Thursday, October 26, 2017 #1

## Reflection & Transmission at a conducting surface

Recall boundary conditions:

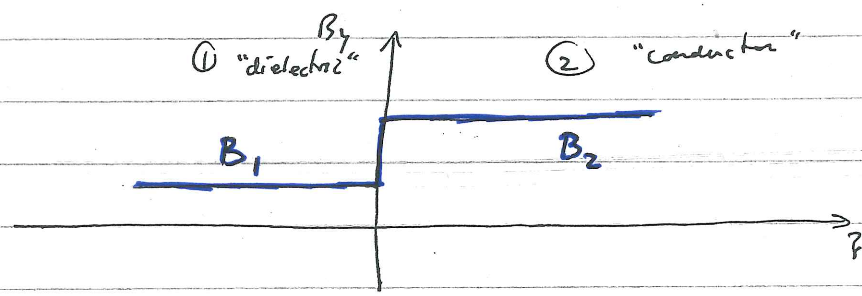
1) $\varepsilon_1 E_{1,\perp} - \varepsilon_2 E_{2,\perp} = \sigma_f = 0$  2) $\vec{E}_{1,\parallel} = \vec{E}_{2,\parallel}$

3) $B_{1,\perp} = B_{2,\perp}$  4) $\frac{1}{\mu_1}\vec{B}_{1,\parallel} - \frac{1}{\mu_2}\vec{B}_{2,\parallel} = \vec{K}_f \times \hat{n}$

$\boxed{\vec{K}_f = 0}$ (i.e. no free surface current ... however $\vec{J}_f \neq 0$)

↳ why? ... suppose $\mu_1 \simeq \mu_2 \simeq \mu_0$

if $\vec{K}_f \neq 0$, then $\vec{B}$ is discontinuous across the boundary




$\Rightarrow \dfrac{\partial B_y}{\partial z} = \underbrace{\beta\,\delta(z)}_{\text{surface current}} \Rightarrow \vec{E}_{\text{surface}} = +\infty$ (if you believe Ohm's law)

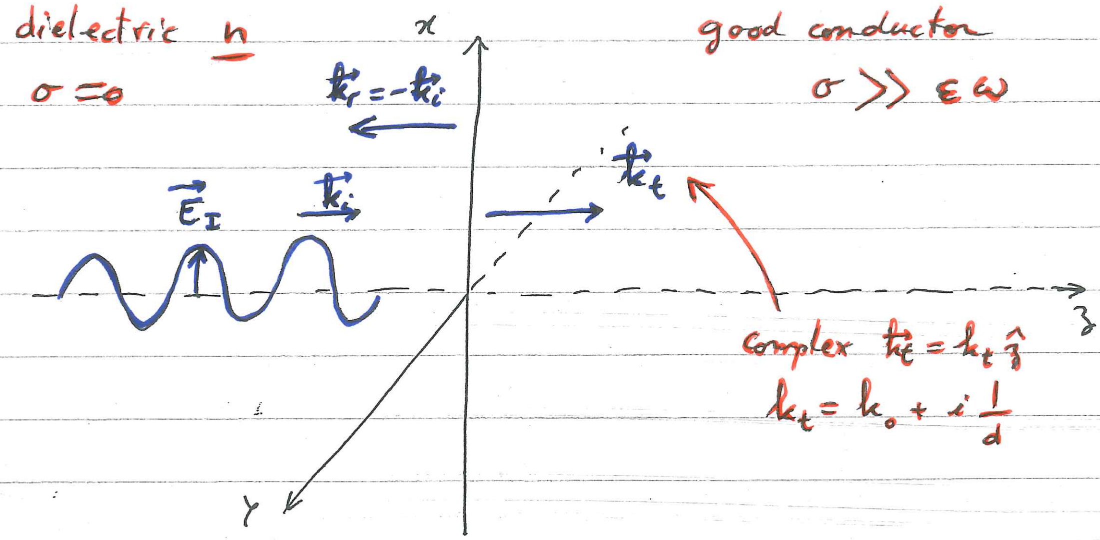
Ampère's law: $\vec{\nabla} \times \vec{B} = \mu\sigma\vec{E} + \mu\varepsilon\dfrac{\partial \vec{E}}{\partial t}$  focus on this term  leave out

$$\big(\cancel{\partial_y B_z}^{\,0} - \underbrace{\partial_z B_y}_{\beta\,\delta(z)},\ \cancel{\partial_z B_x}^{\,0} - \cancel{\partial_x B_z}^{\,0},\ \cancel{\partial_x B_y}^{\,0} - \cancel{\partial_y B_x}^{\,0}\big) = \big(\mu\sigma + \cancel{\mu\varepsilon\partial_t}\big)(E_x, E_y, E_z)$$

$\Rightarrow E_x = +\infty$ (assuming $\sigma$ is finite)

$\Rightarrow$ also $\vec{K} \neq 0$ implies a surface $E_{\parallel}$ → potential problem with b.c.

