Week 0: Thursday, January 25, 2024

Classical E-M quantities:

$$\text{Energy} = \frac{1}{2}\int_{\substack{\text{all}\\\text{space}}} \left[\varepsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right] d^3r \overset{?}{=} H$$

$$\text{Momentum} = \vec{P} = \varepsilon_0 \int_{\substack{\text{all}\\\text{space}}} \vec{E}\times\vec{B}\, d^3r$$

$\underbrace{\vec{E} \;\&\; \vec{B}}_{\text{Operators}}$: Wavefunctions or Operators?

- obey the superposition principle
- oscillate, wavelike, spreadout over space
- Measurable quantities $\longrightarrow$ operators

From Relativistic Mechanics, we know

for photons: $E^2 = p^2c^2 + \cancel{m_0^2 c^4}$ $= 0$ for photon

E = energy

$$\Rightarrow \quad E = pc \qquad \left[\text{in E\&M: } \underset{\text{energy density}}{\mathcal{E}} = \underset{\text{momentum density}}{p}c\right]$$

In QM, particles have momentum $\boxed{\vec{p} = \hbar\vec{k}}$ $\qquad k = \frac{2\pi}{\lambda} = 2\pi\frac{f}{c}$ $\quad \lambda f = c$

$$\Rightarrow p = \hbar\frac{2\pi f}{c} = \hbar\frac{\omega}{c}$$

$$\Rightarrow E = pc = \hbar\frac{\omega}{c}\cdot c \Rightarrow \boxed{E = \hbar\omega} \overset{?}{=} H_{photon}$$

Semi-Classical Atom-light interaction

(I) Motivation

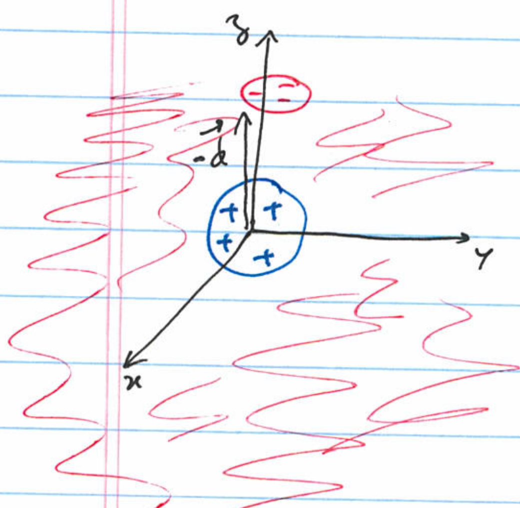
- Classical physics often contains the essential physics and then there are "quantum corrections".

- Later, when we do the quantum treatment, we will be able to distinguish more easily between classical & quantum properties of atom-light interactions

- Easier than full QM treatment.

(II) Model Atom

We consider a "semi-classical atom" with the following properties:



- nucleus $\longrightarrow$ "+" charge $= +e$
  $1\ e^-$ $\longrightarrow$ "−" charge $= -e$

- no preferred orientation of dipole:
  $\vec{d} = -e\,\vec{r}$
  so we will use an isotropic e^- cloud (nucleus is very heavy)
  $\langle \vec{d} \rangle = 0 = -e \langle \vec{r} \rangle$

- Mysteriously, e^- does not spiral into nucleus, but remains at an equilibrium distance $\sqrt{\langle \vec{r}^{\,2} \rangle} = r_0$, but $\langle \vec{r} \rangle = 0$.

- If the e^- is moved away from it's equilibrium position, then there is a linear restoring force:

(at origin on average)

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2} = -k\vec{r}$$

$$\Leftrightarrow \quad \ddot{\vec{r}} + \frac{k}{m}\vec{r} = 0 \quad \Leftrightarrow \quad \ddot{\vec{r}} + \omega^2 \vec{r} = 0$$

with $\omega = \sqrt{\frac{k}{m}}$

(also $\ddot{\vec{d}} + \frac{k}{m}\vec{d} = 0$)

$\Rightarrow$ "simple harmonic atom"

solution: $\vec{r}(t) = \vec{r}_0 \sin(\omega t) + \vec{r}_0{}' \cos(\omega t)$

with $\vec{r}_0$ and $\vec{r}_0{}'$ defined by the initial conditions.

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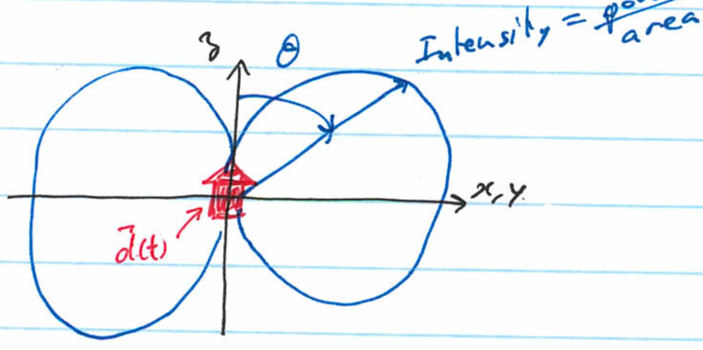
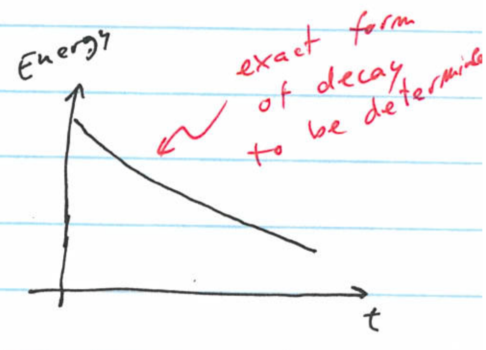
III Radiation Damping

Accelerating charges and oscillating dipoles (i.e. antennas) emit radiation.





$$\boxed{\text{Intensity} = \langle \text{Poynting vector} \rangle = \langle \vec{S} \rangle = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right) = \frac{d_0^2}{32\pi \varepsilon_0} \frac{\omega^4}{c^3} \frac{\sin^2\theta}{r^2} \hat{r}}$$