rho_{copper} = 1.68 \times 10^{-8}\ \Omega.m$

semiconductor: $\rho_{silicon} = 2.5 \times 10^{3}\ \Omega.m$

insulator: $\rho_{glass} = 10^{10} - 10^{14}\ \Omega.m$

ohm's law is valid over 22 orders of magnitude!!!

ohm's law again: $\boxed{\vec{J} = \sigma \vec{E}}$ physicist's version of ohm's law

integrate over volume of wire

$$\underbrace{\int_A da}_{I} \underbrace{\int_L dz}_{L} \vec{J} = \underbrace{\int_A da}_{A} \int_L dz\ \sigma \vec{E} \qquad -\Delta V = V_i - V_f$$

assume constant

constant

assume constant

very good approx. for DC current (not AC!)

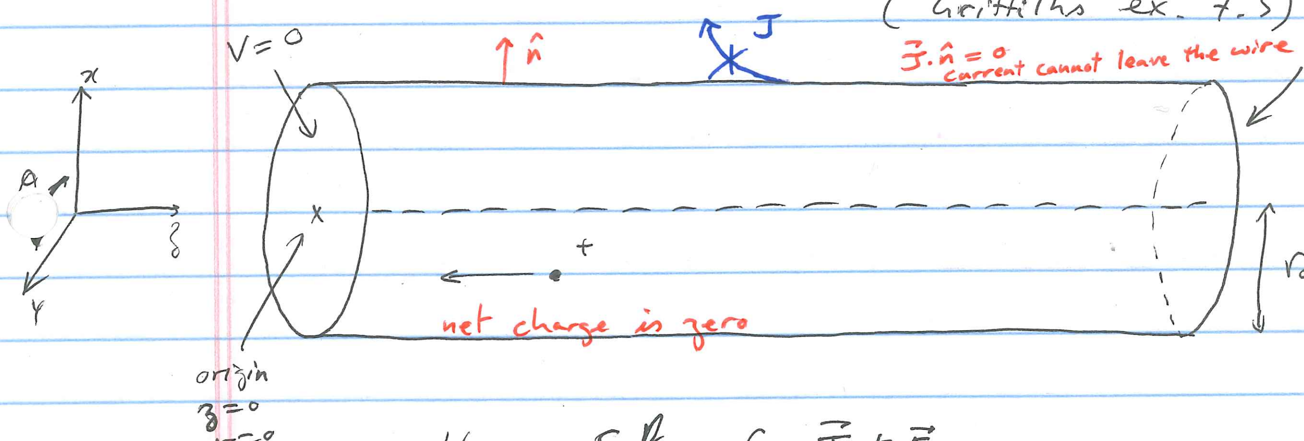
$$\Rightarrow IL = A\underset{\uparrow\ 1/\rho}{\sigma}\underbrace{(V_i - V_f)}_{V} \Leftrightarrow V = \underbrace{\left(\frac{\rho L}{A}\right)}_{\text{resistance } R} I$$

$\Rightarrow$ $\boxed{V = IR}$ Electrician's version of Ohm's Law

$\Rightarrow$ also $\boxed{R = \frac{\rho L}{A}}$

Electric field in a DC current carrying conductor

(Griffiths ex. 7.3)



problem: Solve for $\vec{J}$ & $\vec{E}$

$$\vec{J}\cdot\hat{n} = 0 \text{ on conductor surface} \Leftrightarrow \sigma\vec{E}_{surface}\cdot\hat{n} = 0$$

$$\Leftrightarrow \vec{E}_{surface}\cdot\hat{n} = 0$$

definition of directional derivative

$$\Leftrightarrow (-\vec{\nabla}V)\cdot\hat{n} = 0$$

$$\Leftrightarrow \left.\frac{\partial V}{\partial n}\right|_{surface} = 0 \Leftrightarrow \left.\frac{\partial V}{\partial r}\right|_{r=r_0} = 0$$

Ansatz: for $r < r_0$: $V = \frac{V_0 z}{L}$

Verify boundary condition:

$\nabla_r V = \frac{\partial V}{\partial r} = 0$ on surface (and everywhere)

$\Rightarrow$ proposed solution satisfies $\begin{cases} \text{Laplace's equation } \nabla^2 V = 0 \\ \quad \text{(no free charges)} \\ \text{boundary condition} \end{cases}$

$\Rightarrow$ so it must be the solution (uniqueness theorem)

$\vec{E} = -\vec{\nabla} V = -\frac{V_0}{L}\hat{z}$ $\Rightarrow$ Uniform E-field inside conductor

↳ sort of like an idealized // - plate capacitor.

⚠ Determining the E-field outside the conductor is is much harder (see problem 7.57 [7.43])

- In general, the E-field lines follow the wire shape.
- E-field can be non-uniform in a bend.

Q: How fast does charge travel in a conductor?

