Week #2 Problem Set
Due date: Friday, February 5, 2010.

First and Second Order Coherence

1. Sargent and Meystre, problem 1.15, p. 33.

2. Wiener-Khintchine Theorem: In class, we derived a formula for the Normalized Power Spectral Density, \( F(\omega) \), in terms of the first order correlation function, \( g^{(1)}(\tau) \). Derive an expression for \( g^{(1)}(\tau) \) in terms of \( F(\omega) \).

3. Write a 1 page essay (double-spaced) summarizing the Hanbury-Brown and Twiss paper and the associated letter by E. Purcell – you may choose to concentrate on one or two aspects of the papers rather than the entire papers.

Extra graduate student problem

4. Gaussian lineshape: Consider a gas of identical atoms that emit light at a resonant frequency of \( \omega_0 \) (when at rest). The atoms in the gas have a spread of velocities due to their finite temperature, \( T \), which leads to a spread in the resonant frequency of the atoms due to the Doppler effect. The probability for an atom to emit light at a frequency \( \omega \) close to \( \omega_0 \) in a given direction is then given by

\[
P(\omega) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(\omega - \omega_0)^2}{\sigma^2}\right) \quad \text{with} \quad \sigma = \omega_0 \sqrt{\frac{kT}{mc^2}},
\]

where \( m \) is the mass of the atom, \( c \) is the speed of light, and \( k \) is Boltzmann's constant.

We will assume that the atom is excited by some mechanism, and we look at the radiated light in a given direction. The emitted electric field in this direction is then given by

\[
E(t) = E_0 \sum_{i=1}^{N} \exp(-i\omega_i t + i\phi_i)
\]

where the sum is over the \( N \) atoms in the gas and the \( \phi_i \) are random stationary phases.

a. Show that \( \left\langle E^*(t)E(t+\tau) \right\rangle = E_0^2 \sum_{i=1}^{N} \exp(-i\omega_i \tau) \).

b. Show that \( g^{(1)}(\tau) = \exp\left(-i\omega_0 \tau - \frac{1}{2} \sigma^2 \tau^2\right). \)