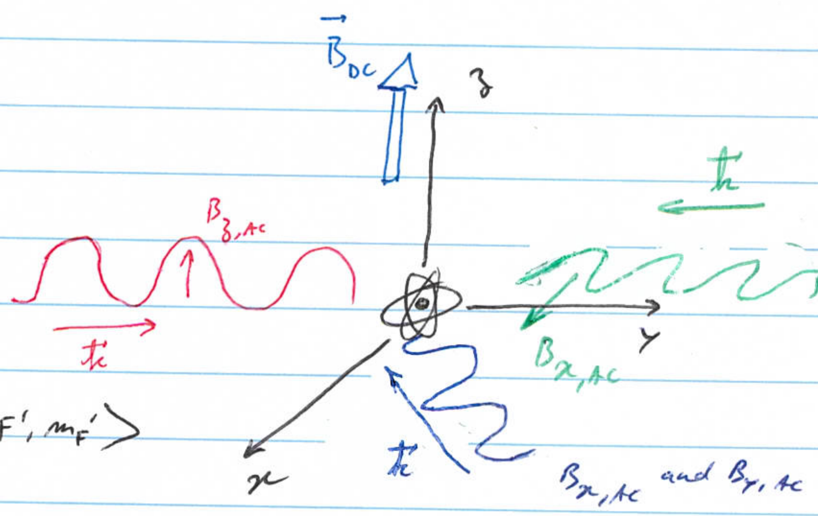
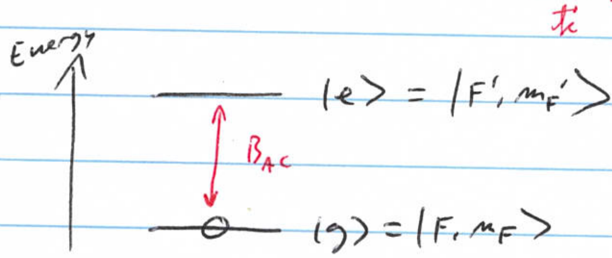
Tuesday, April 9, 2024

Transition Selection Rules

Consider M1 transitions at low DC $\vec{B}_{DC}$-field. These are driven by an oscillating magnetic field $\vec{B}_{AC}$.

$\vec{B}_{DC} = B_{DC}\hat{z}$ defines the quantization axis.





The Rabi frequency is $\boxed{\Omega = \dfrac{\langle e|\, H_{Zeeman,AC}\, |g\rangle}{\hbar}}$

$$\Rightarrow \Omega = \langle F', m_F' |\ \frac{\mu_B}{\hbar^2} g_F\ \vec{F}\cdot\vec{B}_{AC}\ | F, m_F\rangle$$

Case 1: π-polarization : $\vec{B}_{AC} = B_{AC}\hat{z}$

$$\Omega = \frac{\mu_B}{\hbar^2} g_F |\vec{B}_{AC}| \underbrace{\langle F, m_F' |\ F_z\ | F, m_F\rangle}_{\text{Wigner-Eckart theorem assumed } F=F'}$$

$$= \frac{\mu_B}{\hbar} g_F m_F |\vec{B}_{AC}|\ \delta_{FF'}\ \delta_{m_F m_F'}$$

π - transition requires $\Delta m_F = 0 \Rightarrow |g\rangle = |e\rangle$

↳ not really a transition

For $F \neq F'$ ($l = 0$ here)

$$\Omega = \frac{\mu_B}{\hbar^2} g_s |\vec{B}_{AC}| \underbrace{\langle F', m_F' | S_z | F, m_F \rangle}_{\underbrace{cst}\ \delta_{F, F'+1}\ \delta_{m_F m_F'}}$$

↳ how do you calculate the constant?

↳ Use Clebsch-Jordan decomposition

$$\boxed{\Rightarrow \underbrace{\pi\text{-transition}} \text{ requires } \Delta m_F = 0,\ \Delta F = \pm 1 \quad [\vec{B}_{AC} \parallel z\text{-axis or } \vec{B}_{AC} \parallel \vec{B}_{DC}]}$$

<u>Case 2:</u> σ-polarization $\vec{B}_{AC} \perp \vec{B}_{DC}$ (or $\hat{z}$)

$$\Rightarrow \Omega = \frac{\mu_B}{\hbar^2} g_F \langle F, m_F' | F_x B_x + F_y B_y | F, m_F \rangle$$

but $F_x = \dfrac{F_+ + F_-}{2}$ and $F_y = \dfrac{F_+ - F_-}{2i}$

so $$\Omega = \frac{\mu_B}{\hbar^2} g_F \langle F, m_F' | \left(\frac{F_+ + F_-}{2}\right) B_x + \left(\frac{F_+ - F_-}{2i}\right) B_y | F, m_F \rangle$$

$$\Rightarrow \Omega = \frac{\mu_B}{\hbar^2} g_F \langle F, m_F' | F_+ \underbrace{\left(\frac{B_x - iB_y}{2}\right)}_{B_-} + F_- \underbrace{\left(\frac{B_x + iB_y}{2}\right)}_{B_+} | F, m_F \rangle$$

$$\Leftrightarrow \Omega = \frac{\mu_B}{\hbar^2} g_F \langle F, m_F' | F_+ B_- + F_- B_+ | F, m_F \rangle$$

where $\begin{cases} B_{+_{AC}} = \dfrac{B_x + iB_y}{2} = \sigma^+ \text{ polarization} = \text{right circularly polarized} \\ B_{-_{AC}} = \dfrac{B_x - iB_y}{2} = \sigma^- \text{ polarization} = \text{left circularly polarized} \end{cases}$

⚠ Note: definition does not depend on whether wave is going in $+\vec{k}$ or $-\vec{k}$ direction.

Recall that

$$\begin{cases} F_+ |F, m_F\rangle = \hbar\sqrt{F(F+1) - m_F(m_F+1)}\, |F, m_F+1\rangle \\ F_- |F, m_F\rangle = \hbar\sqrt{F(F+1) - m_F(m_F+1)}\, |F, m_F-1\rangle \end{cases}$$

Thus $\Omega \neq 0$ only if $\begin{cases} \Delta m_F = m_F - m_{F'} = +1 & \text{for } B_+ \\ \Delta m_F = m_F - m_{F'} = -1 & \text{for } B_- \end{cases}$

If $F \neq F'$, then $(L=0)$

$$\Omega = \frac{\mu_B g_s}{\hbar^2} \langle F', m_F' | S_+ B_{-_{AC}} + S_- B_{+_{AC}} | F, m_F\rangle$$

then one can have $\Delta F = 0, \pm 1$

<u>Summary for M1 transitions:</u>

$\boxed{\text{For } \pi\text{-polarization} \longrightarrow \Delta F = \pm 1,\ \Delta m_F = 0}$

$\boxed{\text{For } \sigma\text{-polarization} \longrightarrow \Delta F = 0, \pm 1,\ \Delta m_F = \pm 1}$