## Normal incidence




$$\vec{E}_I = E_0 e^{i(kz-\omega t)} \hat{x} \quad \text{and} \quad \vec{B}_I = \frac{nE_0}{c} e^{i(kz-\omega t)} \hat{y}$$

$$\vec{E}_R = E_{0,R} e^{i(-kz-\omega t)} \hat{x} \quad \text{and} \quad \vec{B}_R = -\frac{nE_{0,R}}{c} e^{i(-kz-\omega t)} \hat{y}$$

$$\vec{E}_T = E_{0,t} e^{i(k_t z-\omega t)} \hat{x} \quad \text{and} \quad \vec{B}_T = \underbrace{\frac{n(\omega)}{c} E_{0,t}}_{\frac{k_t E_{0,t}}{\omega}} e^{i(k_t z-\omega t)} \hat{y}$$

complex $k_t = k_0 + i\frac{1}{d}$

### Boundary conditions

1) $E_{1,\perp} = 0 = E_{2,\perp}$

2) $\vec{E}_{1,\parallel} = \vec{E}_{2,\parallel} \Rightarrow E_0 + E_{0,R} = E_{0,t}$

3) $B_{1,\perp} = 0 = B_{2,\perp}$

4) $\vec{B}_{1,\parallel} = \vec{B}_{2,\parallel} \Rightarrow \frac{nE_0}{c} - \frac{nE_{0,R}}{c} = \frac{n(\omega) E_{0,t}}{c}$

$$\Leftrightarrow \frac{k}{\cancel{\omega}}\left(E_{0} - E_{0,R}\right) = \frac{k_t}{\cancel{\omega}} E_{0,t}$$

complex

$$\Leftrightarrow \quad E_0 - E_{0,R} = \frac{k_t}{k} E_{0,t} \qquad (4)$$

$$(2) + (4) \Rightarrow \quad 2E_0 = \left(1 + \frac{k_t}{k}\right) E_{0,t}$$

$$\Leftrightarrow \quad \boxed{t = \frac{E_{0,t}}{E_0} = \frac{2}{1 + \frac{k_t}{k}}}$$

complex

plug back into (2) $\Rightarrow E_{0,R} = E_{0,t} - E_0 = \left(\frac{2}{1+\frac{k_t}{k}} - 1\right) E_0$

$$\Leftrightarrow \quad \boxed{r = \frac{E_{0,R}}{E_0} = \frac{1 - k_t/k}{1 + k_t/k}}$$

complex

$t \,\&\, r \in \mathbb{C} \Rightarrow$ there are non-trivial phases between $\vec{E}_I$, $\vec{E}_R$, and $\vec{E}_t$

<u>Perfect Conductor Limit</u>: $\sigma \rightarrow +\infty$

In this limit $k_t = k_0 + i\frac{1}{d} \longrightarrow$ "$+\infty$"
$(\sigma \rightarrow +\infty)$

$$= \frac{n\omega}{c}\sqrt{\frac{1}{2}\left(1 + \sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2}\right)} + i\omega\sqrt{\frac{\mu\varepsilon}{2}}\sqrt{\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1}$$

$$\longrightarrow +\infty + i\,\infty$$

thus $\lim_{\sigma \to +\infty} t = 0$ & $\lim_{\sigma \to +\infty} r = -1$

$\Rightarrow E_{0,t} = 0 \Rightarrow$ b.c. 2 becomes $\vec{E}_{1,\parallel} = 0$

$$\Leftrightarrow \vec{E}_0 + \vec{E}_{0,R} = 0$$

$$\Leftrightarrow \vec{E}_0 = -\vec{E}_{0,R}$$

Note: In the $\sigma \to +\infty$ limit, you can get a surface current since $\vec{B}_{t,\parallel} = 0 \longrightarrow$ b.c. 4 $\Rightarrow \frac{B_0}{\mu}\hat{y} - \left(-\frac{B_0}{\mu}\hat{y}\right) = \vec{K}_f \times (-\hat{z})$

$$\Rightarrow \left(\frac{n E_0}{c\mu} + \frac{n E_0}{c\mu}\right)\hat{x} = \vec{K}_f$$

$$\Leftrightarrow \boxed{\vec{K}_f = \frac{2n\vec{E}_0}{c\mu}}$$

$\vec{E}_0$ does not diverge, because $\sigma \to +\infty$

Boundary conditions for a perfect conductor: $\boxed{\vec{E}_\parallel = 0,\ \vec{B}_\perp = 0}$

same as DC case.

