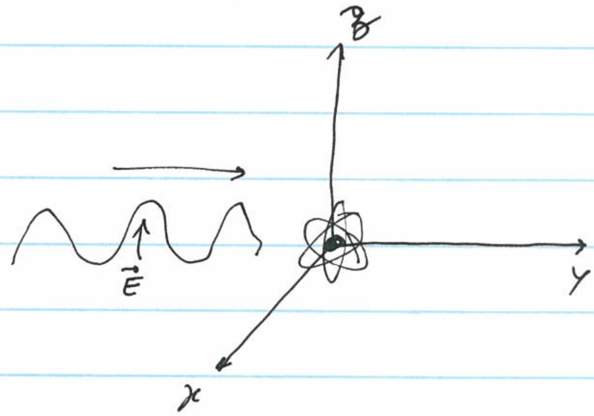
Tuesday, February 6, 2024

Electromagnetically Driven Atom

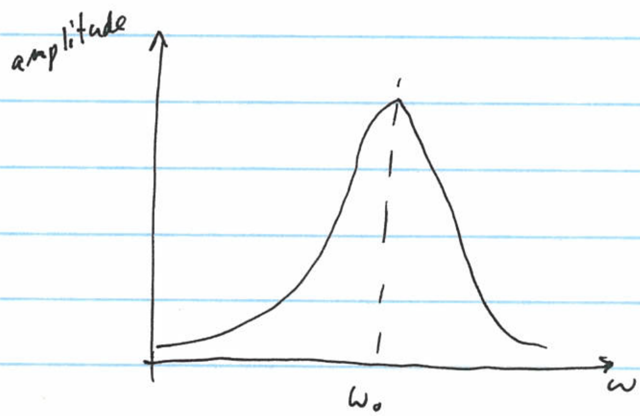


equation of motion for atom's electron is

$$\ddot{z} + 2\gamma\dot{z} + \omega_0^2 z = -\frac{e}{m} E_0 \cos(\omega t)$$

The solution is:

$$\boxed{z(t) = \underbrace{\frac{\left(-e/m\right) E_0}{\sqrt{\left(\omega_0^2-\omega^2\right)^2 + 4\omega^2\gamma^2}}}_{\text{amplitude of oscillation}} \cos(\omega t - \phi) \quad \text{with } \phi = \arctan\left(\frac{2\omega\gamma}{\omega_0^2-\omega^2}\right)}$$



Let's look at the radiated power, i.e. scattered {photons / light}

dipole radiation result

$$\langle \text{power} \rangle = \left\langle \frac{d}{dt} E_{Ho} \right\rangle = \frac{1}{4\pi\varepsilon_0} \frac{e^2 \omega^4}{3c^3} z_0^2$$

z_0 ← amplitude of $z(t)$

$$= \frac{1}{4\pi\varepsilon_0} \frac{e^2}{3c^3} \frac{e^2 E_0^2}{m^2} \underbrace{\frac{\omega^4}{(\omega_0^2-\omega^2)^2 + 4\omega^2\gamma^2}}_{\text{complicated function}}$$

↳ let's simplify it

Let's look at the near resonance behavior $\omega \simeq \omega_0$

↳ define the detuning as $\delta = \omega - \omega_0 \Rightarrow \omega = \omega_0 + \delta$

↳ Add an assumption : $\omega \simeq \omega_0 \gg \delta$

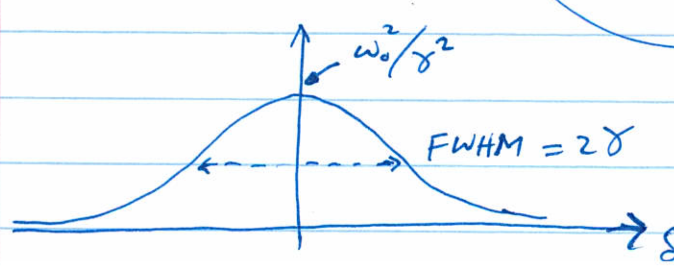
ω_0: 10^{14} Hz, δ: 10^9 Hz

$$\langle \text{power} \rangle_{\text{radiated}} = \frac{E_0^2}{4\pi\varepsilon_0} \frac{e^4}{3m^2c^3} \frac{\cancel{\omega^4} \simeq \omega_0^4}{(\cancel{\omega_0^2} - \cancel{\omega_0^2} - \underbrace{\cancel{\delta^2}}_{\text{neglect (extra small)}} - 2\delta\omega_0)^2 + 4\underbrace{\cancel{\omega^2}}_{\simeq \omega_0^2}\gamma^2}$$

