Week #2 Problem Set
Due date: Friday, February 5, 2010.

First and Second Order Coherence

1. Sargent and Meystre, problem 1.15, p. 33.

2. *Wiener-Khintchine Theorem:* In class, we derived a formula for the Normalized Power Spectral Density, $F(\omega)$, in terms of the first order correlation function, $g^{(1)}(\tau)$. Derive an expression for $g^{(1)}(\tau)$ in terms of $F(\omega)$.

3. Write a 1 page essay (double-spaced) summarizing the Hanbury-Brown and Twiss paper and the associated letter by E. Purcell – you may choose to concentrate on one or two aspects of the papers rather than the entire papers.

Extra graduate student problem

4. *Gaussian lineshape:* Consider a gas of identical atoms that emit light at a resonant frequency of ω_0 (when at rest). The atoms in the gas have a spread of velocities due to their finite temperature, T, which leads to a spread in the resonant frequency of the atoms due to the Doppler effect. The probability for an atom to emit light at a frequency ω close to ω_0 in a given direction is then given by

$$P(\omega) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(\omega-\omega_0)^2}{\sigma^2}\right) \quad \text{with} \quad \sigma = \omega_0\sqrt{\frac{kT}{mc^2}},$$

where m is the mass of the atom, c is the speed of light, and k is Boltzmann's constant.

We will assume that the atom is excited by some mechanism, and we look at the radiated light in a given direction. The emitted electric field in this direction is then given by

$$E(t) = E_0 \sum_{i=1}^{N} \exp(-i\omega_i t + i\phi_i)$$

where the sum is over the N atoms in the gas and the ϕ_i are random stationary phases.

a. Show that $\left\langle E^*(t)E(t+\tau)\right\rangle = E_0^2 \sum_{i=1}^{N} \exp(-i\omega_i \tau)$.

b. Show that $g^{(1)}(\tau) = \exp\left(-i\omega_0\tau - \frac{1}{2}\sigma^2\tau^2\right)$.