Tuesday, April 23, 2024

I Magnetic traps (continued)

$$H_{Zeeman} = -\vec{\mu}\cdot\vec{B} = -|\vec{\mu}||\vec{B}|\underbrace{\cos\theta}_{\substack{\text{constant}\\ \text{for adiabatic motion}}}$$

classical expression

(typically satisfied for ultracold atoms)

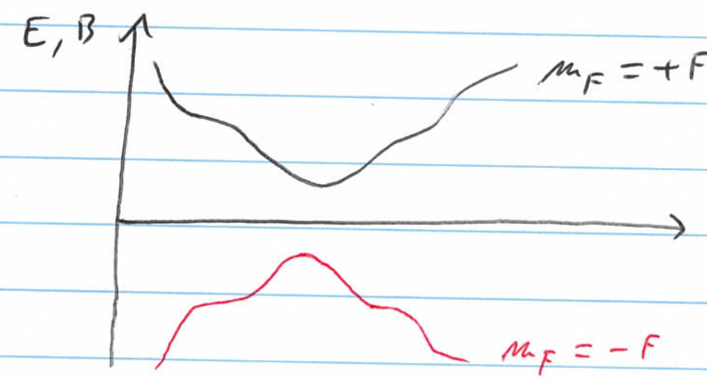
$$= m_F g_F \mu_B |\vec{B}|$$

quantum expression [for adiabatic motion]

with $\mu_B = h \times 1.4\ \mathrm{MHz/G}$

$\Rightarrow$ $\boxed{\text{Magnetic potential energy is proportional to } |\vec{B}|}$

<u>note</u>: If $m_F = +F$ is trapped, then $m_F = -F$ is anti-trapped



II Earnshaw's Theorem for magnetic fields

[electrostatic version proved in 1842, magnetostatic case proved and rediscovered in 1980s]

There are no static magnetic field maxima in current free regions of space.

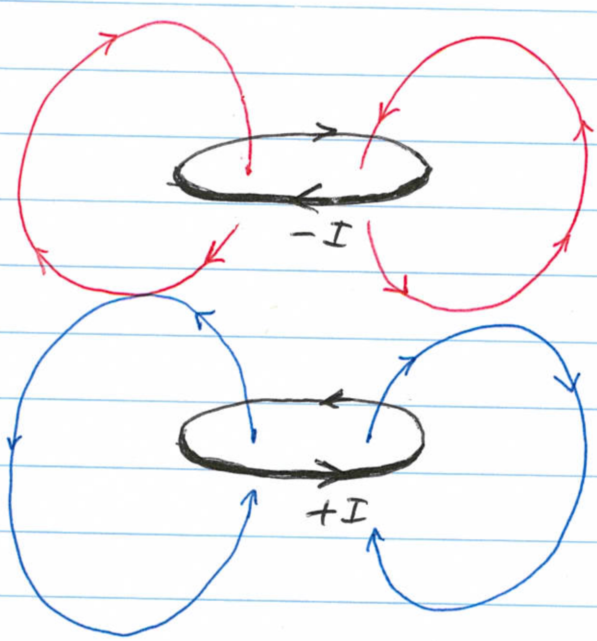
[Magnetic field minima are possible]

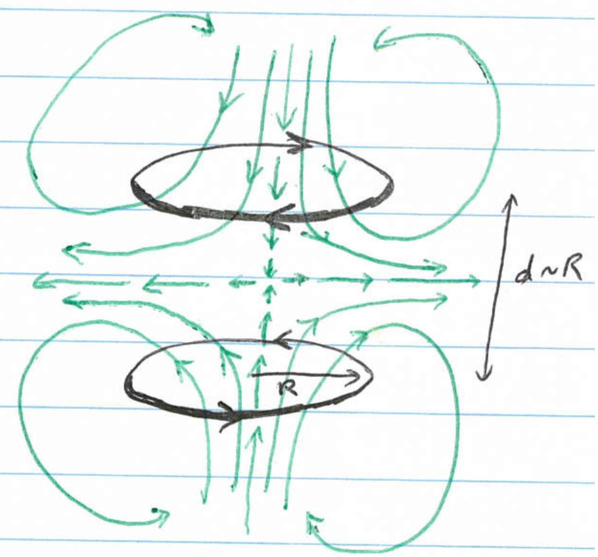
<u>Corollary</u>: states with $m_F < 0$ (i.e. $m_s = \downarrow$) <u>cannot</u> be magnetically trapped.

