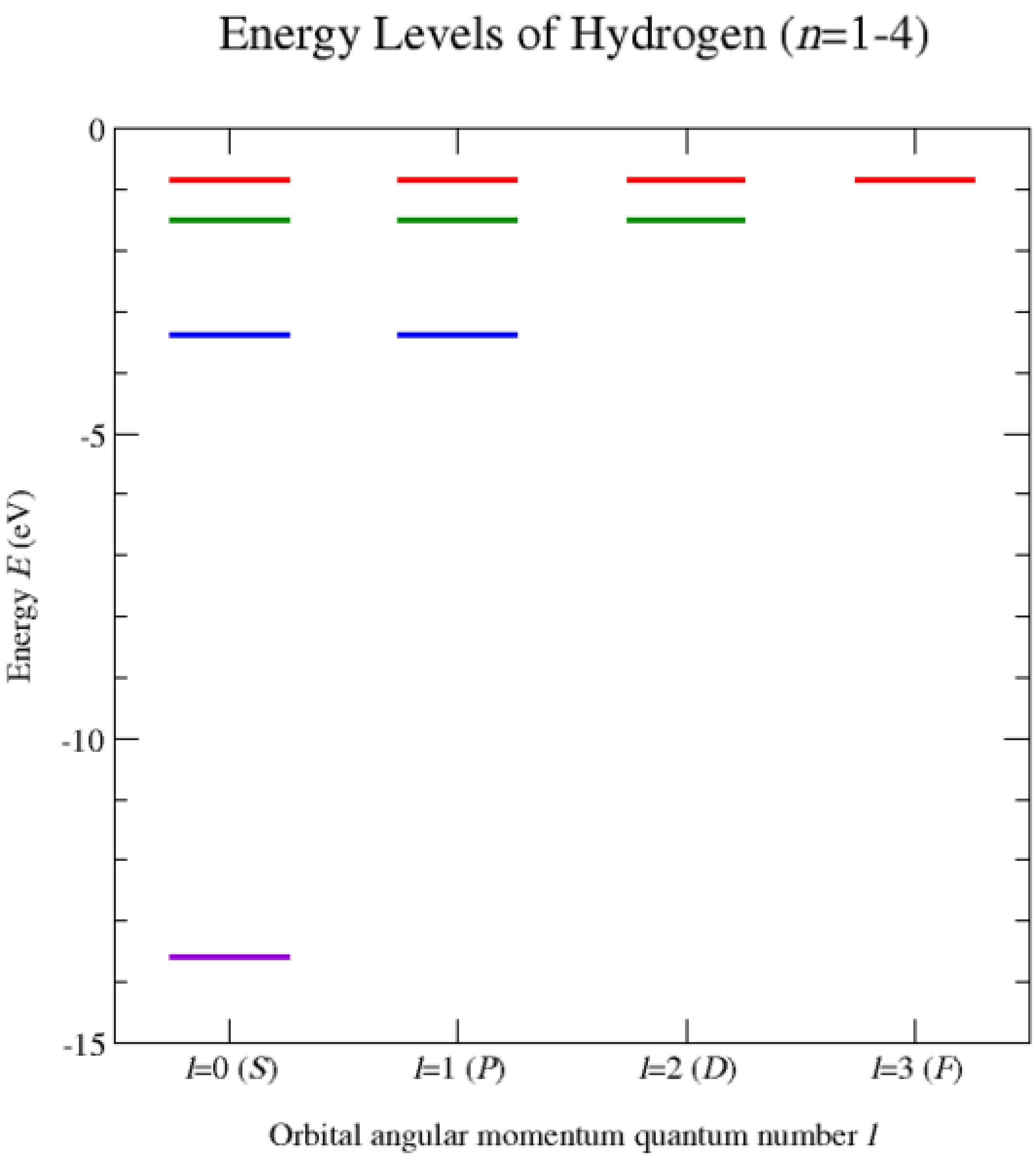
Basic Energy Levels

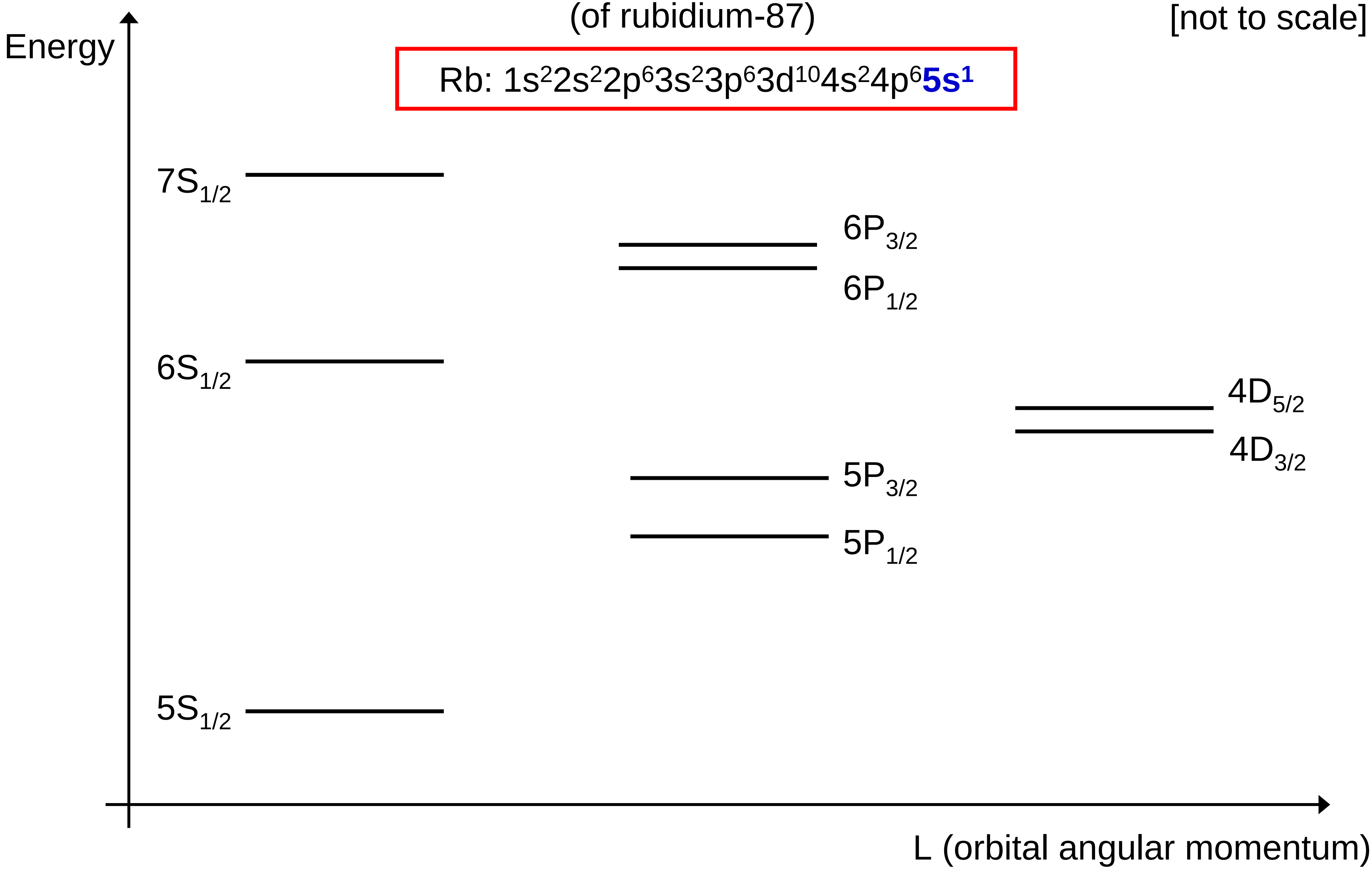


[Figure from wikimedia.org]

