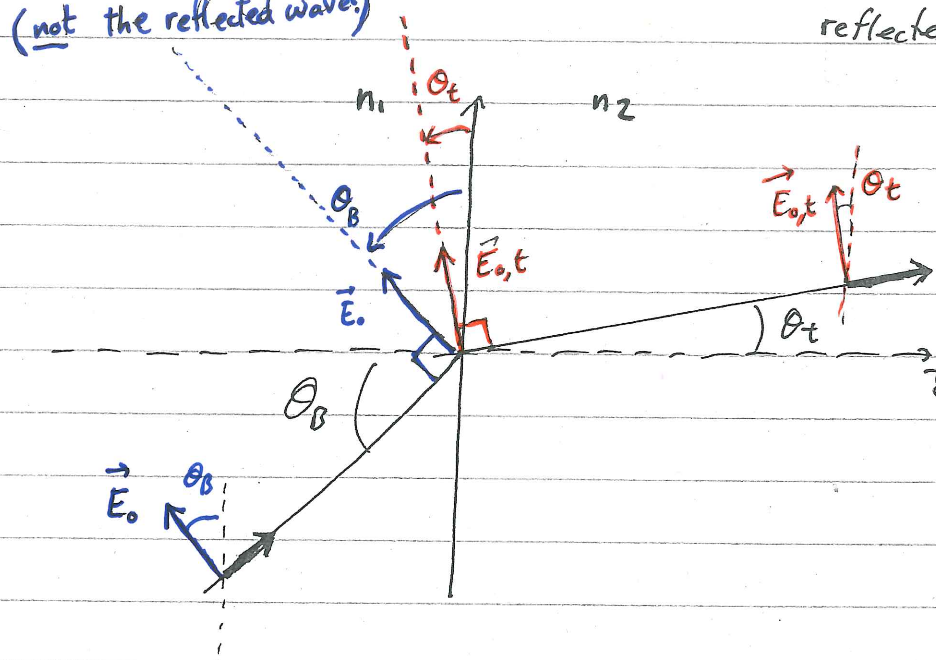
Mid term on Thursday, October 19, 2017

Thursday, October 12, 2017

Brewster's Angle : Physics Approach

Summary of Math approach: $\tan \theta_B = \frac{n_2}{n_1}$

No reflected light $\Leftrightarrow$ $R = 0$ $\Leftrightarrow$ $r = 0$ $\Leftrightarrow$ boundary ~~conditions~~ can be satisfied without reflected E-field



boundary conditions

1) $\varepsilon_1 E_{o,\perp} = \varepsilon_2 E_{o,t,\perp} \Leftrightarrow \underbrace{\mu_0 \varepsilon_1}_{\mu_1 \left(\frac{n_1}{c}\right)^2} E_o \sin\theta_B = \underbrace{\mu_0 \varepsilon_2}_{\mu_2 \left(\frac{n_2}{c}\right)^2} E_{o,t} \sin\theta_t$

$\Leftrightarrow n_1^2 E_o \cancel{\sin\theta_B} = n_2^2 E_{o,t} \cancel{\sin\theta_t}$

Snell's law

$$\Rightarrow \boxed{t_B = \frac{E_{o,t}}{E_o} = \frac{n_1}{n_2} = \frac{\sin\theta_t}{\sin\theta_B} \quad (1)}$$

Snell's law again

2) $\vec{E}_{1,\parallel} = \vec{E}_{2,\parallel} \iff E_0 \cos\theta_B = E_{0,t} \cos\theta_t$

$$\Rightarrow \text{thus} \quad \boxed{t_B = \frac{E_{0,t}}{E_0} = \frac{\cos\theta_B}{\cos\theta_t} \qquad (2)}$$

so we must have (1) = (2) :

