Comment on HW #3 : Capacitor problem, last part. #1

Tuesday, October 3, 2017

## Electromagnetic Waves in Matter

Maxwell's equations in matter with no free charges and no free currents:

1) $\vec{\nabla}\cdot\vec{D} = 0$ 2) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

3) $\vec{\nabla}\cdot\vec{B} = 0$ 4) $\vec{\nabla}\times\vec{H} = -\frac{\partial\vec{D}}{\partial t}$

We consider a linear medium:

$\vec{D} = \varepsilon\vec{E}$ and $\vec{H} = \frac{1}{\mu}\vec{B}$

more general linear medium: $\varepsilon$ = tensor/matrix

$\mu^{-1}$ = tensor/matrix

In this case, Maxwell's equations become:

1) $\rightarrow$ $\vec{\nabla}\cdot\vec{E} = 0$ 2) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

3) $\vec{\nabla}\cdot\vec{B} = 0$ 4) $\vec{\nabla}\times\vec{B} = \underbrace{\mu\varepsilon}_{\frac{1}{c_m}}\frac{\partial\vec{E}}{\partial t}$

$c_m$ = speed of light in medium

$= \frac{1}{\sqrt{\varepsilon\mu}} < c$

this is the only difference with the vacuum version of Maxwell's eq.

$\Rightarrow$ the wave equations are $\begin{cases} \vec{\nabla}^2\vec{E} = \frac{1}{c_m^2}\frac{\partial^2}{\partial t^2}\vec{E} \\ \vec{\nabla}^2\vec{B} = \frac{1}{c_m^2}\frac{\partial^2}{\partial t^2}\vec{B} \end{cases}$

<u>Definition:</u> the index of refraction $\underline{\underline{n}}$ of a medium "m" is defined as $\boxed{c_m = \frac{c}{n}}$ $\Leftrightarrow$ $n = \frac{c}{c_m}$

In most dielectrics $\mu \simeq \mu_0$ [difference at the part in $10^5 - 10^6$ level]

↳ In this case, $n = \frac{c}{c_m} = \sqrt{\frac{\varepsilon \cancel{\mu}}{\varepsilon_0 \cancel{\mu_0}}} \simeq \sqrt{\frac{\varepsilon_0(1+\chi_e)}{\varepsilon_0}} \simeq \sqrt{1+\chi_e}$

$\simeq \sqrt{\varepsilon_r}$

$\Rightarrow n \simeq \sqrt{\varepsilon_r}$

where $\varepsilon_r$ = relative permittivity or dielectric constant $= \frac{\varepsilon}{\varepsilon_0}$

<u>examples</u>

$n_{vacuum} = 1.0$

$n_{air} = 1.00029 \simeq 1.0$

$n_{H_2O}(20°C) = 1.33$

$n_{H_2O}\left(\begin{smallmatrix}0°C \\ \text{"ice"}\end{smallmatrix}\right) = 1.31$

$n_{quartz} = 1.46$

$n_{BK7} = 1.51$

$n_{diamond} = 2.4$

$n_{sapphire} = 1.7$

$n_{silicon} = 4.1$

$n_{AlN} = 3$

used in microwave electronics

Generally, $n$ and $\varepsilon_r$ are <u>(weak) functions</u> of $\lambda$, $f$.

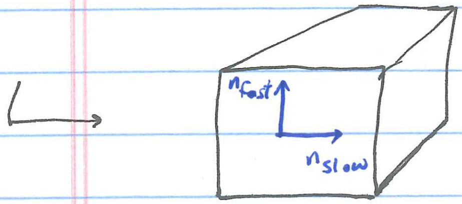
↳ e.g. glass in a prism

<u>In matter</u>: Energy density: $u_{EM} = \frac{1}{2}\left(\varepsilon \vec{E}^2 + \frac{1}{\mu}\vec{B}^2\right)$

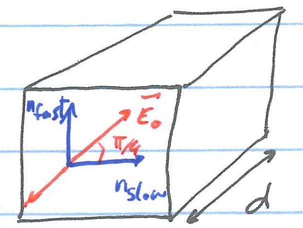
Poynting vector: $\vec{S} = \frac{1}{\mu}\vec{E} \times \vec{B}$

Intensity $= \frac{1}{2}\, \underset{\uparrow\, \frac{c}{n}}{c_m}\, \underset{\uparrow\, n^2\varepsilon_0}{\varepsilon}\, \vec{E}_0^2 \simeq \frac{1}{2} n c \varepsilon_0 \vec{E}_0^2$

<u>Birefringent materials</u>: In birefringent materials, the index of refraction depends on polarization.




How do you make circularly polarized light?