Fine Structure

(of rubidium-87)

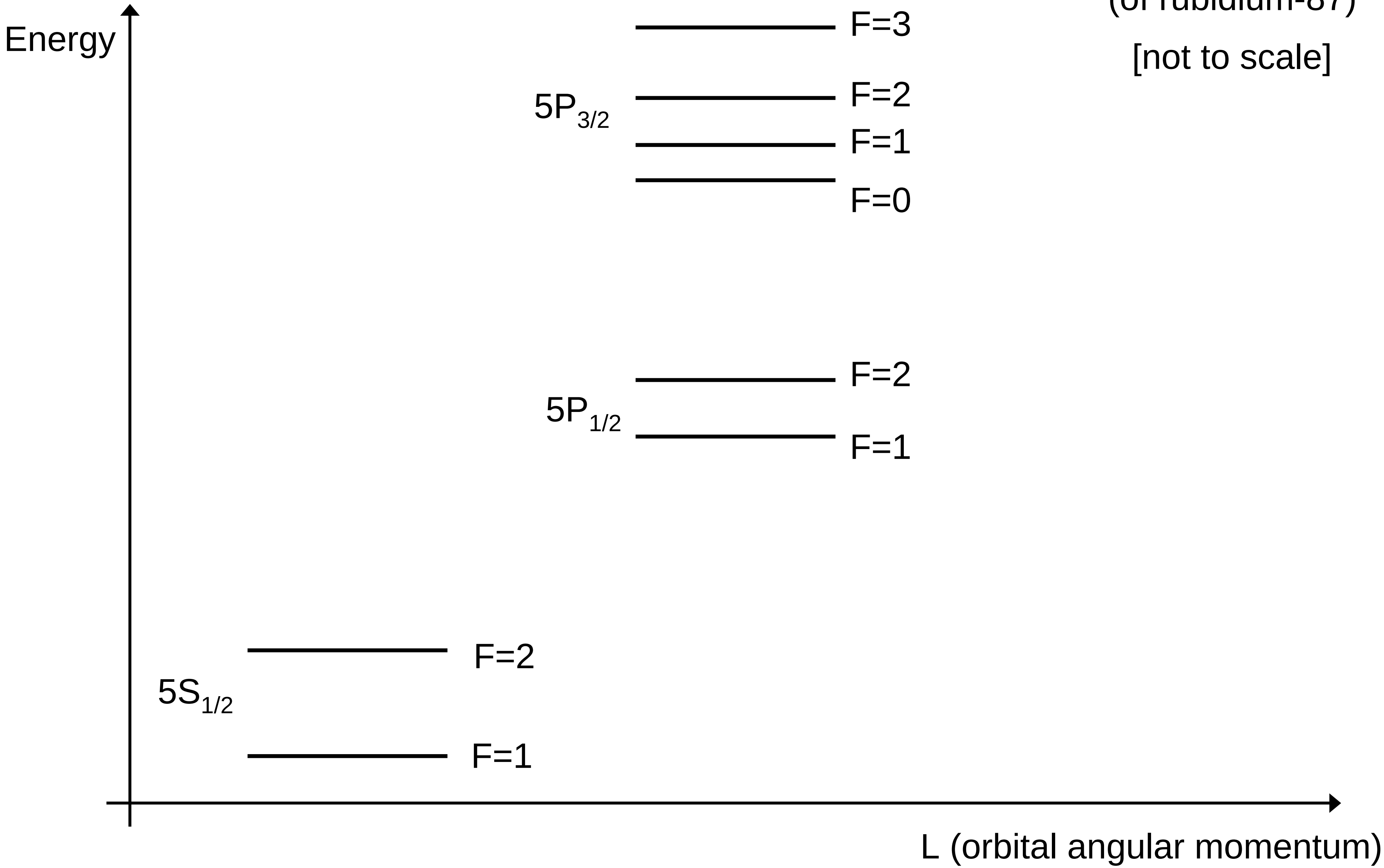
Rb: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6$ **$5s^1$**