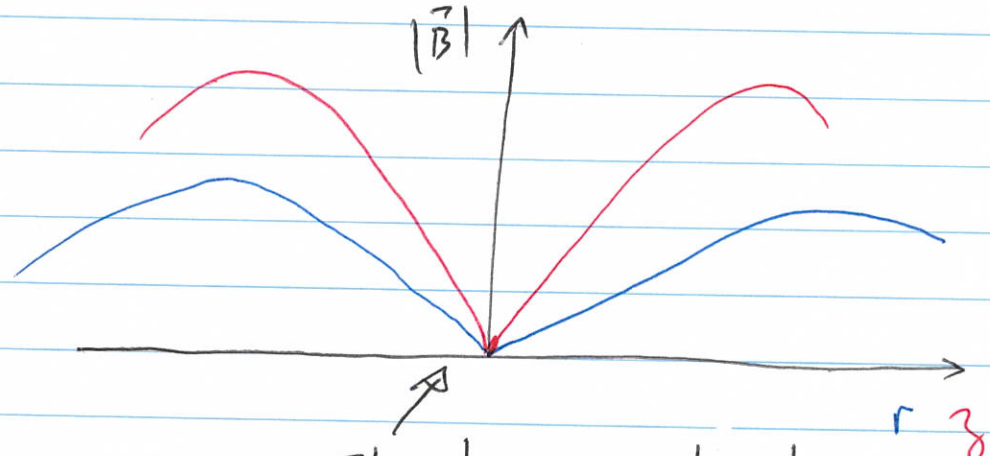
<u>Take home message</u>:

states with $m_F > 0$ are low magnetic field seekers.
states with $m_F < 0$ are high magnetic field seekers.

III <u>Quadrupole Trap</u> (anti-Helmholtz coil trap)







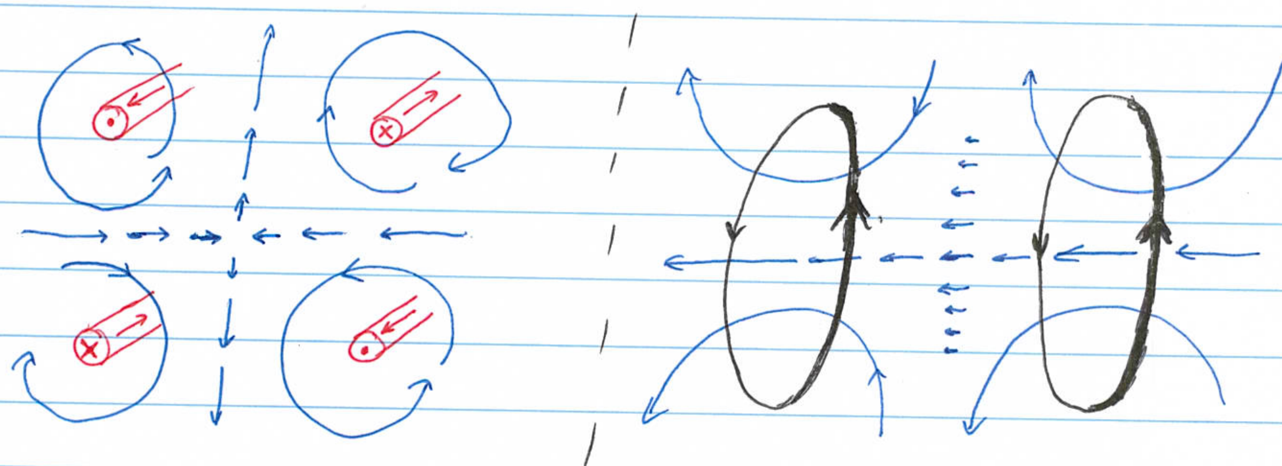
$$\text{slope}\big|_z = 2\,\text{slope}\big|_r$$

Note: the adiabaticity condition $\left[\frac{H_{Zeeman}}{\hbar} \gg \frac{1}{B}\left(\frac{dB}{dt}\right)\right]$ cannot be satisfied at the very bottom of the quadrupole trap, so typically you get a spin-flip as atom passes through the trap bottom $|\uparrow\rangle \longrightarrow |\downarrow\rangle$. The $|\downarrow\rangle$ state is anti-trapped and ejected (i.e. trap loss).

