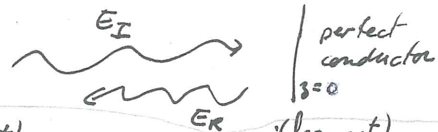
Tuesday, October 31, 2017

#1

Standing waves:



$$\boxed{\vec{E}_{\parallel} \overset{\text{b.c.}}{=} 0}$$

Incident plane wave: $\vec{E}_I = E_0 e^{i(kz-\omega t)}\hat{x}$, $\vec{B}_I = B_0 e^{i(kz-\omega t)}\hat{y}$

Reflected plane wave: $r = \frac{E_{0,r}}{E_0} = -1$ so $\vec{E}_R = -E_0 e^{i(-kz-\omega t)}\hat{x}$, $\vec{B}_R = B_0 e^{i(-kz-\omega t)}\hat{y}$

from b.c.

$$\Rightarrow \vec{E}_{total} = \vec{E}_I + \vec{E}_R = E_0\left[e^{i(kz-\omega t)} - e^{i(-kz-\omega t)}\right]\hat{x} = E_0 e^{-i\omega t}\, 2i\left[\frac{e^{ikz} - e^{-ikz}}{2i}\right]\hat{x}$$

$$= E_0\, 2i\, e^{-i\omega t}\sin(kz)\,\hat{x}$$

take the real part $\Rightarrow$ $\boxed{\vec{E}_{total} = 2E_0 \sin(kz)\sin(\omega t)\,\hat{x}}$

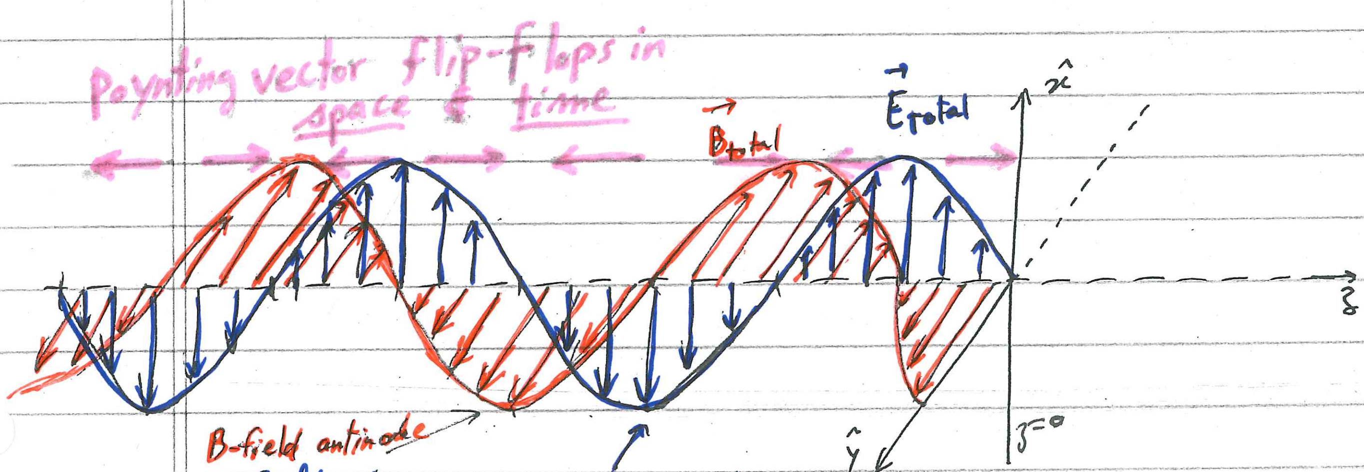
Also, $\vec{B}_{total} = \vec{B}_I + \vec{B}_R = B_0\left[e^{i(kz-\omega t)} + e^{i(-kz-\omega t)}\right]\hat{y}$

$$= B_0 e^{-i\omega t}\, 2\left[\frac{e^{ikz} + e^{-ikz}}{2}\right]\hat{y}$$

$$= 2B_0 e^{-i\omega t}\cos(kz)\,\hat{y}$$

$\rightarrow$ take real part $\Rightarrow$ $\boxed{\vec{B}_{total} = 2B_0\cos(kz)\cos(\omega t)\,\hat{y}}$

Results:
1) $\vec{E}$ & $\vec{B}$ fields are $\pi/2$ out of phase in space and time.
2) $\vec{E}$ & $\vec{B}$ fields are standing waves (they don't travel).

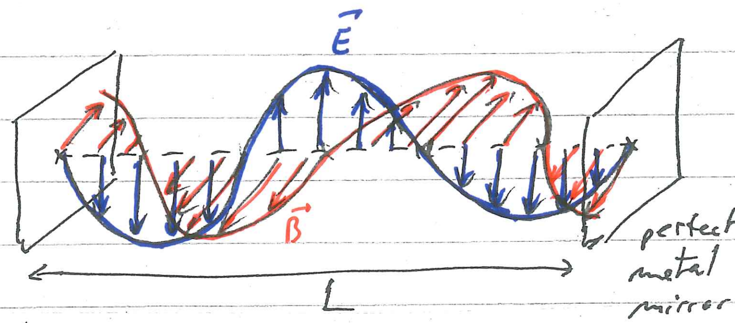


⚠ note: $\vec{E}$ & $\vec{B}$ do not crest at the same time.