Hyperfine Structure

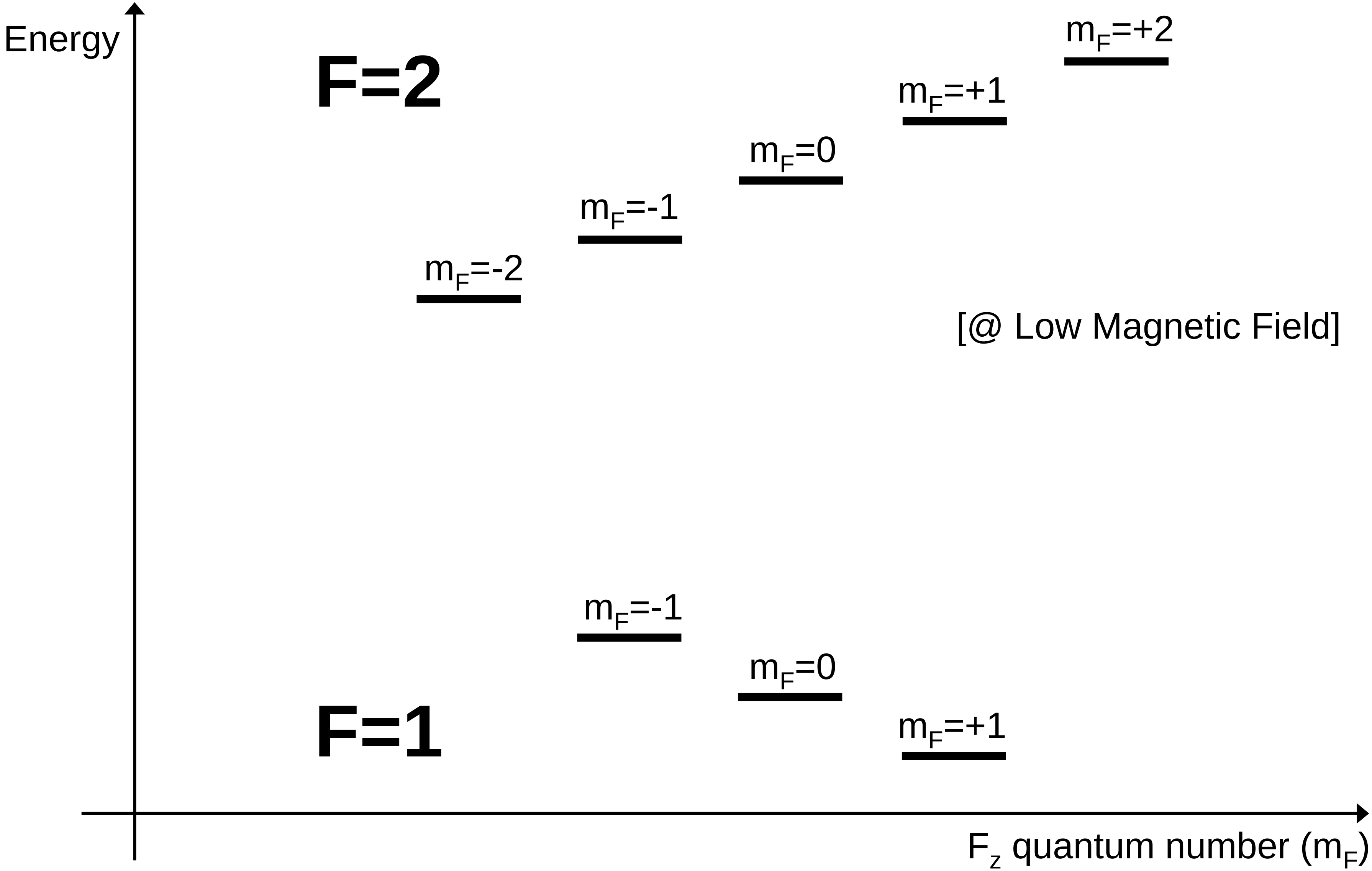
(of rubidium-87)

[not to scale]