$\hookrightarrow$ Not a major problem for a 3D trap, but this loss becomes more appreciable ("Majorana loss") as atoms congregate at the bottom of trap when they are cooled down.

(IV) Ioffe-Pritchard Trap (bottom of trap is at a finite $|\vec{B}|$)

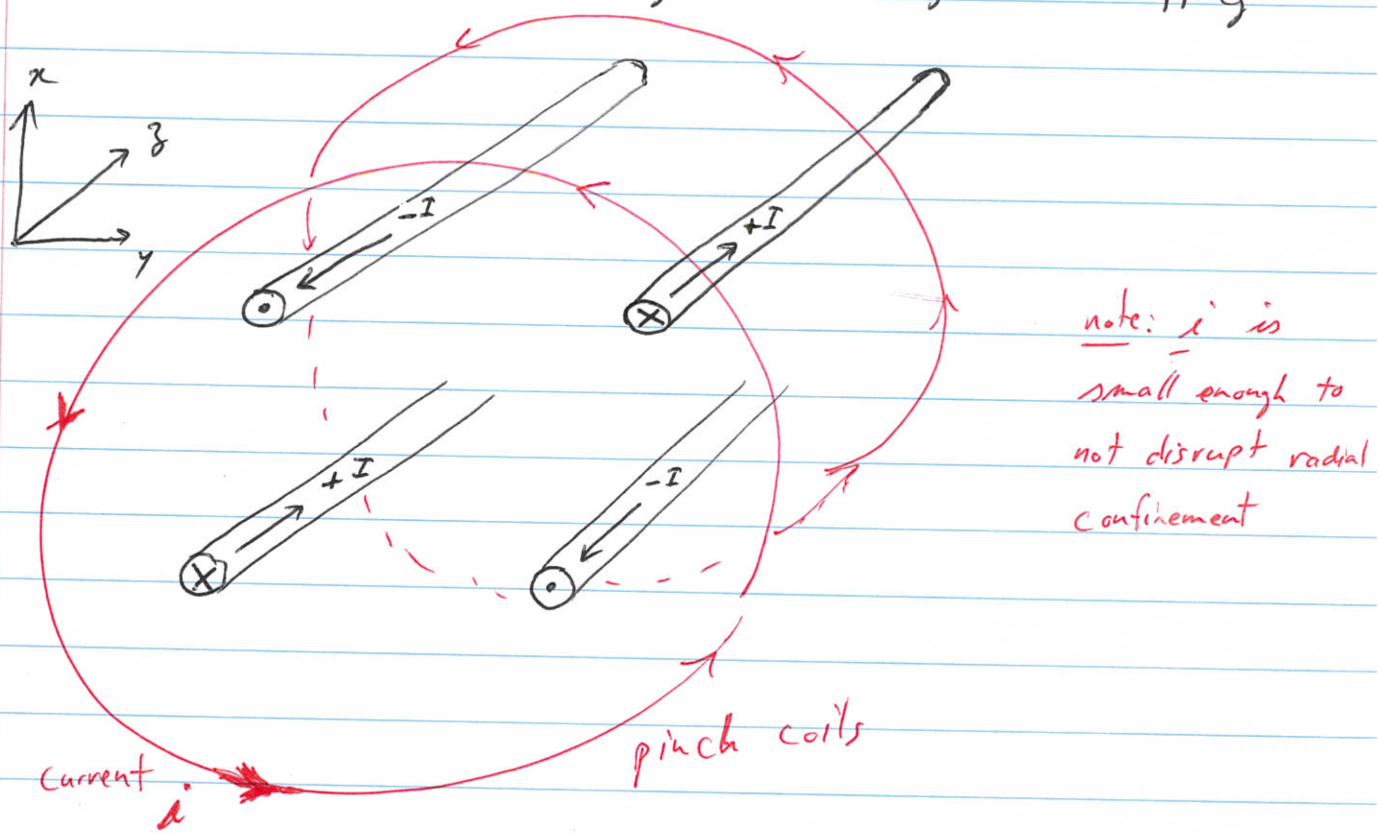
Consider the current arrangements



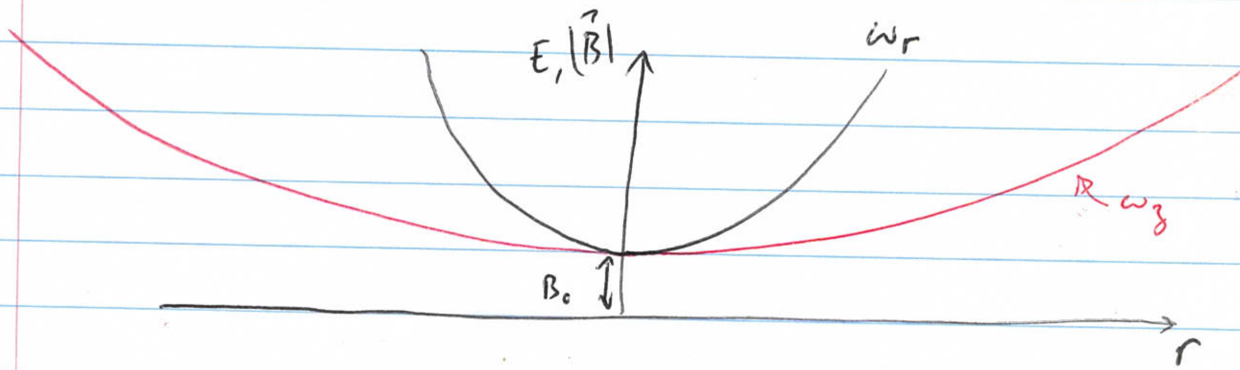
$\Rightarrow$ local minimum in plane,

local minimum along main axis, but not along radial directions. (saddle point)

Combine both current configurations to get 3D trapping



The Ioffe-Pritchard potential is harmonic in all 3 dimensions