The Fabry-Perot cavity

Simple approach:

compare to problem 9.36 in Griffiths



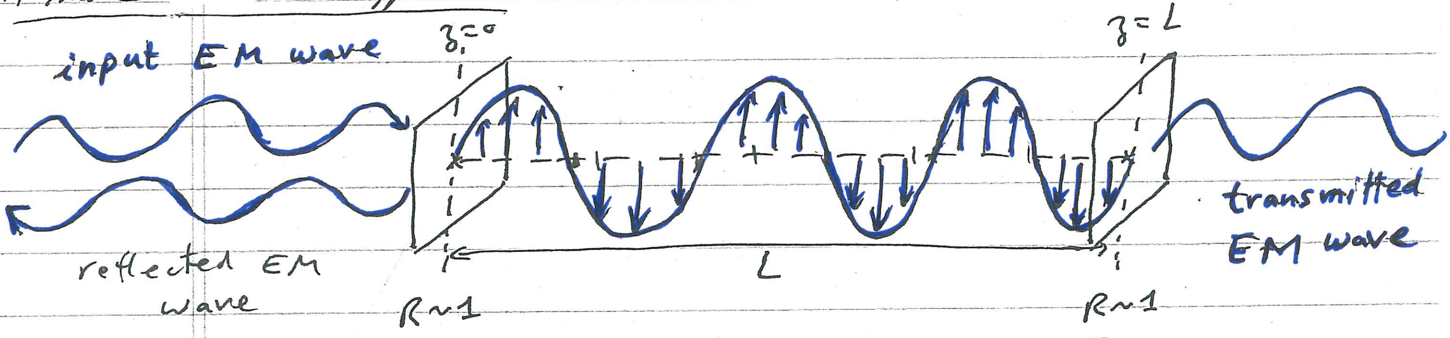
boundary conditions $\begin{cases} E_{\parallel} = 0 \\ B_{\perp} = 0 \end{cases}$

$\Rightarrow$ the only allowed wavelengths are those such that $\boxed{\frac{n\lambda}{2} = L}$
$n \in \mathbb{N}$

How do you get E-M waves into the cavity to start with?

- antenna source $\rightarrow$ produces loss (exception of a gain medium in a laser)
- thermal excitation $\rightarrow$ low circulating power.

A more approach:



$$E_I = E_0 e^{i(kz-\omega t)}$$

$$E_R = r E_I = \sqrt{R}\, E_I$$

$$E_{in} = E_t = t E_I = \sqrt{T} E_I$$

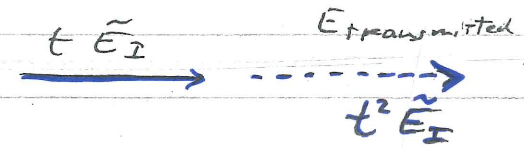
note: we will neglect the vector nature of the $\vec{E}$-field since it is not important for the basic physics of the cavity.

recall: for $n_1 = n_2$ $\quad R = |r|^2, \; T = |t|^2, \; R + T = 1$

we neglect phases incurred in mirrors, but assume both mirrors are identical.

<u>1^{st} pass</u>

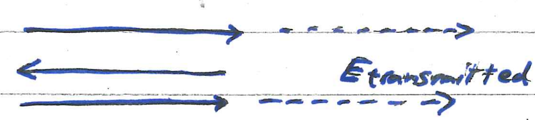
$$E_{transmitted,1} = t^2 \tilde{E}_I$$



<u>2^{nd} pass:</u>

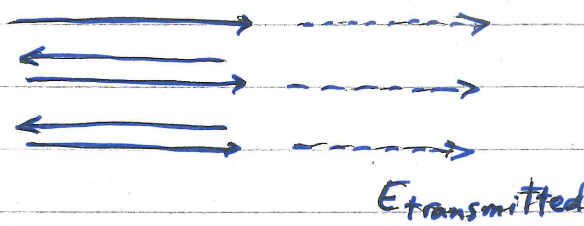
$$E_{transmitted,2} = t^2 \tilde{E}_I + r^2 t^2 \tilde{E}_I e^{ik2L}$$

$$= t^2 \tilde{E}_I + (r^2 e^{i2kL})\, t^2 \tilde{E}_I$$



<u>3^{rd} pass:</u>

$$E_{transmitted,3} = t^2 \tilde{E}_I + (r^2 e^{i2kL})^1 t^2 \tilde{E}_I + (r^2 e^{i2kL})^2 t^2 \tilde{E}_I$$



