Thursday, September 28, 2017 #1

<u>EM planewave</u>: $\vec{E} = E_0 \cos(kz - \omega t)\,\hat{x}$ (Solution of $\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$)

<u>Q</u>: Does planewave satisfy Maxwell's equations?

Last time, we saw that 1) $\nabla \cdot \vec{E} = 0$ ← planewave

~~Note that $\vec{E} = E_0 \cos(kz - \omega t)\,\hat{x}$ satisfies the wave equation but ...~~

~~→ the planewave must be transverse.~~

~~i.e. $\vec{E}$ and $\vec{B}$ must be perpendicular to the direction of propagation~~

2) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$(\partial_x, \partial_y, \partial_z) \times (E_x, E_y, E_z) = \left(\cancelto{0}{\partial_y E_z} - \cancelto{0}{\partial_z E_y},\ \partial_z E_x - \cancelto{0}{\partial_x E_z},\ \cancelto{0}{\partial_x E_y} - \cancelto{0}{\partial_y E_x}\right) = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \underbrace{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}}, \quad \underbrace{0 = \frac{\partial B_x}{\partial t} \to B_x = cst, \quad 0 = \frac{\partial B_z}{\partial t} \to B_z = cst}_{\text{not wave like}}$$

$$-E_0 k \sin(kz - \omega t) = -\frac{\partial}{\partial t} B_y$$

$$\Rightarrow B_y = E_0 \underbrace{\frac{k}{\omega}}_{1/c} \cos(kz - \omega t) + cst$$

⇒ Maxwell's equations (Faraday's law) require an accompanying B-field planewave (exactly in phase):

$$\boxed{\vec{B} = E_0 \frac{k}{\omega} \cos(kz - \omega t)\,\hat{y} = \frac{E_0}{c} \cos(kz - \omega t)\,\hat{y} = B_0 \cos(kz - \omega t)\,\hat{y} \quad \text{with } B_0 = E_0/c}$$

we note that $\boxed{\hat{E} \times \hat{B} = \text{propagation direction}}$ ← also poynting vector direction

$$\hat{x} \times \hat{y} = +\hat{z}$$

↳ this is very general: $\vec{B}$ cannot be $\vec{B} \neq -E_0 \cos(kz - \omega t)\hat{y}$

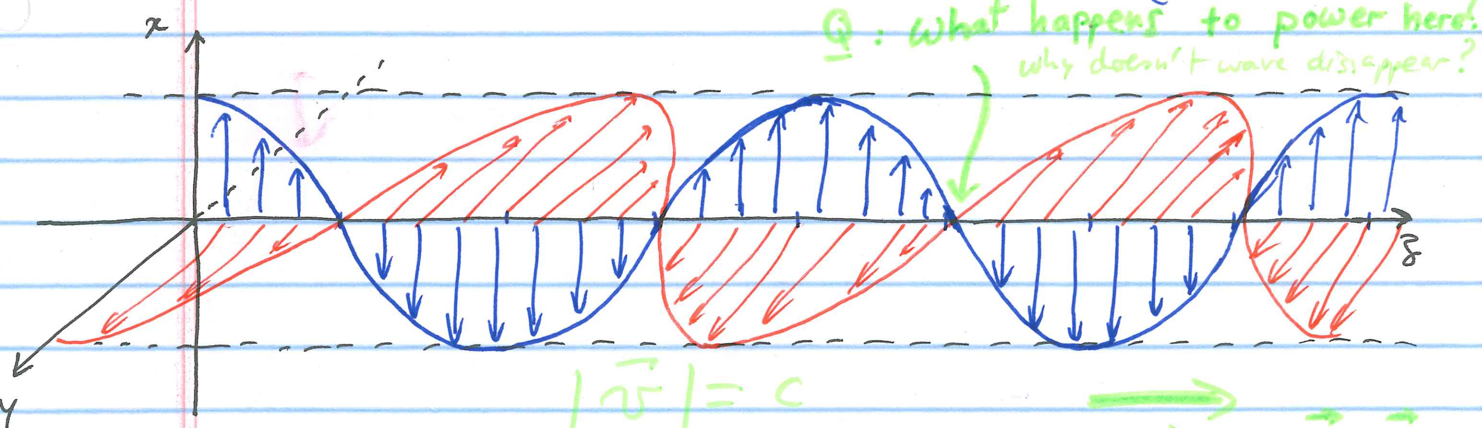
forbidden ~~impossible~~

⇒ Electrodynamics (in vacuum) is right-handed

(this does not violate parity)