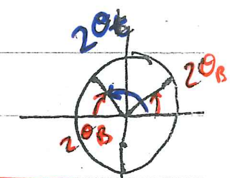
$$\iff \frac{\cos\theta_B}{\cos\theta_t} = t_B = \frac{\sin\theta_t}{\sin\theta_B}$$

$$\iff 2\sin\theta_B \cos\theta_B = 2\sin\theta_t \cos\theta_t$$

$$\Rightarrow \sin(2\theta_B) = \sin(2\theta_t)$$

$\Rightarrow$ solutions are
$$\begin{cases} \cancel{\theta_B = \theta_t} \\ 2\theta_B = \pi - 2\theta_t \end{cases}$$

not true according to Snell's law (unless $n_1 = n_2$)



$\hookrightarrow$ $\boxed{\theta_B + \theta_t = \pi/2}$

⚠ Remarkable result → does not depend on n_1 or n_2 (directly)
$\hookrightarrow$ "deep physics here".

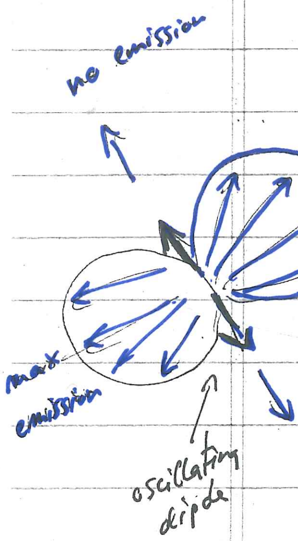
(Snell's law) Also $\dfrac{\sin\theta_B}{\sin\theta_t} = \dfrac{n_2}{n_1} \Rightarrow \dfrac{\sin\theta_B}{\underbrace{\sin(\pi/2 - \theta_B)}_{\cos\theta_B}} = \dfrac{n_2}{n_1}$

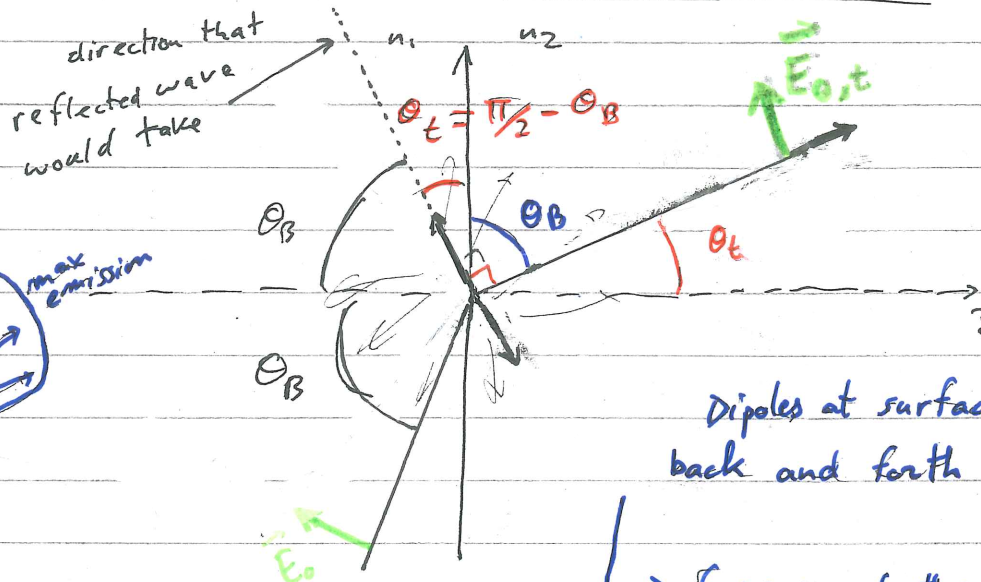
$$\iff \boxed{\tan\theta_B = \frac{n_2}{n_1}}$$

note: $n_2 = 1.5$ glass, $n_1 = 1.0$ $\Rightarrow$ $\boxed{\theta_B \doteq 56°}$

$\hookrightarrow$ you can use Brewster's angle to measure index of refraction.

Explanation for the physics of $\theta_B + \theta_t = \pi/2$





Dipoles at surface oscillate back and forth (like an antenna)

↳ source of the transmitted wave

Source of the reflected wave

↳ but there is no perpendicular emission

⚠ ~~He-Ne laser tube (demo) with brewster windows~~

~~↳ polarizes the laser light~~

did not do this demo, equipment partially missing

- unpolarized light reflecting off of a surface will tend to be polarized $\perp$ to the plane of incidence i.e. $\parallel$ to surface (s-polarization)
  - ↳ polarized sunglasses use vertical polarizers.