$\lambda/4$ or quarterwave plate

$$\lambda f = c \Rightarrow \begin{cases} \lambda_{fast} = \dfrac{c}{n_{fast}} \dfrac{1}{f} \Rightarrow \phi_{fast} = 2\pi \dfrac{d}{\lambda_{fast}} = k_{fast} d \\ \\ \lambda_{slow} = \dfrac{c}{n_{slow}} \dfrac{1}{f} \Rightarrow \phi_{slow} = 2\pi \dfrac{d}{\lambda_{slow}} = k_{slow} d \end{cases}$$

(accumulated phase)

We want the <u>accumulated phase difference</u> at exit to be $\frac{\pi}{2} + 2\pi m$

integer

$\pi/2$ needed for circularity

$$\Delta\phi = 2\pi m + \pi/2 = \phi_{fast} - \phi_{slow}$$
$$= 2\pi d \frac{f}{c} (n_{fast} - n_{slow})$$
$$= \frac{2\pi d}{\lambda_{air/vacuum}} (n_{fast} - n_{slow})$$

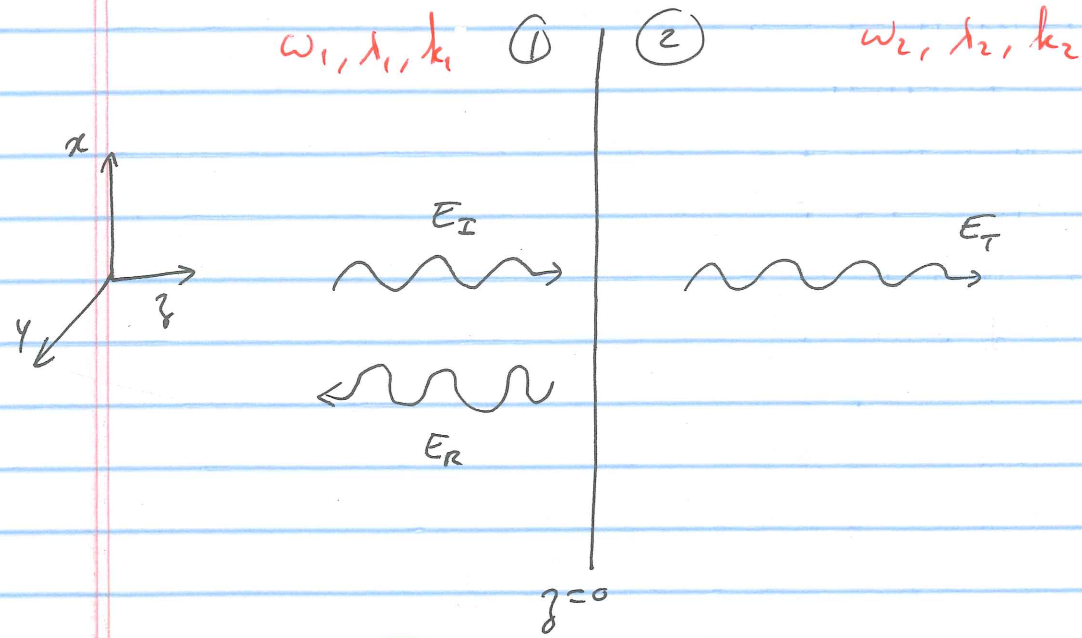
integer

if $d = 1\,mm$, then $n_{fast} - n_{slow} \simeq 10^{-4}$ for a <u>zero order</u> waveplate ($m=0$)

Q: what does a $\lambda/2$ waveplate do?

A: It rotates a linear polarization by 90°!

## Reflection & Transmission @ normal incidence




$$\boxed{\omega_1 = \omega_2 = \omega \quad , \quad \lambda_1 \neq \lambda_2 \quad , \quad k_1 \neq k_2}$$

$$\begin{cases} \vec{E}_I = E_0 \cos(k_1 z - \omega t)\,\hat{x} \longrightarrow \vec{E}_I = E_0 \exp[i(k_1 z - \omega t)]\,\hat{x} \\ \vec{B}_I = \dfrac{n_1 E_0}{c} \exp[i(k_1 z - \omega t)]\,\hat{y} \end{cases}$$

↳ take the real part at the end of calculation!

$$\begin{cases} \vec{E}_R = E_{0,R} \exp[i(-k_1 z - \omega t + \phi_R)]\,\hat{x} \\ \vec{B}_R = -\dfrac{n_1 E_{0,R}}{c} \exp[i(-k_1 z - \omega t + \phi_R)]\,\hat{y} \end{cases}$$

⚠ Poynting vector requirement

$$\begin{cases} \vec{E}_T = E_{0,T} \exp[i(k_2 z - \omega t + \phi_t)]\,\hat{x} \\ \vec{B}_T = \dfrac{n_2 E_{0,T}}{c} \exp[i(k_2 z - \omega t + \phi_t)]\,\hat{y} \end{cases}$$

Recall boundary conditions: (no free charges or currents)