Some metamaterials are left-handed { Veselago (1967), Pendry, Smith (2000)

Q: what happens to power here? why doesn't wave disappear?



$$|\vec{v}| = c$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

Theorem: Planewaves form a basis $\{\vec{k}, \omega, \hat{e}\}$ for all solutions of Maxwell's equations in vacuum (no charges & no currents)

(even in static limit: $\omega \to 0$)

$\vec{k}$: direction; ω: frequency; $\hat{e}$: polarization

<u>Polarization</u>: The direction/axis of the E-field (always transverse to the propagation direction) defines the polarization direction.

EM waves are generally <u>linearly polarized</u>

i.e. for $\hat{z}$ propagation: $\vec{E}(\vec{r},t) = E_0 \cos(kz - \omega t + \varphi)(\cos\phi\,\hat{x} + \sin\phi\,\hat{y})$

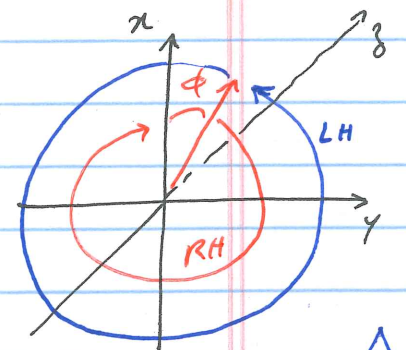
↳ ϕ = polarization direction angle.

EM waves can also be <u>circularly polarized</u>

↳ the polarization $\vec{E}$ vector rotates around the propagation direction (but does not change in magnitude!)

$$\vec{E} = \underbrace{E_0 \cos(kz - \omega t)\hat{x} + E_0 \overbrace{\cos\left(kz - \omega t + \tfrac{\pi}{2}\right)\hat{y}}^{-\sin(kz-\omega t)\,\hat{y}}}_{\text{superposition of two linearly polarized planewaves in which the } \hat{y}\text{-wave is "delayed" by } \lambda/4 \text{ or } T/4}$$

if these are different then you get elliptically polarized light!



⚠ <u>Application</u>: 3D movie glasses uses circular polarizers (i.e. only transmit one polarization LH or RH)

<u>Polarization basis</u>: you can use either a linearly polarized basis $\hat{e} = \{\hat{x}, \hat{y}\}$ or a circularly polarized basis

$$\hat{e} = \left\{ \hat{e}_+ = \frac{\hat{x} + i\hat{y}}{\sqrt{2}},\ \hat{e}_- = \frac{\hat{x} - i\hat{y}}{\sqrt{2}} \right\}$$

to describe any state of polarization of an EM planewave (propagating along $\hat{z}$)

Energy, Momentum, and Intensity of EM waves

Electric Energy Density : $u_E = \frac{1}{2}\varepsilon_0 E_0^2 \cos^2(kz-\omega t)$
(of a planewave)

⚠ the total energy of a planewave is infinite, since it extends to infinity (in space .... and time)

Magnetic Energy Density : $u_B = \frac{1}{2}\frac{1}{\mu_0}\underbrace{B_0^2}\cos^2(kz-\omega t)$

$$\frac{E_0^2}{c^2} = \frac{E_0^2}{\left(\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)^2} = \varepsilon_0\mu_0 E_0^2$$

$$= \frac{1}{2}\frac{1}{\cancel{\mu_0}}\varepsilon_0\cancel{\mu_0}E_0^2\cos^2(kz-\omega t)$$

$$= u_E$$

$\boxed{\Rightarrow \text{ the energy of an EM wave is equally shared between } \vec{E} \;\&\; \vec{B} \text{ fields.}}$

