Test on boundary conditions at the interface of two dielectrics mediums on Tuesday

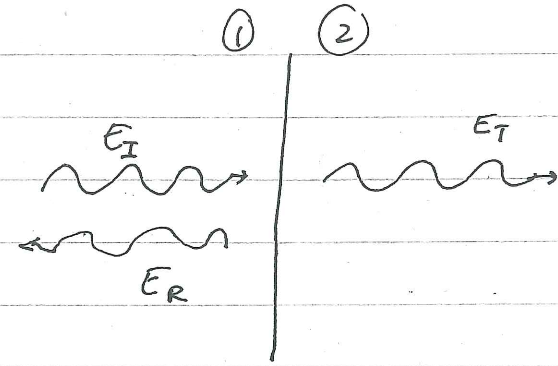
#1

Thursday, October 5, 2017

Summary of normal incidence reflection & transmission based on boundary conditions:

$$r = \frac{E_R}{E_I} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t = \frac{E_T}{E_I} = \frac{2n_1}{n_1 + n_2}$$

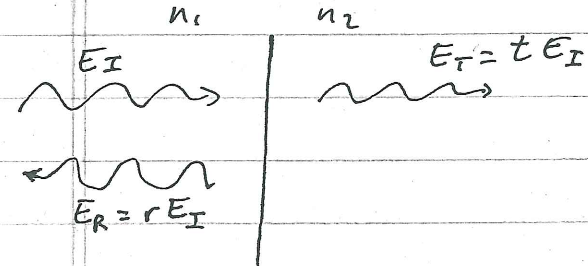


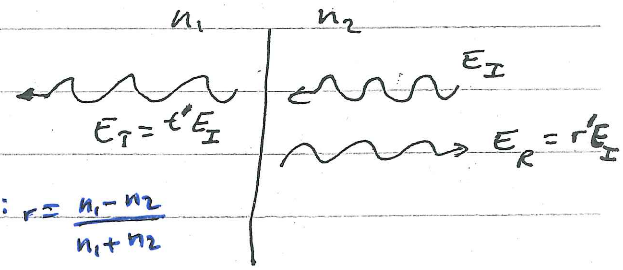
$$R = \left(\frac{E_R}{E_I}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \; ; \; T = \frac{n_2}{n_1}\left(\frac{E_T}{E_I}\right)^2 = \frac{4n_1 n_2}{(n_1+n_2)^2}$$

$R + T = 1$ ← Should be true regardless of boundary conditions

Question: Are there any other "big picture"/fundamental physics considerations that can give us insight into the reflection - transmission problem?

Stoke's Relations: A broader view of reflection-transmission (only works if there is no absorption)

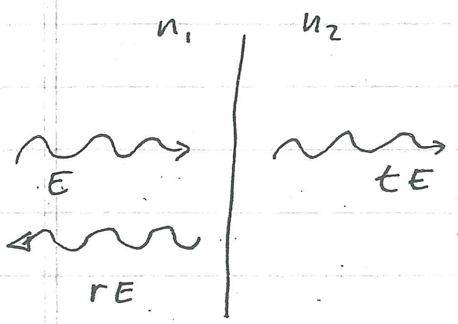




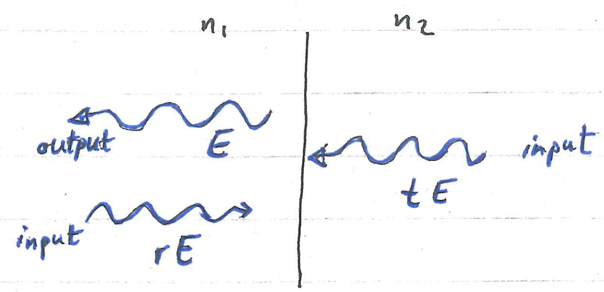
recall: $r = \frac{n_1 - n_2}{n_1 + n_2}$

Stokes's relations: $\boxed{r = -r'}$ and $\boxed{tt' + r^2 = 1}$

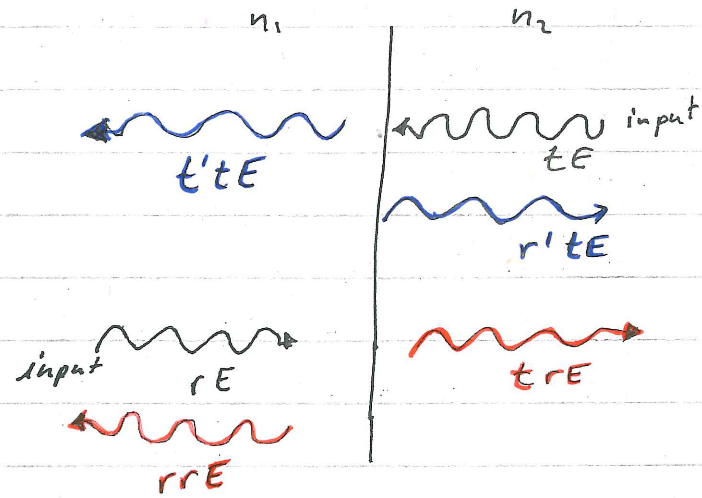
Proof based on time-reversal symmetry:



time reversal №1



time reversal №2



for №1 $\equiv$ №2, we require:

