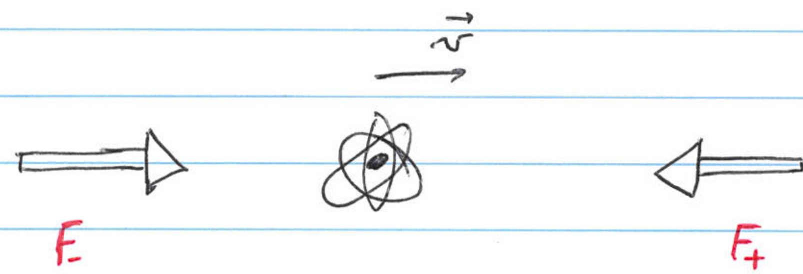
Tuesday, April 16, 2024

Semi-classical model of Doppler Cooling



$$\vec{F} = \frac{d\vec{p}}{dt} = \text{momentum} \times \frac{\text{kicks}}{\text{secs}}$$

$$= \hbar \vec{k}\, \gamma_{\text{scattering}}$$

$$= \hbar k \frac{s_0}{1 + s_0 + \left(\frac{2\delta}{\gamma}\right)^2} \frac{\gamma}{2}$$

1D model: We will do the calculation in the low intensity limit ($s_0 = \frac{I}{I_{sat}} \ll 1$) so that the atoms do not have to choose between the two beams, i.e., saturation is not important.

detuning: $\delta = + \delta_\ell \pm \underbrace{\omega_{\text{Doppler}}}_{\vec{k}\cdot\vec{v}} = + \delta_\ell \pm kv$
($\delta_\ell = \omega_\ell - \omega_a$) laser direction

in 1D: $F_{total} = -F_+ + F_-$

$$= \hbar k \frac{\gamma}{2} s_0 \left\{ \frac{-1}{1 + s_0 + \left[\frac{2(\delta_\ell + kv)}{\gamma}\right]^2} + \frac{1}{1 + s_0 + \left[\frac{2(\delta_\ell - kv)}{\gamma}\right]^2} \right\}$$

low v damping force:

assume $|kv| \ll \delta$

$$F \simeq \frac{\hbar k \gamma s_0}{2}\left\{ \frac{-1}{1+s_0+\frac{4\delta_\ell^2}{\gamma^2}+\frac{8kv\delta_\ell}{\gamma^2}+\cancel{4\left(\frac{kv}{\gamma}\right)^2}} + \frac{1}{1+s_0+\frac{4\delta_\ell^2}{\gamma^2}-\frac{8kv\delta_\ell}{\gamma^2}+\cancel{4\left(\frac{kv}{\gamma}\right)^2}}\right\}$$

neglect

neglect

$$\simeq \frac{\hbar k \gamma s_0}{2}\left\{ \frac{-1}{\left(1+s_0+4\frac{\delta_\ell^2}{\gamma^2}\right)\underbrace{\left(1+\frac{8kv\delta_\ell/\gamma^2}{1+s_0+4\delta_\ell^2/\gamma^2}\right)}_{\frac{1}{1+\varepsilon}\simeq 1-\varepsilon}} + \frac{1}{\left(1+s_0+4\frac{\delta_\ell^2}{\gamma^2}\right)\underbrace{\left(1-\frac{8kv\delta_\ell/\gamma^2}{1+s_0+4\delta_\ell^2/\gamma^2}\right)}_{\frac{1}{1-\varepsilon}\simeq 1+\varepsilon}}\right\}$$

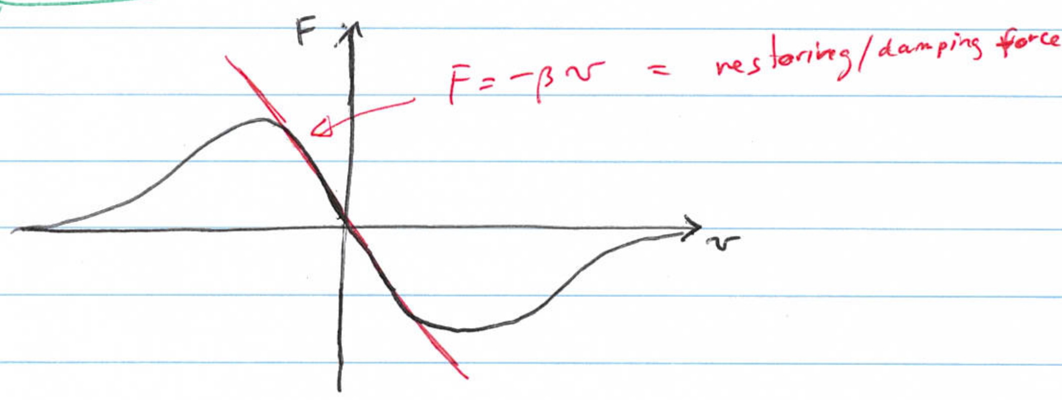
$$\simeq \frac{\hbar k \gamma s_0}{2}\,\frac{1}{1+s_0+4\delta_\ell^2/\gamma^2}\left\{\cancel{-1}+\underbrace{\frac{8kv\delta_\ell/\gamma^2}{1+s_0+4\delta_\ell^2/\gamma^2}}+\cancel{1}+\underbrace{\frac{8kv\delta_\ell/\gamma^2}{1+s_0+4\delta_\ell^2/\gamma^2}}\right\}$$

$$\simeq \frac{\hbar k \gamma s_0}{\left(1+s_0+4\delta_\ell^2/\gamma^2\right)^2}\,\frac{8kv\delta_\ell}{\gamma^2}$$

$$\Longleftrightarrow \quad \boxed{F \simeq \frac{8\hbar k^2 \delta_\ell s_0 v}{\underbrace{\gamma\left(1+s_0+4\delta_\ell^2/\gamma^2\right)}_{-\beta}} = -\beta v}$$

damping coefficient

negative for red detuning ($\omega_\ell < \omega_a \Rightarrow \delta_\ell < 0$)



Doppler Temperature

The final temperature is determined by the equilibrium condition:

cooling rate = heating rate

What's the cooling rate?