For $\omega_z \ll \omega_r \Rightarrow$ cigar shaped trap.

$$H_{I\text{-}P} = \frac{1}{2} m \omega_r^2 r^2 + \frac{1}{2} m \omega_z^2 z^2 = V_{\text{Ioffe-pritchard}}$$

why is the radial trapping potential harmonic?

$$|\vec{B}_{total}| = \sqrt{B_z^2 + B_r^2} = \sqrt{B_z + (\alpha r)^2} \quad \text{with } \alpha = \frac{d}{dr} B_r$$

$$= |B_z| \sqrt{1 + \frac{(\alpha r)^2}{B_z^2}}$$

Taylor expand around $r = 0$

$$\simeq |B_z| \left(1 + \frac{1}{2} \frac{(\alpha r)^2}{B_z^2} + \cdots \right)$$

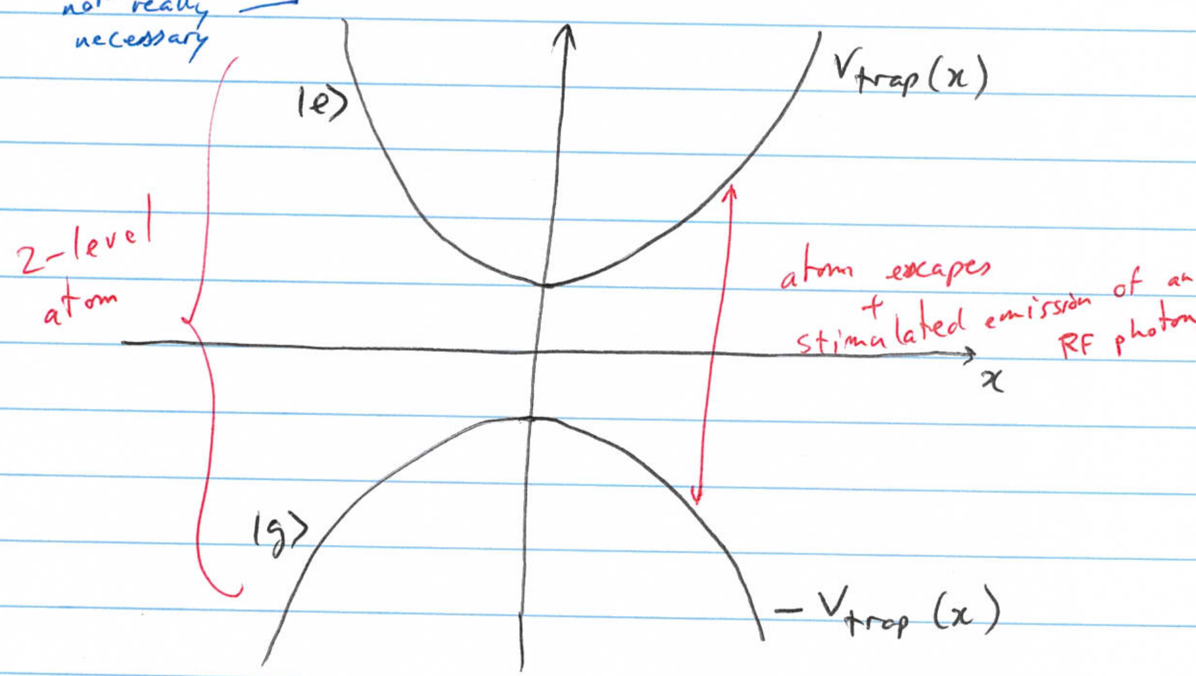
$$\simeq |B_z| + \frac{1}{2} \frac{\alpha^2 r^2}{|B_z|} + \cdots$$