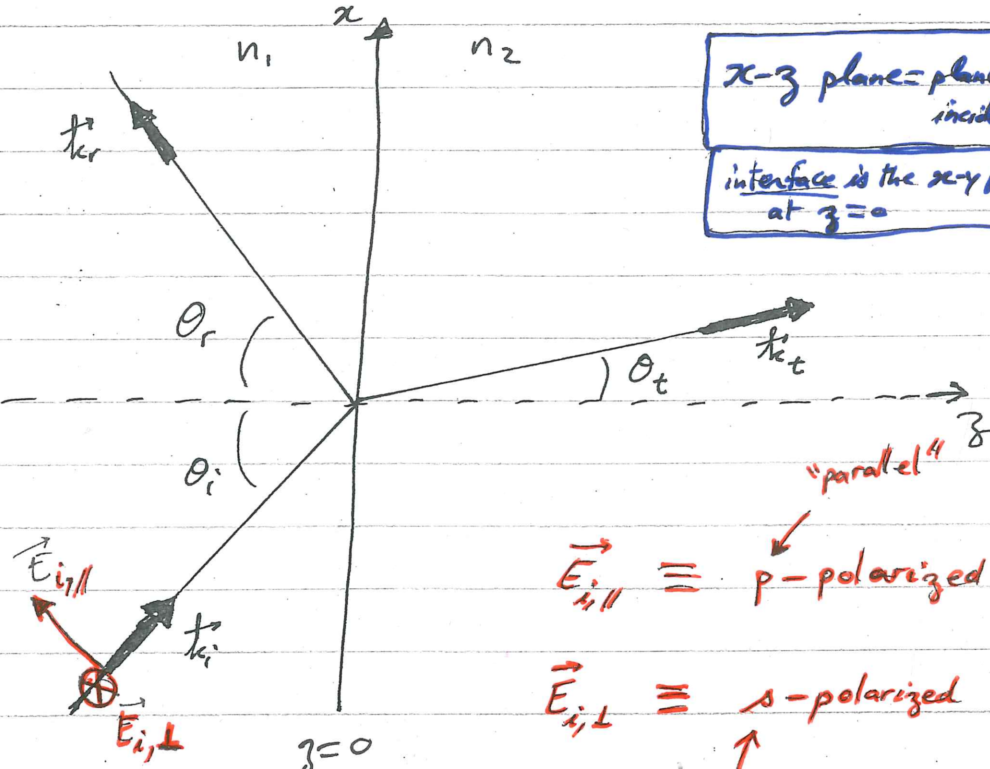
in n_1: $t'tE + rrE = E$

$$\Rightarrow \boxed{t't + r^2 = 1}$$

in n_2: $r'tE + trE = 0$

$$\Rightarrow \boxed{r = -r'}$$

Reflection & Transmission at oblique incidence



x-z plane = plane of incidence

interface is the x-y plane at z=0

2 options for $\vec{E}_i$:

// to plane of incidence

$\perp$ to plane of incidence

$\vec{E}_{i,\parallel} \equiv$ p-polarized ("parallel")

$\vec{E}_{i,\perp} \equiv$ s-polarized (German: "senkrecht" = $\perp$)

General considerations

Stokes's relations still hold : $r = -r'$

(apply carefully) $t't + r^2 = 1$

Incident field : $\vec{E}_i = \vec{E}_0 \, e^{i(\vec{k}_i \cdot \vec{r} - \omega_i t)}$

Reflected field : $\vec{E}_r = \vec{E}_{0,r} \, e^{i(\vec{k}_r \cdot \vec{r} - \omega_r t)}$

transmitted field : $\vec{E}_t = \vec{E}_{0,t} \, e^{i(\vec{k}_t \cdot \vec{r} - \omega_t t)}$

At the interface, the boundary conditions will take the general form:

$$\underbrace{[\,?\,]}_{\varepsilon_1,\ \frac{1}{\mu_1},\ \cos\theta_I,\ \sin\theta_I} \overset{\vec{B}_0}{\vec{E}_0}\, e^{i(\vec{k}_i\cdot\vec{r}-\omega_i t)} + \underbrace{[\,??\,]}_{\varepsilon_1,\ \frac{1}{\mu_1},\ \cos\theta_r} \overset{\vec{B}_{0,r}}{\vec{E}_{0,r}}\, e^{i(\vec{k}_r\cdot\vec{r}-\omega_r t)}$$

$$= \underbrace{[\,???\,]}_{\varepsilon_2,\ \frac{1}{\mu_2},\ \cos\theta_t,\ \sin\theta_t} \overset{\vec{B}_{0,t}}{\vec{E}_{0,t}}\, e^{i(\vec{k}_t\cdot\vec{r}-\omega_t t)}$$

boundary conditions

Boundary conditions must be true for all $\begin{cases} \vec{r} \text{ at interface} \\ \text{(i.e. all } (x,y) \text{ with } z=0) \\ t \end{cases}$