The energy flow/flux density is given by the Poynting vector:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = [\text{power per unit area}]\,\hat{k}$$

$$= \frac{1}{\mu_0}\frac{\varepsilon_0}{\varepsilon_0}E_0\frac{E_0}{c}\cos^2(kz-\omega t)\,\hat{k}$$

$\hat{z}$

$$= \varepsilon_0 c E_0^2\cos^2(kz-\omega t)\,\hat{k}$$

$$= \underbrace{(u_E + u_B)}_{u_{total}}\, c\,\hat{k} = \underbrace{\text{Energy density} \times \text{speed} \times \text{direction}}$$

$\vec{S}$ is like $\vec{J}$ but for EM energy "current" density (flux)

$\vec{J} = \rho\vec{v}$

planewave → $\vec{S} = u_{EM}\,\underbrace{c\hat{k}}_{\vec{v}}$

average intensity = "intensity" = $\langle \text{power per unit area} \rangle_{time}$

$$= \langle |\vec{S}| \rangle_{time}$$

$$= c\,\varepsilon_0 E_0^2 \underbrace{\langle \cos^2(kz-\omega t) \rangle_{time}}_{1/2}$$

$$\Rightarrow \text{Intensity} = \frac{1}{2} c\,\varepsilon_0 E_0^2 = \langle |\vec{S}| \rangle_t$$

note: max E-field produced with pulsed lasers $10^{10} - 10^{12}$ V/cm

momentum density is $\vec{p}_{EM} = \varepsilon_0 \mu_0 \vec{S} = \frac{1}{c^2} \vec{S}$

$$\Rightarrow \langle \vec{p}_{EM} \rangle = \frac{1}{c^2} \langle \vec{S} \rangle_{time}$$

$$= \frac{1}{2} \frac{\varepsilon_0}{c} E_0^2 = \frac{1}{c^2} \langle \text{Intensity} \rangle_t$$

note: $\vec{p}_{EM} = \frac{1}{c^2} \vec{S} = \frac{u_{EM}\, c\, \hat{k}}{c^2} = \frac{u_{EM}}{c} \hat{k}$

$$= \frac{\text{"}E\text{"}}{c} \hat{k}$$

$$\Rightarrow \boxed{p_{EM} = \frac{u_{EM}}{c} \hat{k}}$$ ← for a photon: $p = E/c$

no QM!

Stopped here

Electromagnetic waves in matter

Maxwell's equations in matter with no free charges and no free currents:

1) $\vec{\nabla} \cdot \vec{D} = 0$

2) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

3) $\vec{\nabla} \cdot \vec{B} = 0$

4) $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$

We consider a linear medium:

$$\vec{D} = \varepsilon \vec{E} \quad \text{and} \quad \vec{H} = \frac{1}{\mu} \vec{B}$$

more general linear medium : ε = tensor/matrix

μ^{-1} = tensor/matrix

In this case, Maxwell's equations become:

1) $\to$ $\vec{\nabla} \cdot \vec{E} = 0$

2) $\to$ $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

3) $\to$ $\vec{\nabla} \cdot \vec{B} = 0$

4) $\to$ $\vec{\nabla} \times \vec{B} = \underbrace{\mu \varepsilon}_{1/c_m^2} \frac{\partial \vec{E}}{\partial t}$

c_m = speed of light in medium $= \frac{1}{\sqrt{\varepsilon \mu}} \leq c$

this the only difference with vacuum version of maxwell's eq.

$\Rightarrow$ the wave equations are

$$\begin{cases} \vec{\nabla}^2 \vec{E} = \frac{1}{c_m^2} \frac{\partial^2}{\partial t^2} \vec{E} \\ \vec{\nabla}^2 \vec{B} = \frac{1}{c_m^2} \frac{\partial^2}{\partial t^2} \vec{B} \end{cases}$$

<u>Definition:</u> the index of refraction $\underline{\underline{n}}$ of a medium "m" is defined as $\boxed{c_m = \frac{c}{n}} \Leftrightarrow n = \frac{c}{c_m}$

In most dielectrics $\mu \simeq \mu_0$ [difference at the the part in $10^5 - 10^6$ level]