<u>n^{th} pass:</u>

$$E_{transmitted,n} = t^2 \tilde{E}_I \Big[\underbrace{1 + r^2 e^{i2kL} + (r^2 e^{i2kL})^2 + \dots (r^2 e^{i2kL})^{n-1}}_{\text{geometric series}} \Big]$$

$$= \frac{1 - (r^2 e^{i2kL})^n}{1 - r^2 e^{i2kL}}$$

$$E_{transmitted} = \lim_{n \to +\infty} E_{transmitted,n} = t^2 \tilde{E_I} \frac{1}{1 - r^2 e^{i2kL}}$$

$$= \frac{T \tilde{E_I}}{1 - R e^{i2kL}}$$

Thus $I_{transmitted} = \frac{1}{2} c \varepsilon \left| E_{transmitted} \right|^2$

$$= \frac{1}{2} c \varepsilon \, T^2 \frac{|E_I|^2}{|1 - R e^{i2kL}|^2}$$

$$= \frac{1}{2} c \varepsilon \, |E_I|^2 \frac{T^2}{|1 - R\cos(2kL) - iR\sin(2kL)|^2}$$

a little bit of algebra

$$= I_{incident} \cdot \frac{(1-R)^2}{1+R^2 - 2R\cos(2kL)}$$

On resonance (i.e $\cos(2kL) = +1$): if $\cancel{2}kL = n\cancel{2}\pi \iff \frac{2\cancel{\pi}}{\lambda} L = n\cancel{\pi}$

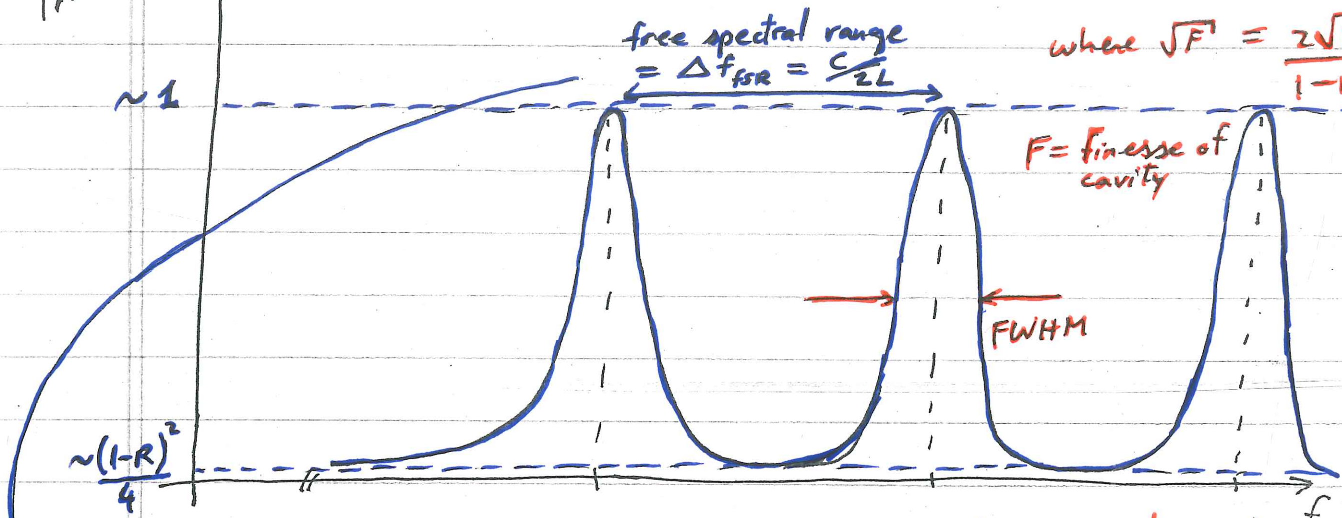
$$\iff \boxed{L = \frac{n\lambda}{2}}$$

then $I_{transmitted} = I_{incident} \dfrac{\cancel{(1-R)^2}}{\underbrace{1+R^2 - 2R}_{\cancel{(1-R)^2}}} = I_{incident}$

$\Rightarrow$ the waves interfere constructively (on transmission) to allow all the light to pass through the Fabry–Perot cavity even if $R \simeq 1$. No light is reflected off the input face of the cavity (destructive interference on reflection)

Off resonance, most of the light is reflected off the cavity and there is little ~~t~~ transmission

$$\text{Transmission} = \frac{I_{transmitted}}{I_{incident}}$$



$$FWHM = 4\arcsin\left(\frac{1}{\sqrt{F}}\right)$$

where $\sqrt{F} = \frac{2\sqrt{R}}{1-R}$

as $R \to 1 \Rightarrow$ peaks get narrower

$$L = \frac{n\lambda_1}{2} = \frac{(n+1)\lambda_2}{2} \Rightarrow \begin{cases} \lambda_1 = \frac{2L}{n} \\ \lambda_2 = \frac{2L}{n+1} \end{cases} \Rightarrow \begin{cases} \frac{f_1}{c} = \frac{n}{2L} \\ \frac{f_2}{c} = \frac{n+1}{2L} \end{cases}$$

$\lambda f = c$

$\frac{1}{\lambda} = \frac{f}{c}$

$$\Delta f_{fsr} = f_2 - f_1 = \frac{c}{2L}$$

$$\boxed{\text{note: the cavity transmission spectrum gives the Fourier/frequency spectrum of the incident light field (modulo } \Delta f_{fsr})}$$