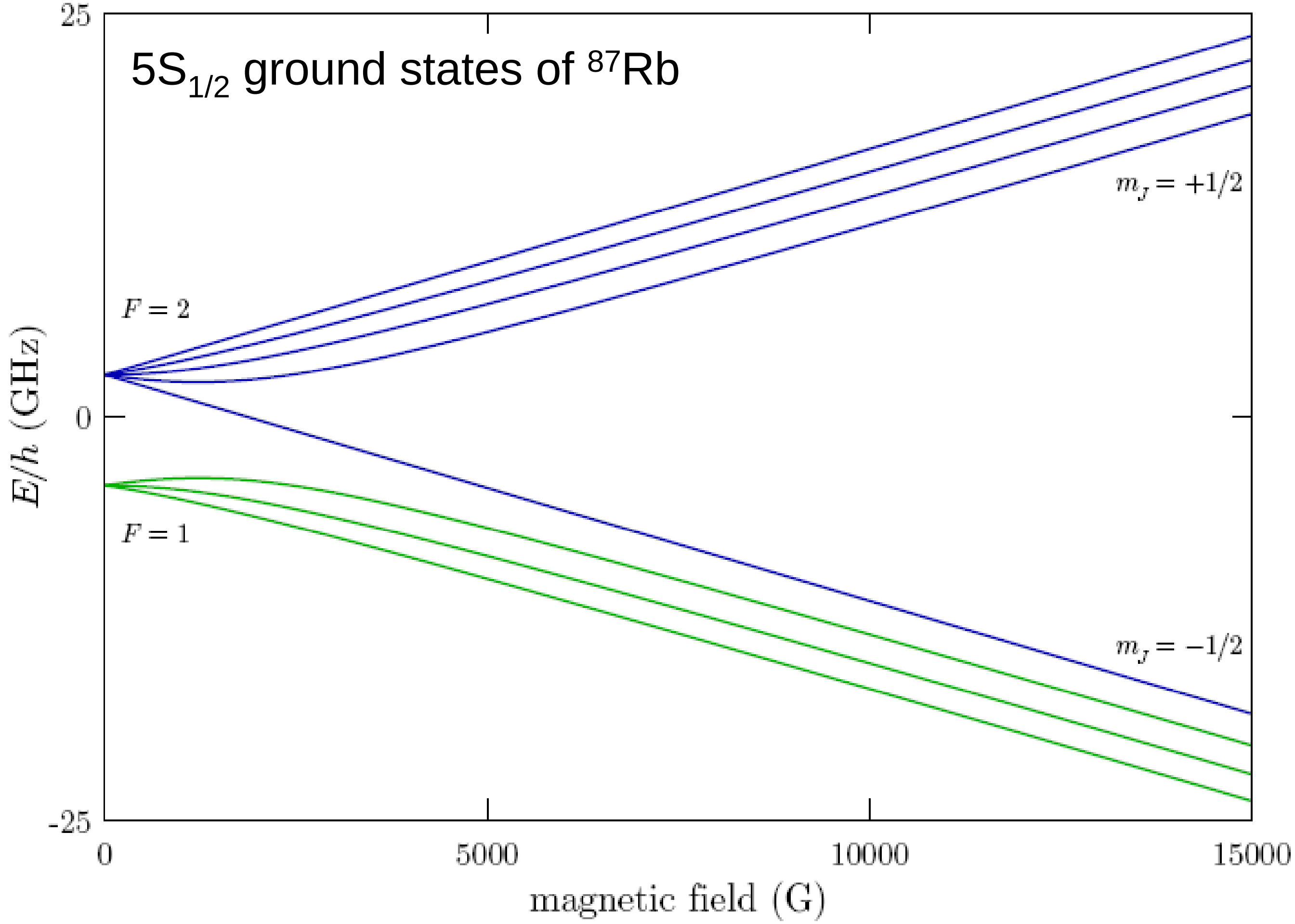


Zeeman Sub-Structure

[^{87}Rb, ^{39}K, ^{41}K]