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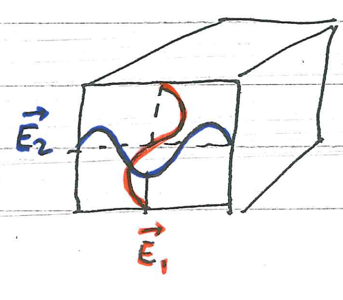
Cavities (or Resonators):

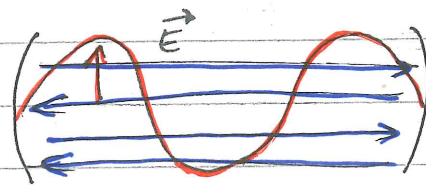
Definition: A container for EM waves.

ex 1: A metal box

↳ similar to a particle in a box (QM)



Ex 2: 2 mirrors, i.e. a Fabry–Perot cavity.

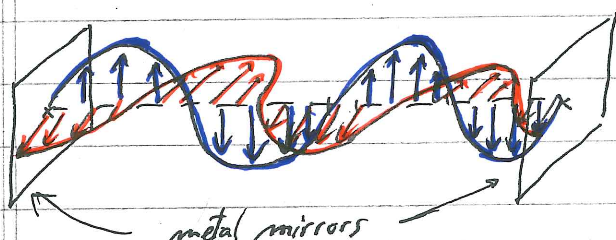



Most cavities have a part that is not fully reflective, but that is slightly transmissive.

example: a laser cavity
↳ light can get out on one side.

counter-example: Superconducting microwave cavity used by the S. Haroche group (ENS Paris) for 2012 Nobel prize work.
↳ How do they get the microwave photons in?
↳ black body radiation! (or atom beam)

boundary conditions (perfect conductor)

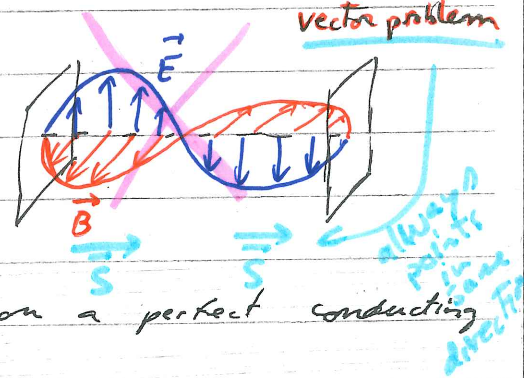



$\vec{E}_{\parallel, \text{mirror}} = 0$

$B_{\perp, \text{mirror}} = 0$

$\overset{?}{\Longrightarrow}$ $\cancel{\vec{B}_{\parallel, \text{mirror}} = 0}$

↳ poynting vector problem

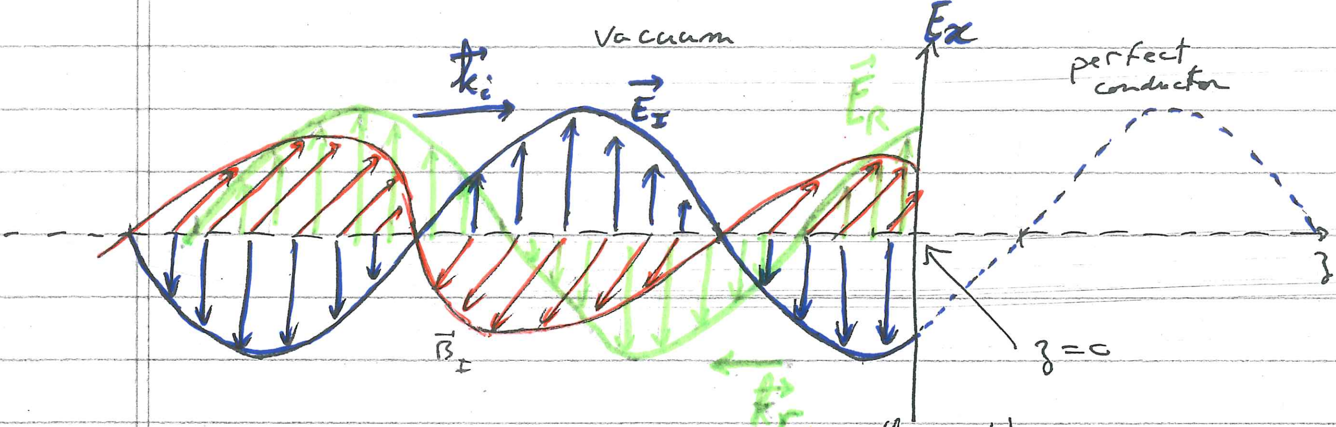



Standing waves:

Consider a plane wave incident on a perfect conducting mirror (normal incidence):

Standing waves

Consider a plane wave incident a perfect conducting mirror (normal incidence)




Incident plane wave: $\vec{E}_I = E_0 e^{i(kz-\omega t)}\hat{x}$

$\vec{B}_I = B_0 e^{i(kz-\omega t)}\hat{y}$

Reflected planewave: $\vec{E}_R = -E_0 e^{i(-kz-\omega t)}\hat{x}$

$\left(r = \frac{E_{0,r}}{E_0} = -1\right)$ $\vec{B}_R = B_0 e^{i(-kz-\omega t)}\hat{y}$

Stopped here

$$\Rightarrow \vec{E}_{total} = \vec{E}_I + \vec{E}_R = E_0\left[e^{i(kz-\omega t)} - e^{i(-kz-\omega t)}\right]\hat{x}$$

$$= E_0 e^{-i\omega t} 2i\left(\frac{e^{ikz} - e^{-ikz}}{2i}\right)\hat{x}$$

$$= E_0 2i e^{-i\omega t} \sin(kz)\hat{x}$$

take the real part $\Rightarrow$ $\boxed{\vec{E}_{total} = 2E_0 \sin(kz)\sin(\omega t)\hat{x}}$

Also, $\vec{B}_{total} = \vec{B}_I + \vec{B}_R = B_0 \left[ e^{i(kz-\omega t)} + e^{i(-kz-\omega t)} \right] \hat{y}$

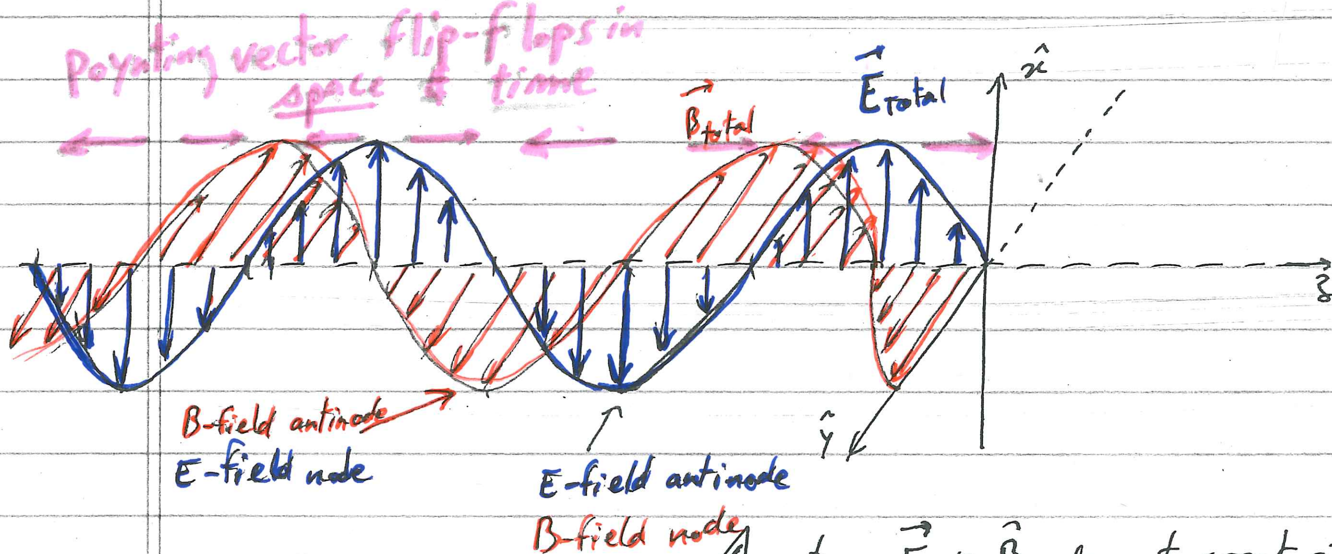
$$= B_0 e^{-i\omega t}\, 2 \left[ \frac{e^{-ikz} + e^{ikz}}{2} \right] \hat{y}$$

$$= 2 B_0 e^{-i\omega t} \cos(kz)\, \hat{y}$$

$\rightarrow$ take real part $\Rightarrow$ $\boxed{\vec{B}_{total} = 2B_0 \cos(kz)\cos(\omega t)\, \hat{y}}$

Results: 1) $\vec{E}$ & $\vec{B}$ fields are $\pi/2$ out of phase in space and time.

2) $\vec{E}$ & $\vec{B}$ fields are standing waves (they don't travel).




⚠ note: $\vec{E}$ & $\vec{B}$ do not crest at the same time