B_0 = magnetic field at bottom of the trap

Dressed atom picture of evaporation (1D, spin-1/2)

Assume a harmonic trapping potential : $V_{trap}(x)$

not really necessary



$$H = \begin{array}{c c} & \begin{array}{cc} |g\rangle & |e\rangle \end{array} \\ \begin{array}{c} \langle g| \\ \langle e| \end{array} & \begin{pmatrix} -V_{trap}(x) & 0 \\ 0 & +V_{trap}(x) \end{pmatrix} \end{array} + \underbrace{\hbar\omega_{rf}\begin{pmatrix} N+1 & 0 \\ 0 & N \end{pmatrix}}_{\text{reset energy offset: } \hbar\omega_{rf}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}} + \hbar\begin{pmatrix} 0 & \Omega/2 \\ \Omega^*/2 & 0 \end{pmatrix}$$

$$\Rightarrow H = \begin{pmatrix} \hbar\omega_{rf} - V_{trap}(x) & \hbar\Omega/2 \\ \hbar\Omega^*/2 & +V_{trap}(x) \end{pmatrix}$$

let's calculate the eigenenergies: $\det(H - \lambda I) = 0$

$$\Rightarrow \begin{vmatrix} \hbar\omega_{rf} - V_{trap}(x) - \lambda & \hbar\Omega/2 \\ \hbar\Omega^*/2 & V_{trap}(x) - \lambda \end{vmatrix} = 0$$

$$\Leftrightarrow \left(\hbar\omega_{rf} - V_{trap}(x) - \lambda\right)\left(V_{trap}(x) - \lambda\right) - \frac{\hbar^2|\Omega|^2}{4} = 0$$

$$\Leftrightarrow \lambda^2 - \left(\hbar\omega_{rf} - \cancel{V_{trap}(x)} + \cancel{V_{trap}(x)}\right)\lambda + \left(\hbar\omega_{rf} - V_{trap}(x)\right)V_{trap}(x) - \frac{\hbar^2|\Omega|^2}{4} = 0$$

$$\Leftrightarrow \lambda^2 - \hbar\omega_{rf}\lambda + \left(\hbar\omega_{rf} - V_{trap}\right)V_{trap} - \frac{\hbar^2|\Omega|^2}{4} = 0$$

thus $E_\pm = \lambda_\pm = \dfrac{\hbar\omega_{rf}}{2} \pm \dfrac{1}{2}\sqrt{\underbrace{\hbar^2\omega_{rf}^2 - 4\left(\hbar\omega_{rf} - V_{trap}\right)V_{trap}}_{\hbar^2\omega_{rf}^2 - 4\hbar\omega_{rf}V_{trap} + 4V_{trap}^2 = \underbrace{\left(\hbar\omega_{rf} - 2V_{trap}\right)^2}_{\text{"detuning"}}} + \hbar^2|\Omega|^2}$

$$\Rightarrow \boxed{E_\pm = \frac{\hbar\omega_{rf}}{2} \pm \frac{1}{2}\sqrt{\left(\hbar\omega_{rf} - 2V_{trap}(x)\right)^2 + \hbar^2|\Omega|^2}}$$