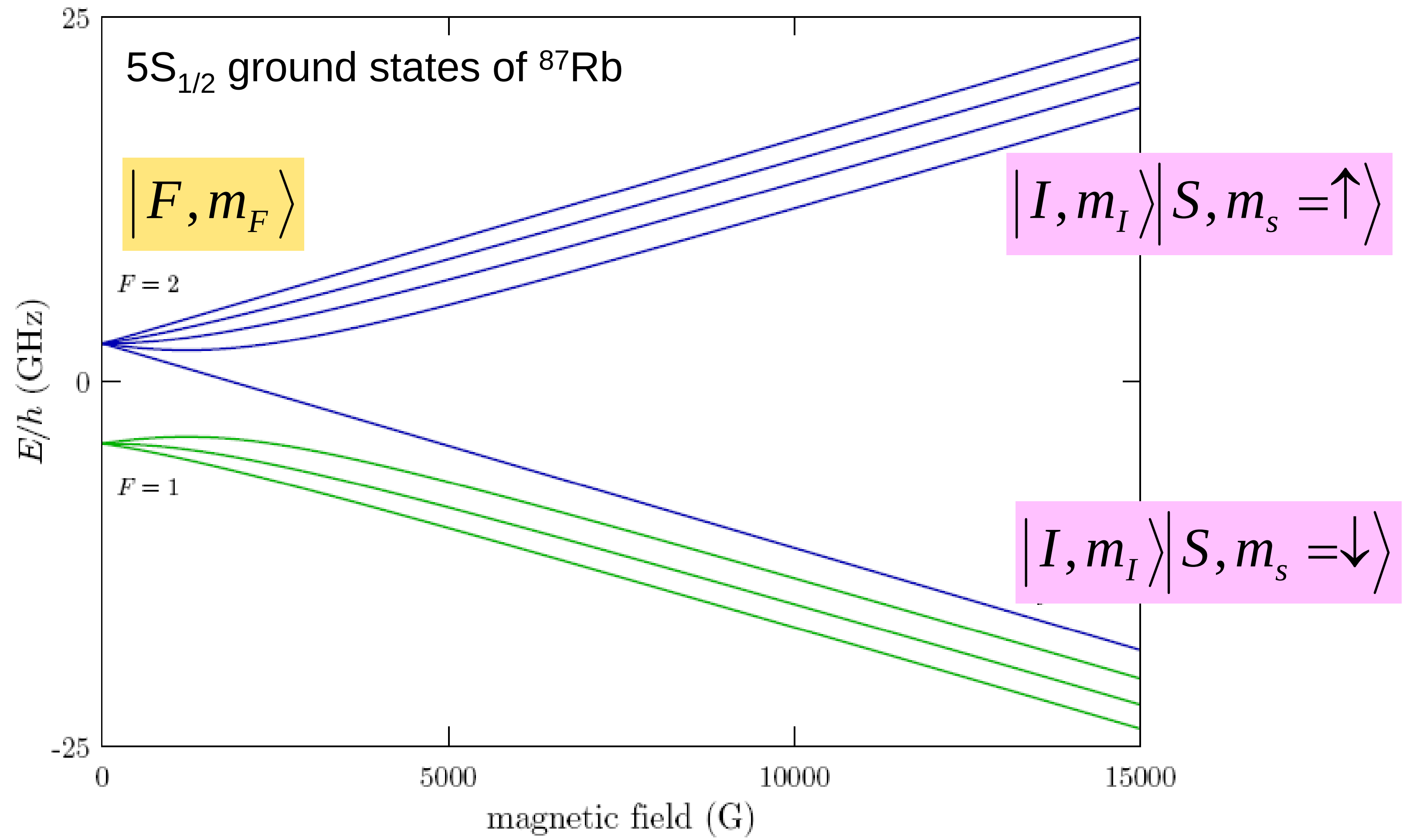


Zeeman Sub-Structure at High B-field



[Figure adapted from steck.us by Prof. Dan Steck, U. of Oregon (2010)]

Zeeman Sub-Structure at High B-field



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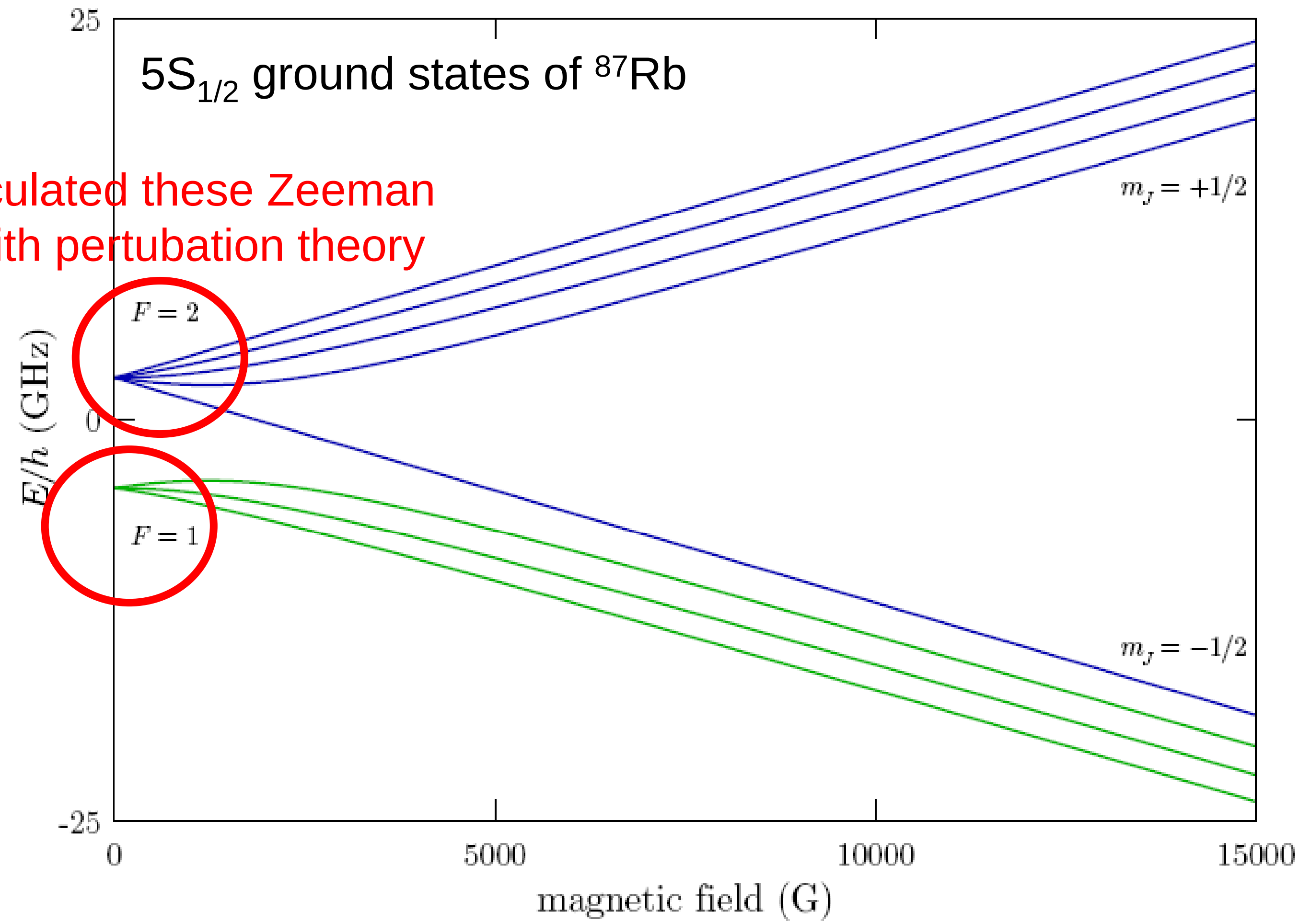
Zeeman Sub-Structure at High B-field