$\Rightarrow$ All the exponents must be equal on the boundary $(z=0)$

for all t $\Rightarrow$ requires $\boxed{\omega_i = \omega_r = \omega_t \equiv \omega}$

for all $\vec{r}$ (with $z=0$) $\Rightarrow$ requires $\begin{cases} \vec{k}_i\cdot\vec{r} = \vec{k}_r\cdot\vec{r} = \vec{k}_t\cdot\vec{r} \\ \hookrightarrow\ k_{i,x} = k_{r,x} = k_{t,x} \\ \hookrightarrow\ k_{i,y} = k_{r,y} = k_{t,y} \end{cases}$

does not say anything about $k_{i,z}$, $k_{r,z}$, $k_{t,z}$ (since $z=0$)

$\hookrightarrow$ if $k_{i,y}=0 \Rightarrow \begin{cases} k_{r,y}=0 \\ k_{t,y}=0 \end{cases}$

$\Rightarrow$ incident, reflected, and transmitted $\vec{k}$'s are in the same plane.

$$k_{i,x} = k_{r,x} = k_{t,x}$$

$$\Leftrightarrow \quad k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t$$

recall

$k = \frac{\omega}{c}$

$\lambda f = c$

$\lambda = c/f$

$$\underbrace{\frac{\omega}{c_1} = \frac{n_1\omega}{c} \qquad \frac{\omega}{c_1} = \frac{n_1\omega}{c}} \qquad \frac{\omega}{c_2} = \frac{n_2\omega}{c}$$

$$\frac{n_1\omega}{c}\sin\theta_i = \frac{n_1\omega}{c}\sin\theta_r$$

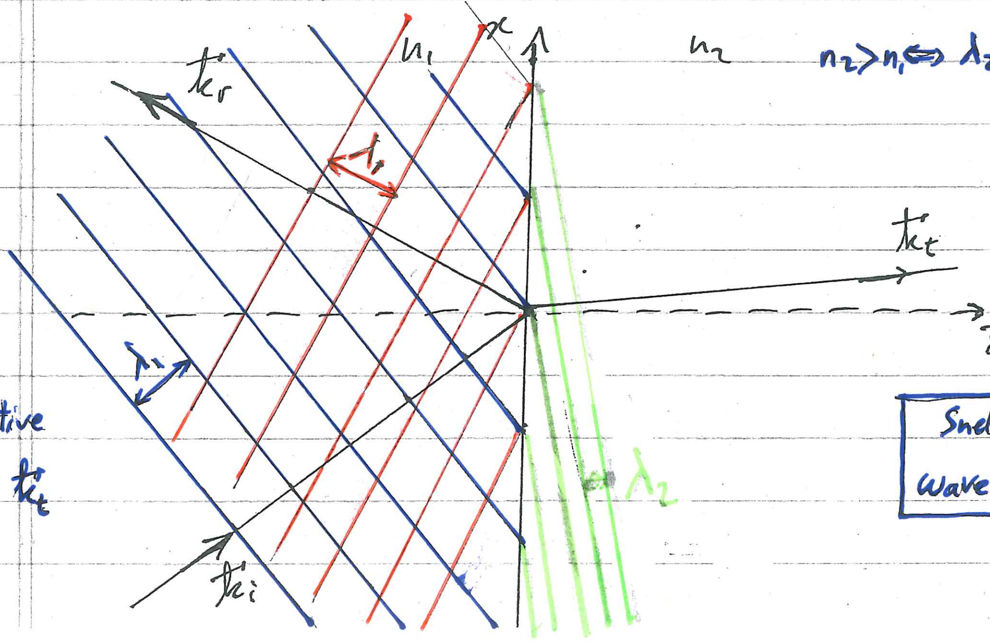
$$\Rightarrow \boxed{\theta_i = \theta_r}$$ Law of reflection

these laws are true for all wave phenomena at a planar interface

$$\frac{n_1\omega}{c}\sin\theta_i = \frac{n_2\omega}{c}\sin\theta_t \Rightarrow \boxed{n_1\sin\theta_i = n_2\sin\theta_t}$$

Law of refraction or Snell's law

what's the Physics Snell's Law? or "why does light bend when it enters another medium?"



$n_2 > n_1 \Leftrightarrow \lambda_2 < \lambda_1$

light "bends" at the interface so as to maintain a continuous phase along both sides of the boundary

$\boxed{\text{Snell's law is due to the wave nature of light}}$

Also: oscillating are constructive in the k_r and k_t directions