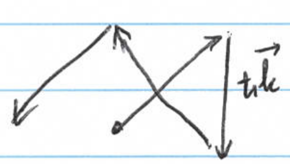
$$\text{Cooling rate} = \frac{dE}{dt} = \text{power out}$$

$$= \vec{F}_{cooling} \cdot \vec{v}$$

$$= \frac{s_0}{\left[1 + s_0 + \left(\frac{2\delta_\ell}{\gamma}\right)^2\right]^2} \; \frac{8\hbar k^2 \delta_\ell}{\gamma} v^2$$

What's the heating rate?

Random direction of absorption (3D) and random fluorescence scattering generates a random walk in momentum each with a step of $\underbrace{\bar{p} = \hbar k}_{\text{absorption or re-emission in 3D}}$



$$\text{heating rate} = \frac{d}{dt}\langle E_{kinetic} \rangle = \frac{d}{dt}\left[\left\langle \frac{P^2(t)}{2m} \right\rangle\right]$$

$$= \frac{1}{2m}\frac{d}{dt}\left\langle \vec{p}_1^{\,2} + \vec{p}_2^{\,2} + \cdots + 2\vec{p}_1\cdot\vec{p}_2 + 2\vec{p}_2\cdot\vec{p}_3 + \cdots \right\rangle_t$$

$$= \frac{1}{2m}\frac{d}{dt}\left\{ \underbrace{\langle \vec{p}_1^{\,2}\rangle_t}_{(\hbar k)^2} + \underbrace{\langle \vec{p}_2^{\,2}\rangle_t}_{(\hbar k)^2} + \cdots + \underbrace{\cancel{2\langle \vec{p}_1\cdot\vec{p}_2\rangle_t} + \cancel{2\langle \vec{p}_2\cdot\vec{p}_3\rangle_t}}_{0 \;+\; \cdots \;0} \right\}$$

no correlation between different scattering event
$\langle \vec{p}_i \cdot \vec{p}_j \rangle_t = 0$
$i \neq j$

$$= \frac{1}{2m}\frac{d}{dt}\left\{ N\,(\hbar k)^2 \right\}$$

$N(t)$

$$= \frac{1}{2m}(\hbar k)^2 \underbrace{\frac{d}{dt}N(t)}_{2\,\times\,\gamma_{scattering}}$$

$1+1 =$ absorption + spontaneous emission

$$\Rightarrow \text{heating rate} = \frac{1}{2m}\,2\,(\hbar k)^2\,\underbrace{\gamma_{scattering}}_{2\,\times\,\frac{s_0 \;\; \gamma/2}{1+s_0+\left(\frac{2\delta_l}{\gamma}\right)^2}}$$

2 laser beams counterpropagating

for $\langle v \rangle = 0$ no Doppler shift on average (neglect)

<u>At equilibrium:</u> heating rate = cooling rate

$$\frac{2^{\hbar}\,\cancel{(\hbar k)^2}}{2m}\;\frac{\cancel{s_0}\;\;\gamma}{\cancel{1+s_0+\left(\frac{2\delta_l}{\gamma}\right)^2}} = \frac{\cancel{s_0}}{\left[1+s_0+\left(\frac{2\delta_l}{\gamma}\right)^2\right]^{\cancel{2}}}\;\frac{8\hbar k^2}{\gamma}\,\delta_l\, v^2$$

$$\Leftrightarrow \quad \frac{2\hbar\gamma}{2m} = \frac{8\,\delta_\ell\, v^2}{\gamma\left[1+\cancel{s_0}+\left(\frac{2\delta_\ell}{\gamma}\right)^2\right]}$$

$\simeq 0$ (neglect)
for low intensity limit

$$\Leftrightarrow \quad \underbrace{\frac{\hbar\gamma}{2\cdot 8}\,\frac{\gamma}{\delta_\ell}\left[1+\left(\frac{2\,\delta_\ell}{\gamma}\right)^2\right]} = \frac{1}{2} m v^2 = \langle E_{kinetic}\rangle$$

This function has a minimum for $\delta_\ell = -\gamma/2$

$$\Rightarrow \quad \frac{\hbar\gamma}{\cancel{16}8}\,\frac{\cancel{\gamma}}{\cancel{\gamma}/\cancel{2}}\left(1+\left(\frac{\cancel{2}\,\cancel{\gamma}/\cancel{2}}{\cancel{\gamma}}\right)^2\right) = \underbrace{\langle E_{kinetic}\rangle}_{=\frac{1}{2}kT}$$

$$\Leftrightarrow \quad \frac{\hbar\gamma}{4} = \frac{1}{2}kT$$

$$\Rightarrow \quad T_{minimum} = \frac{\hbar\gamma}{2k} = \text{Doppler cooling limit}$$

or

Doppler Temperature

<u>Ex:</u> $T \simeq 180\ \mu K$ for ^{87}Rb

with $\hbar = 1.054 \times 10^{-34}$ J.s

$\gamma = 2\pi \times 6$ MHz

$k = 1.38 \times 10^{-23}$ J/K