PHYS 402: Electricity & Magnetism II
Due date: Thursday, November 30, 2017

Problem set #8

Griffiths 4th Ed. [3rd Ed.] problems
11.3 [11.3], 11.4 [11.4], 11.6 [11.6], 11.14 [11.14], 11.25 [11.23]

Lagrangian and Hamiltonian of a non-relativistic charged particle

While the electromagnetic force on a charged particle cannot be derived exclusively from a scalar potential (as required for standard Lagrangian mechanics), in this problem you will show that there exists a Lagrangian functional that produces the correct equations of motion for a charged particle in an arbitrary electromagnetic field.

a) Write down the Lorentz force law for charged particle in an electric field $\vec{E}(\vec{r},t)$ and magnetic field $\vec{B}(\vec{r},t)$. Write down the Lorentz force law in terms of the electric potential $V(\vec{r},t)$ and the vector potential $\vec{A}(\vec{r},t)$.

b) Show that the Lagrangian functional ***L*** below produces the Lorentz force law for a particle of charge ***q*** and mass ***m*** when it is used in conjunction with the Lagrange equation of motion (i.e. the equations that follow from the principle of least action):

$$L = \frac{1}{2} m\dot{\vec{r}}^{\,2} + q\dot{\vec{r}} \cdot \vec{A}(\vec{r},t) - qV(\vec{r},t)$$

Hint: Read up on convective derivatives.

c) Derive an expression for the canonical momentum $\vec{p}$, and show that the Hamiltonian ***H*** for the charged particle can be written as

$$H = \frac{1}{2m}\left(\vec{p} - q\vec{A}(\vec{r},t)\right)^2 + qV(\vec{r},t)\ .$$