Week #3 Problem Set
Due date: Friday, February 8, 2010.

2-level atoms

1. Level crossings

Consider a 2-level atom with ground and excited states $|g\rangle$ and $|e\rangle$, respectively, and energies, E_g and E_e, respectively. These energies depend on a parameter m such that

$$E_g(m) = E_g(0) + \alpha\cdot m \text{ and } E_e(m) = E_e(0) - \alpha\cdot m$$

We modify the basic Hamiltonian of the system, H_0, by adding a generic interaction with Hamiltonian:

$$H_{\text{int}} = \begin{bmatrix} 0 & W \\ W^* & 0 \end{bmatrix}$$

a) Calculate the new eigenenergies of the system with the interaction present, as a function of m.

b) Plot the energy of the system as function of m, with and without the interaction present.

c) Calculate the new eigenstates of the system with the interaction present, and show that one can go continuously from modified state $|g\rangle$ to modified state $|e\rangle$, and vice versa, by adiabatically varying m. What is the condition for adiabaticity? Make a quantitative (and logical) argument. What happens if you ramp m much faster than the adiabatic condition?

2. 2-level atom in a laser field without RWA

In this problem, you will solve the Schrodinger Equation for a 2-level atom in a laser field without recourse to the rotating wave approximation, at least for short times.

As seen in class, the general form for the wavefunction of a level atom is

$$|\psi(t)\rangle = c_g(t)e^{-i\omega_g t}|g\rangle + c_e(t)e^{-i\omega_e t}|e\rangle$$

We will assume that at t=0, $c_g \approx 1$ and $c_e \approx 0$ (i.e. the system is at the bottom of a Rabi flop). The interaction with the laser field is given by $W = \hbar\Omega\sin(\omega t)$ (in the notation of problem 1 … with m=0). You can assume that the laser field is in the vicinity of resonance (i.e. $\omega \approx \omega_{eg}$, though the detuning can still be quite large)

a) Derive two first order coupled differential equations (exact) for $c_g(t)$ and $c_e(t)$ that are valid at all times (as seen in class).

b) Derive an expression for $c_e(t)$ that is valid at short times by directly integrating one of the differential equations.

c) Use the expression for $c_e(t)$ to derive an expression for $c_g(t)$ that is valid at short times.

d) Infer the energy shift of the ground state. What happens if you apply the rotating wave approximation (in spirit, perhaps not the way we saw it in class)? Do your results agree with the AC Stark shift derived in class?