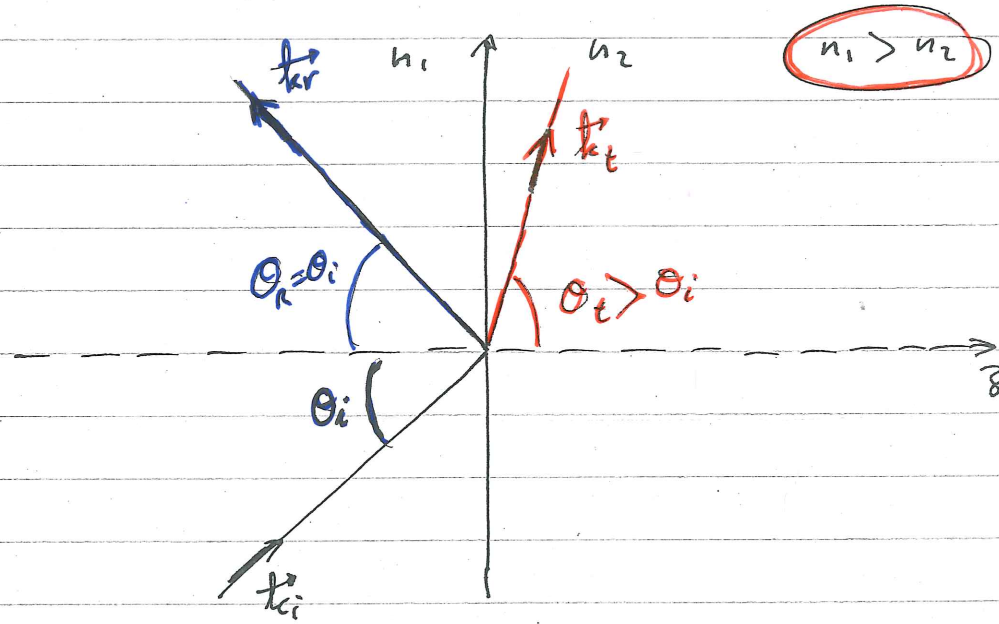
- Aubin lab experience with high power laser reflections.

Total Internal Reflection

If $n_i > n_t$, then total internal reflection is possible (i.e. no transmitted wave)

Show equations on overhead

↳ the "$\sqrt{\quad}$" term in "r" and "t" expressions becomes imaginary $\Rightarrow$ can be interpreted as k_t becoming complex → i.e. wave decays as it "penetrates" into n_2.

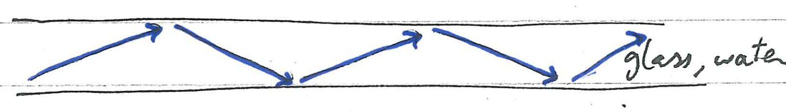


when $\theta_t \big|_c = \pi/2$, you get total internal reflection

Snell's law: $n_i \sin\theta_{i,c} = n_2 \sin(\pi/2) \Rightarrow \sin\theta_{i,c} = \frac{n_t}{n_i}$

↳ total internal reflection for $\theta_i > \theta_{i,c}$

Geometric optics explanation of fiber optic transmission:



EM-waves in conductors

Ohm's law: $\vec{J}_f = \sigma \vec{E}$

conductivity

Maxwell's equations in a conducting medium (linear):

1) $\vec{\nabla}\cdot\vec{E} = \dfrac{\rho_f}{\varepsilon} \Longrightarrow \vec{\nabla}\cdot\vec{E} = 0$ no free charges

metal has a dielectric constant

2) $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

3) $\vec{\nabla}\cdot\vec{B} = 0$

free current ohm's Law

4) $\vec{\nabla}\times\vec{B} = \mu\vec{J}_f + \mu\varepsilon\dfrac{\partial\vec{E}}{\partial t} \Longrightarrow \vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + \mu\varepsilon\dfrac{\partial\vec{E}}{\partial t}$ (4)

