Zeeman Sub-Structure at High B-field

$5S_{1/2}$ ground states of $^{87}\text{Rb}$

[Figure adapted from steck.us by Prof. Dan Steck, U. of Oregon (2010)]
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We calculated these Zeeman shifts with perturbation theory.

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How do you calculate this entire plot?

We calculated these Zeeman shifts with perturbation theory.

$F = 2$

$F = 1$

$E/\hbar$ (GHz)

$F = 2$

$F = 1$

$\frac{m_j}{m} = \pm \frac{1}{2}$

$m_j = +1/2$

$m_j = -1/2$

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Zeeman shifts and the Hyperfine Hamiltonian

In the ground state (S state, so L=0):

\[ H = H_0 + H_{\text{FineStructure}} + H_{\text{Hyperfine}} + H_{\text{Zeeman}} \]
Zeeman shifts and the Hyperfine Hamiltonian

In the ground state (S state, so L=0):

\[ H = H_0 + H_{\text{FineStructure}} + H_{\text{Hyperfine}} + H_{\text{Zeeman}} \]

\[ H = \frac{P^2}{2m} - \frac{e^2}{R} + \frac{e^2}{mc^2} \frac{1}{R^3} (\vec{L} \cdot \vec{S}) + hA(\vec{I} \cdot \vec{J}) + \frac{\mu_B}{\hbar} \left( g_s \vec{S} + g_I \vec{I} \right) \]
The Breit-Rabi formula for the Zeeman shift of atomic ground states is given by:

\[ U(m_F, B) = g_I \mu_B m_F B + \frac{E_{hfs}}{2} \left( \pm (1 + \frac{4m_Fx}{2I+1} + x^2)^{1/2} - \frac{1}{2I+1} \right), \]

where the ± is used for the \( F = I \pm J \) state, respectively, and

\[ x = \frac{(g_J - g_I) \mu_B B}{E_{hfs}}. \]
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\[ |F, m_F\rangle \]

\[ |I, m_I\rangle |S, m_s = \uparrow\rangle \]

\[ |I, m_I\rangle |S, m_s = \downarrow\rangle \]

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$$|F, m_F\rangle$$

$F = 2$

$F = 1$

No good quantum numbers

$$|I, m_I\rangle |S, m_s = \uparrow\rangle$$

$$|I, m_I\rangle |S, m_s = \downarrow\rangle$$

[Figure adapted from steck.us by Prof. Dan Steck, U. of Oregon (2010)]
F=2 with I=3/2, S=1/2

\[ |F = 2, m_F = +2\rangle = |I = 3/2, m_I = +3/2\rangle \uparrow \]

\[ |F = 2, m_F = +1\rangle = \frac{1}{2} |m_I = +3/2\rangle \downarrow + \frac{\sqrt{3}}{2} |m_I = +1/2\rangle \uparrow \]

\[ |F = 2, m_F = 0\rangle = \frac{1}{\sqrt{2}} |m_I = +1/2\rangle \downarrow + \frac{1}{\sqrt{2}} |m_I = -1/2\rangle \uparrow \]

\[ |F = 2, m_F = -1\rangle = \frac{1}{2} |m_I = -3/2\rangle \uparrow + \frac{\sqrt{3}}{2} |m_I = -1/2\rangle \downarrow \]

\[ |F = 2, m_F = -2\rangle = |m_I = -3/2\rangle \downarrow \]
Clebsch-Gordan Coefficients: I=3/2, S=1/2

F=1 with I=3/2, S=1/2

\[ |F = 1, m_F = +1\rangle = \frac{\sqrt{3}}{2} |m_I = +3/2\rangle \downarrow - \frac{1}{2} |m_I = +1/2\rangle \uparrow \]

\[ |F = 1, m_F = 0\rangle = \frac{1}{\sqrt{2}} |m_I = +1/2\rangle \downarrow - \frac{1}{\sqrt{2}} |m_I = -1/2\rangle \uparrow \]

\[ |F = 2, m_F = -1\rangle = -\frac{\sqrt{3}}{2} |m_I = -3/2\rangle \uparrow + \frac{1}{2} |m_I = -1/2\rangle \downarrow \]
General Formula...

\[ |F_+ = I + S, m_F\rangle = \]
\[ \frac{\sqrt{F_+ + m_F}}{\sqrt{2I + 1}} |m_I = m_F - 1/2\rangle \uparrow + \frac{\sqrt{F_+ - m_F}}{\sqrt{2I + 1}} |m_I = m_F + 1/2\rangle \downarrow \]

\[ |F_- = I - S, m_F\rangle = \]
\[ -\frac{\sqrt{F_+ - m_F}}{\sqrt{2I + 1}} |m_I = m_F - 1/2\rangle \uparrow + \frac{\sqrt{F_+ + m_F}}{\sqrt{2I + 1}} |m_I = m_F + 1/2\rangle \downarrow \]
Why are Alkalis “2-level atoms”? (of rubidium-87)

Energy

5S_{1/2}  5P_{3/2}

F=0  F=1  F=2  F=3

L (orbital angular momentum)
Why are Alkalis "2-level atoms"?

(of rubidium-87)

[not to scale]
The D2 line Cycling Transition

Energy

5P_{3/2}

F=3

m_F=-3 \quad m_F=-2 \quad m_F=-1 \quad m_F=0 \quad m_F=+1 \quad m_F=+2 \quad m_F=+3

5S_{1/2}

F=2

m_F=-2 \quad m_F=-1 \quad m_F=0 \quad m_F=+1 \quad m_F=+2

L (orbital angular momentum)
The D2 line Cycling Transition

$5S_{1/2}$  
$F=2$  
$m_F=-2$  $m_F=-1$  $m_F=0$  $m_F=+1$  $m_F=+2$

$5P_{3/2}$  
$F=3$  
$m_F=-3$  $m_F=-2$  $m_F=-1$  $m_F=0$  $m_F=+1$  $m_F=+2$  $m_F=+3$

$\Delta m_F = -2$  

L (orbital angular momentum)