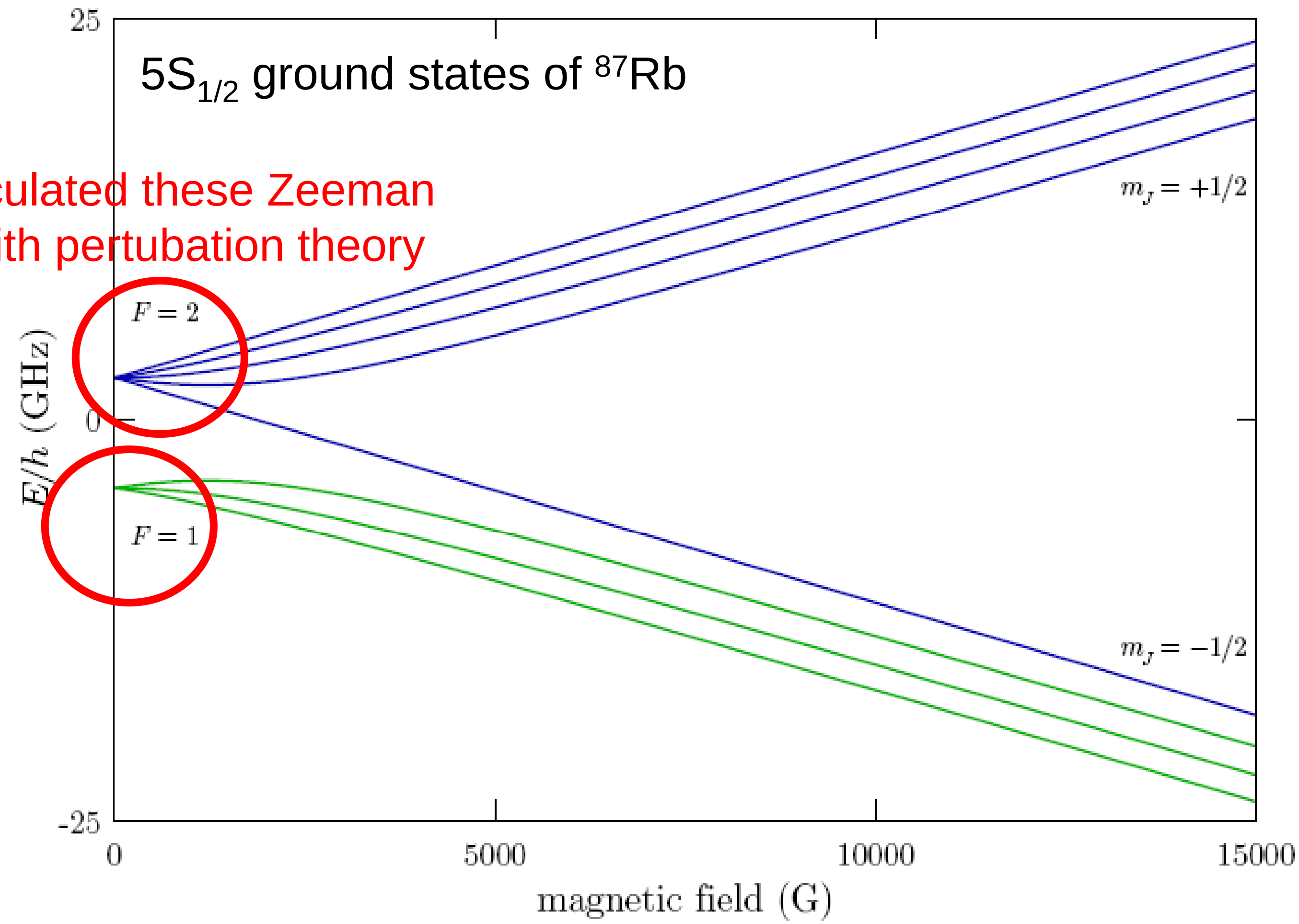
We calculated these Zeeman shifts with pertubation theory



[Figure adapted from steck.us by Prof. Dan Steck, U. of Oregon (2010)]

Zeeman Sub-Structure at High B-field

How do you calculate this entire plot?



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Zeeman shifts and the Hyperfine Hamiltonian

In the ground state (S state, so L=0):

$$H = H_0 + H_{FineStructure} + H_{Hyperfine} + H_{Zeeman}$$

Zeeman shifts and the Hyperfine Hamiltonian

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$$H = H_0 + H_{FineStructure} + H_{Hyperfine} + H_{Zeeman}$$

$$H = \frac{P^2}{2m} - \frac{e^2}{R} + \frac{e^2}{mc^2}\frac{1}{R^3}\left(\vec{L}\cdot\vec{S}\right) + hA\left(\vec{I}\cdot\vec{J}\right) + \frac{\mu_B}{\hbar}\left(g_s\vec{S} + g_I\vec{I}\right)\cdot\vec{B}$$

perturbation

Breit-Rabi Formula

The Breit-Rabi formula for the Zeeman shift of atomic ground states is given by:

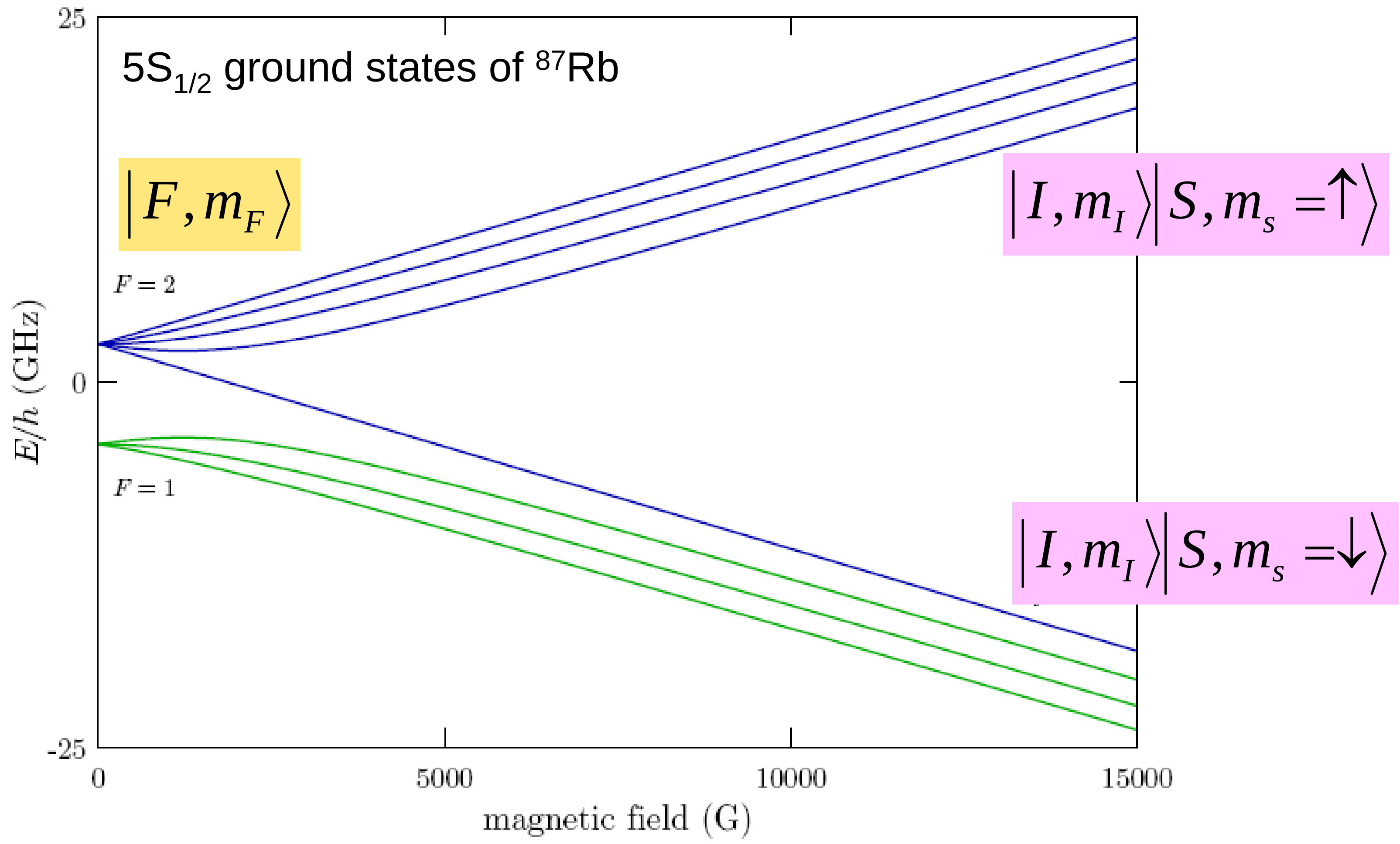
$$U(m_F, B) = g_I \mu_B m_F B + \frac{E_{hfs}}{2}\left(\pm\left(1 + \frac{4 m_F x}{2I+1} + x^2\right)^{1/2} - \frac{1}{2I+1}\right),$$

where the $\pm$ is used for the $F = I \pm J$ state, respectively, and

$$x \equiv \frac{(g_J - g_I)\mu_B B}{E_{hfs}}.$$

PhD Thesis of J. Obrecht, U. of Colorado (2007)]