We search for a wave-type equation:

$$\vec{\nabla}\times(2) \Longrightarrow \underbrace{\vec{\nabla}\times(\vec{\nabla}\times\vec{E})}_{\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{=0}) - \vec{\nabla}^2\vec{E}} = -\frac{\partial}{\partial t}\underbrace{(\vec{\nabla}\times\vec{B})}_{\text{equation 4}}$$

$$\Longrightarrow -\vec{\nabla}^2\vec{E} = -\frac{\partial}{\partial t}\left[\mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right]$$

$$\Longleftrightarrow \boxed{\vec{\nabla}^2\vec{E} = \underbrace{\mu\varepsilon}_{\frac{n^2}{c^2}}\frac{\partial^2\vec{E}}{\partial t^2} + \mu\sigma\frac{\partial\vec{E}}{\partial t}}$$

wave equation + $\dfrac{\partial\vec{E}}{\partial t}$ term similarly, $\vec{\nabla}\times(4) \Rightarrow$ $$\boxed{\vec{\nabla}^2\vec{B} = \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} + \mu\sigma\frac{\partial\vec{B}}{\partial t}}$$

Ansatz: Assume a plane wave solution, propagating in $+z$-direction

$$\vec{E}(z,t) = \vec{E}_0 e^{i(kz-\omega t)} \quad \text{and} \quad \vec{B}(z,t) = \vec{B}_0 e^{i(kz-\omega t)}$$

$\hookrightarrow$ plug into conductor wave equation

$$\left(\cancel{\frac{\partial^2}{\partial x^2}}^{0} + \cancel{\frac{\partial^2}{\partial y^2}}^{0} + \frac{\partial^2}{\partial z^2}\right)\vec{E}_0 e^{i(kz-\omega t)} = \mu\varepsilon\overbrace{\frac{\partial^2}{\partial t^2}\left[\vec{E}_0 e^{i(kz-\omega t)}\right]}^{\cancel{\vec{E}_0}(-\omega^2)\cancel{e^{i(kz-\omega t)}}} + \mu\sigma\underbrace{\frac{\partial}{\partial t}\left[\vec{E}_0 e^{i(kz-\omega t)}\right]}_{\cancel{\vec{E}_0}(-i\omega)\cancel{e^{i(kz-\omega t)}}}$$

$$\underbrace{\qquad}_{\cancel{\vec{E}_0}(-k^2)\cancel{e^{i(kz-\omega t)}}}$$

$$\Rightarrow -k^2 = \mu\varepsilon(-\omega^2) + \mu\sigma(-i\omega)$$

$$\Leftrightarrow \boxed{k^2 = \mu\varepsilon\omega^2 + i\mu\sigma\omega}$$

example of a dispersion relation.

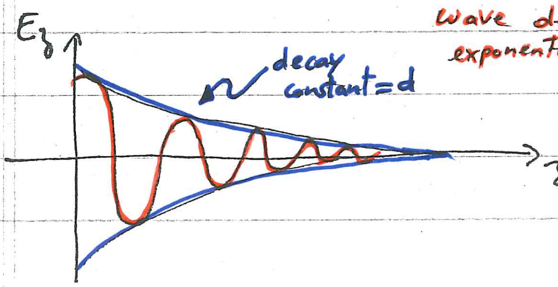
Stopped here

other examples: $k = \frac{\omega}{c}$ (light in vacuum)

$\left(\frac{\hbar k}{2m}\right)^2 = \hbar\omega$ (QM) $\Rightarrow k^2 = \frac{2m\omega}{\hbar}$

$\hookrightarrow$ k is complex: $k = k_0 + i\frac{1}{d}$

thus $\vec{E}(z,t) = \vec{E}_0 \underbrace{e^{-\frac{z}{d}}}_{\text{wave decays exponentially}} e^{i(k_0 z-\omega t)}$



$\boxed{d > 0 \rightarrow \text{wave decays}}$

$d < 0 \rightarrow$ wave blows up

$\hookrightarrow$ energy cannot be conserved