(normal incidence)

1) $\epsilon_1 \cancelto{0}{E_{\perp,1}} - \epsilon_2 \cancelto{0}{E_{\perp,2}} = \cancelto{0}{\sigma_f}$

$\Rightarrow \epsilon_1 E_{\perp,1} = \epsilon_2 E_{\perp,2}$

2) $\vec{E}_{\parallel,1} = \vec{E}_{\parallel,2}$

3) $\cancelto{0}{B_{\perp,1}} = \cancelto{0}{B_{\perp,2}}$ (normal incidence)

4) $\frac{1}{\mu_1} \vec{B}_{\parallel,1} - \frac{1}{\mu_2} \vec{B}_{\parallel,2} = \cancelto{0}{\vec{K}_f} \times \hat{n}$

$\Rightarrow \frac{\vec{B}_{\parallel,1}}{\mu_1} = \frac{\vec{B}_{\parallel,2}}{\mu_2}$

since $\mu_1 \simeq \mu_2 \simeq \mu_0$

$\Rightarrow \vec{B}_{\parallel,1} \simeq \vec{B}_{\parallel,2}$

$e^{-i\omega t}$'s cancel out

2) $\vec{E}_{1,\parallel} = \vec{E}_{2,\parallel} \Rightarrow E_0 + E_{0,R} = E_{0,T}$ for $z=0$

4) $\vec{B}_{1,\parallel} = \vec{B}_{2,\parallel} \Rightarrow B_0 - B_{0,R} = B_{0,T}$ for $z=0$

$\Leftrightarrow \frac{n_1 E_0}{\cancel{c}} - \frac{n_1 E_{0,R}}{\cancel{c}} = \frac{n_2 E_{0,T}}{\cancel{c}}$

$\Leftrightarrow E_0 - E_{0,R} = \frac{n_2}{n_1} E_{0,T}$

We must solve 2 equations for 2 unknowns:

$$\begin{cases} E_0 + E_{0,R} = E_{0,T} & (1) \\ E_0 - E_{0,R} = \frac{n_2}{n_1} E_{0,T} & (2) \end{cases}$$

$(1) \to (2) \Rightarrow E_0 - E_{0,R} = \frac{n_2}{n_1}\left(E_0 + E_{0,R}\right)$

$\Leftrightarrow -E_{0,R}\left(1 + \frac{n_2}{n_1}\right) = E_0\left(\frac{n_2}{n_1} - 1\right)$

$\Leftrightarrow E_{0,R} = \frac{1 - n_2/n_1}{1 + n_2/n_1} E_0 \Leftrightarrow \boxed{E_{0,R} = \overbrace{\left(\frac{n_1 - n_2}{n_1 + n_2}\right)}^{r = \text{reflection amplitude}} E_0}$

GRE question

$\pi$ phase shift for $n_1 < n_2$

no phase shift for $n_1 > n_2$

$$(1)+(2) \implies 2E_0 = E_{0,T} + \frac{n_2}{n_1} E_{0,T}$$

$$\Leftrightarrow \quad E_{0,T} = \frac{2}{1+\frac{n_2}{n_1}}$$

$$\Leftrightarrow \quad \boxed{E_{0,T} = \frac{2n_1}{\underbrace{n_1+n_2}_{t \,=\, \text{transmission amplitude}}} E_0}$$

$\leftarrow$ no phase shift on transmission

Let's calculate Intensities (Energy flow density)

$$I_I = \frac{1}{2} n_1 c \varepsilon_0 E_0^2 \quad , \quad I_R = \frac{1}{2} n_1 c \varepsilon_0 E_{0,R}^2$$

$$I_T = \frac{1}{2} n_2 c \varepsilon_0 E_{0,T}^2$$

$$\text{Reflection coefficient} = R = \frac{I_R}{I_I} = \frac{E_{0,R}^2}{E_0} = r^2 = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2$$

$$\Leftrightarrow \quad \boxed{R = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2} \quad \text{, i.e. } I_R = R\, I_I$$

$$\text{Transmission coefficient} = T = \frac{I_T}{I_I} = \frac{n_2}{n_1}\frac{E_{0,T}^2}{E_0^2} = \frac{n_2}{n_1}\left(\frac{2n_1}{n_1+n_2}\right)^2$$

$$= \left(\frac{n_2}{n_1}\right)|t|^2$$

!

$$\Leftrightarrow \quad \boxed{T = \frac{4n_1 n_2}{(n_1+n_2)^2}} \quad \text{i.e. } I_T = T\, I_I$$

#7

<u>Note:</u> $T + R = 1$ Conservation of Energy/power

$$\left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 + \frac{4 n_1 n_2}{(n_1 + n_2)^2} = \frac{(n_1 + n_2)^2}{(n_1 + n_2)^2} = 1$$