Zeeman Sub-Structure at High B-field



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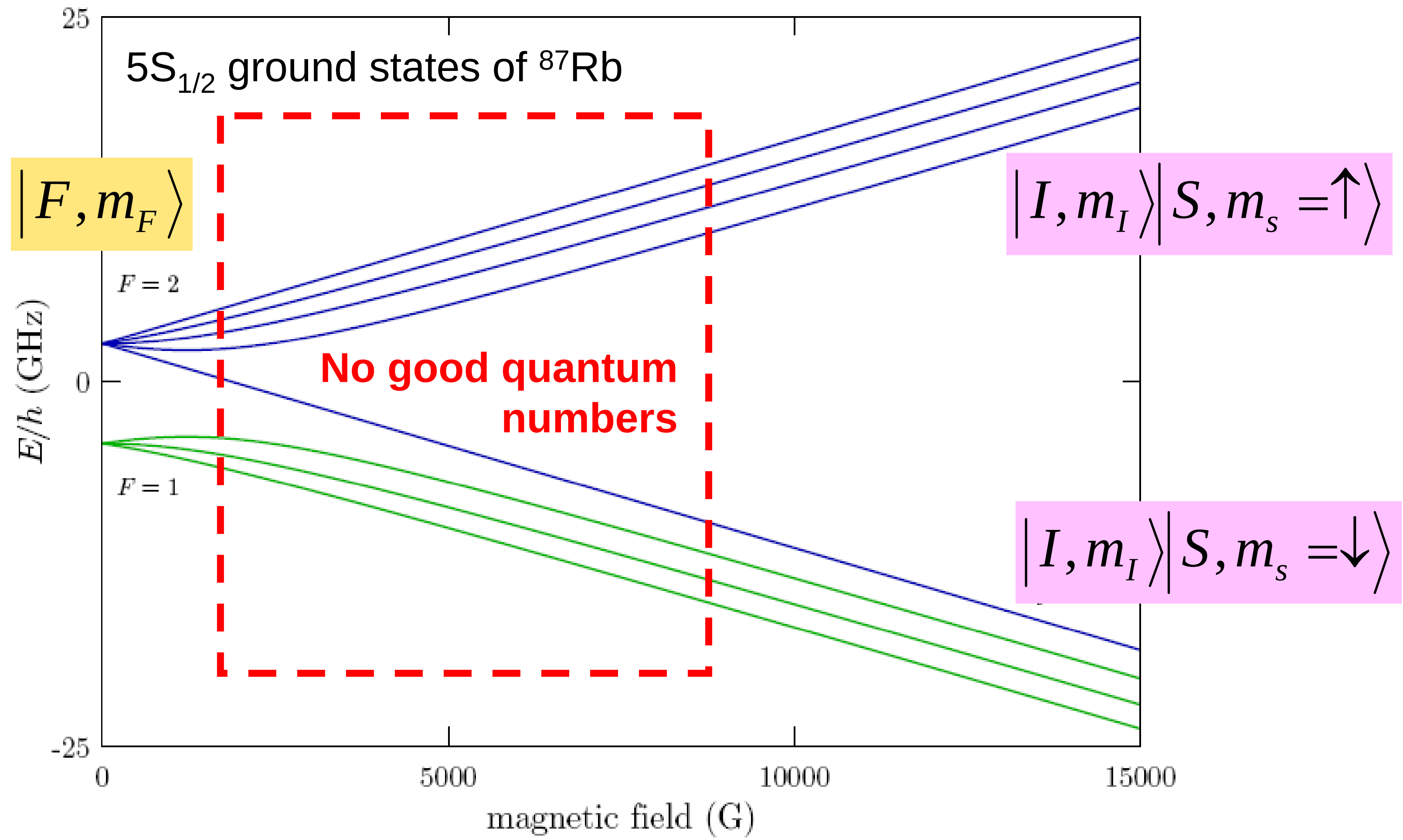
Zeeman Sub-Structure at High B-field



[Figure adapted from steck.us by Prof. Dan Steck, U. of Oregon (2010)]

Clebsch-Gordan Coefficients: I=3/2,S=1/2

F=2 with I=3/2, S=1/2

$$\left|F=2, m_F=+2\right\rangle = \left|I=3/2,\ m_I=+3/2\right\rangle\left|\uparrow\right\rangle$$

$$\left|F=2, m_F=+1\right\rangle = \frac{1}{2}\left|m_I=+3/2\right\rangle\left|\downarrow\right\rangle + \frac{\sqrt{3}}{2}\left|m_I=+1/2\right\rangle\left|\uparrow\right\rangle$$

$$\left|F=2, m_F=0\right\rangle = \frac{1}{\sqrt{2}}\left|m_I=+1/2\right\rangle\left|\downarrow\right\rangle + \frac{1}{\sqrt{2}}\left|m_I=-1/2\right\rangle\left|\uparrow\right\rangle$$

$$\left|F=2, m_F=-1\right\rangle = \frac{1}{2}\left|m_I=-3/2\right\rangle\left|\uparrow\right\rangle + \frac{\sqrt{3}}{2}\left|m_I=-1/2\right\rangle\left|\downarrow\right\rangle$$

$$\left|F=2, m_F=-2\right\rangle = \left|m_I=-3/2\right\rangle\left|\downarrow\right\rangle$$

Clebsch-Gordan Coefficients: I=3/2,S=1/2

F=1 with I=3/2, S=1/2

$$\left|F=1, m_F=+1\right\rangle = \frac{\sqrt{3}}{2}\left|m_I=+3/2\right\rangle\left|\downarrow\right\rangle - \frac{1}{2}\left|m_I=+1/2\right\rangle\left|\uparrow\right\rangle$$

$$\left|F=1, m_F=0\right\rangle = \frac{1}{\sqrt{2}}\left|m_I=+1/2\right\rangle\left|\downarrow\right\rangle - \frac{1}{\sqrt{2}}\left|m_I=-1/2\right\rangle\left|\uparrow\right\rangle$$

$$\left|F=2, m_F=-1\right\rangle = -\frac{\sqrt{3}}{2}\left|m_I=-3/2\right\rangle\left|\uparrow\right\rangle + \frac{1}{2}\left|m_I=-1/2\right\rangle\left|\downarrow\right\rangle$$

General Formula…

$$\left|F_+ = I + S, m_F\right\rangle =$$

$$\frac{\sqrt{F_+ + m_F}}{\sqrt{2I+1}}\left|m_I = m_F - 1/2\right\rangle\left|\uparrow\right\rangle + \frac{\sqrt{F_+ - m_F}}{\sqrt{2I+1}}\left|m_I = m_F + 1/2\right\rangle\left|\downarrow\right\rangle$$

$$\left|F_- = I - S, m_F\right\rangle =$$

$$-\frac{\sqrt{F_+ - m_F}}{\sqrt{2I+1}}\left|m_I = m_F - 1/2\right\rangle\left|\uparrow\right\rangle + \frac{\sqrt{F_+ + m_F}}{\sqrt{2I+1}}\left|m_I = m_F + 1/2\right\rangle\left|\downarrow\right\rangle$$