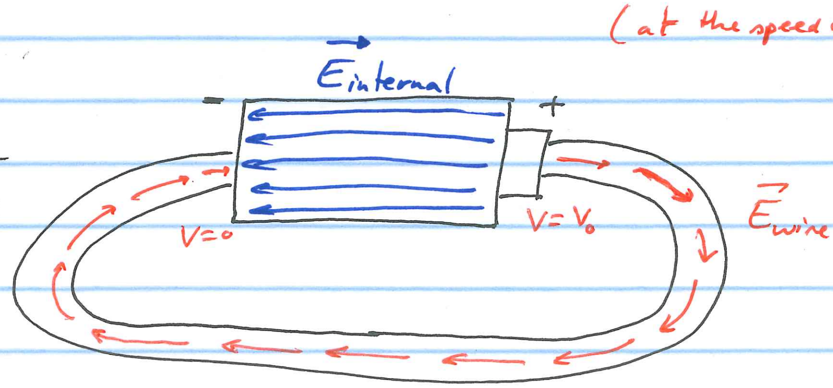
↳ A: drift velocity $\simeq$ 1 - 100 mm/s

Q: Why are electrical signals so fast?

↳ A: All the charges move "simultaneously" under the influence of the E-field, which changes "instantaneously" (at the speed of light)

Electromotive Force

The electromotive force (battery, generator, etc...) moves charges against the internal $\vec{E}$-field.



Force per unit charge: $\vec{f}_{total} = \vec{f}_{source} + \vec{E}$

definition: Electromotive force (EMF) $= \mathcal{E} = \oint_{\substack{\text{current}\\\text{loop}}} \vec{f}_{total} \cdot d\vec{\ell}$

$$= \oint_{\substack{\text{current}\\\text{loop}}} \vec{f}_{source} \cdot d\vec{\ell} + \cancel{\oint_{\substack{\text{current}\\\text{loop}}} \vec{E} \cdot d\vec{\ell}}$$

$= 0$ since $\vec{\nabla} \times \vec{E} = 0$ (Kirchoff's law)

$$= \oint_{\substack{\text{current}\\\text{loop}}} \vec{f}_{source} \cdot d\vec{\ell}$$ = Energy per unit charge delivered by battery

in wire: $\vec{f}_{source} = 0 \Rightarrow \vec{f}_{total} = \vec{E}$

in battery: $\vec{f}_{source} + \vec{E} = 0 = \vec{f}_{total} \Rightarrow \int_-^+ (\vec{f}_{source} + \vec{E}) \cdot d\vec{\ell} = 0$

[net force on charge is zero in battery (resistanceless battery)] $\Rightarrow \mathcal{E} = \int_-^+ \vec{f}_{source} \cdot d\vec{\ell}$

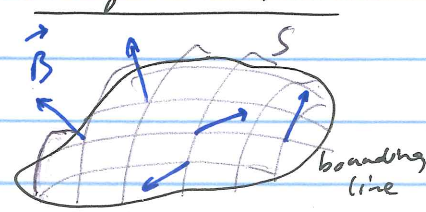
$$= \underbrace{-\int_-^+ \vec{E} \cdot d\vec{\ell}}_{V_0}$$

If there is an internal resistance, then $\mathcal{E} - I \underbrace{R_{internal}}_{Z_{out}} = V_0$

$$\Rightarrow \boxed{\mathcal{E} = V_0}$$

Faraday's Law

definition: Magnetic flux: $\phi = \int_S \vec{B} \cdot d\vec{s}$



<u>Universal flux rule</u>: $\mathcal{E}_{\text{bounding}} = -\frac{d\Phi}{dt}$

<u>note</u>: A changing magnetic flux induces an EMF (i.e. a voltage, <u>not</u> a current)

↳ though generally this happens indirectly.

If an E-field is responsible for f_{source}, then

$$\mathcal{E} = \oint \vec{f}_{\text{total}} \cdot d\vec{l} = \underbrace{\int \vec{E}_{\text{induced}} \cdot d\vec{l} \ \underbrace{+ \oint \vec{E}_{\text{wire}} \cdot d\vec{l}}_{=0}}_{\text{then we must drop the requirement that } \vec{\nabla}\times\vec{E}=0 \text{ (i.e. } \vec{E} = -\vec{\nabla} V)} \not\leftarrow 0$$

$$\mathcal{E}_{\substack{\text{bounding}\\ \text{line}}} = -\frac{d\Phi}{dt} \Rightarrow \oint \vec{E}_{\text{induced}} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s}$$

$$\Rightarrow \int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = \int_S \left(-\frac{d}{dt}\vec{B}\right)\cdot d\vec{s}$$

equality is true for any S $\Rightarrow$ $\boxed{\vec{\nabla}\times\vec{E} = -\frac{d}{dt}\vec{B}}$ Faraday's law or Maxwell's 2nd equation