$$\simeq \frac{E_0^2}{4\pi\varepsilon_0} \frac{e^4}{3m^2c^3} \frac{\omega_0^{\cancel{4}\,2}}{4\delta^2 \cancel{\omega_0^2} + 4\gamma^2 \cancel{\omega_0^2}}$$

$$\simeq \frac{E_0^2}{16\pi\varepsilon_0} \frac{e^4}{3m^2c^3} \frac{\omega_0^2}{\delta^2 + \gamma^2}$$

Lorentzian function



"standard" atomic lineshape

Summary of classical Atom

Pro:

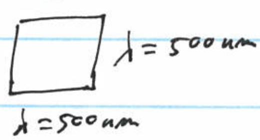
- Simple
- Has the correct lineshape: Lorentzian
- Radiative damping factor (i.e. linewidth = FWHM) is roughly correct (to within a factor of ~2)
- Classical, on-resonance, total scattering cross-section is exactly correct:

$$\sigma_T = \frac{\text{total scattered Power (on resonance)}}{\underbrace{\text{Intensity}}}$$

$\hookrightarrow I = \langle S \rangle = \langle \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \rangle$

$= \frac{1}{2} c \, \varepsilon_0 \, E_0^2$

$$\Rightarrow \boxed{\sigma_{\substack{\text{classical}\\ \text{on-resonance}}} = \frac{3}{2\pi} \lambda^2}$$

$\simeq \frac{1}{2}$ 

note: atom diameter $\simeq 1$ Å

- scattered light has the same optical frequency as the incident light ⟶ true at low intensities

Con:

- Scattering rate can become arbitrarily high ... even at high intensities $\langle P \rangle_{\text{scattered}} \propto \underbrace{E_0^2}_{\hookrightarrow \text{intensity}}$

  ↳ Experiments observe a saturation of $P_{\text{scattered}}$
- Scattered light always has the same optical frequency as incident light. ⟶ not true at high intensity!
- Linewidth is independent of incident intensity.

  ↳ Experimentally, the linewidth becomes broader at high intensity.

Rayleigh Scattering

Let's consider the case of very far off-resonance scattering: $\omega \ll \omega_0$ (i.e. $\delta \sim \omega_0$)

In this case:

$$\langle P_{scattered} \rangle = \frac{1}{4\pi\varepsilon_0} \frac{e^4}{3m^2c^3} E_0^2 \frac{\omega^4}{(\omega_0 - \underbrace{\omega^2}_{\text{small}})^2 + 4\underbrace{\omega^2}_{\text{small}}\gamma^2}$$

$$\simeq \frac{1}{4\pi\varepsilon_0} \frac{e^4}{3m^2c^3} E_0^2 \frac{\omega^4}{\omega_0^4}$$

ω^4 scaling or $\frac{1}{\lambda^4}$ scaling

typically in the UV for air molecules

↳ Higher frequencies scatter much more than lower frequencies

red: $\lambda_R = 650$ nm
blue: $\lambda_B = 475$ nm

$$\frac{P_{scattered, blue}}{P_{scattered, red}} = \left(\frac{\lambda_R}{\lambda_B}\right)^4 = (1.37)^4 = 3.5$$

explains why sky is blue!

multiple scatters in sky
↳ effect is compounded

(I) What's coherence?

Two or more things are said to be coherent if there exists a definite phase relationship between them, so that some form of interference effect can be observed.

ex: 1- Optics: light in two arms of an interferometer.

2- Quantum mechanics: Any superposition of two states

$$|\psi\rangle = \alpha|a\rangle + \beta|b\rangle$$
$$= |\alpha|\,|a\rangle + e^{i\phi}|\beta|\,|b\rangle$$

Multiple particles

$$|\psi\rangle = \left(|a\rangle_1 + e^{i\phi_1}|b\rangle_1\right)\left(|a\rangle_2 + e^{i\phi_2}|b\rangle_2\right)\cdots\left(|a\rangle_n + e^{i\phi_n}|b\rangle_n\right)$$

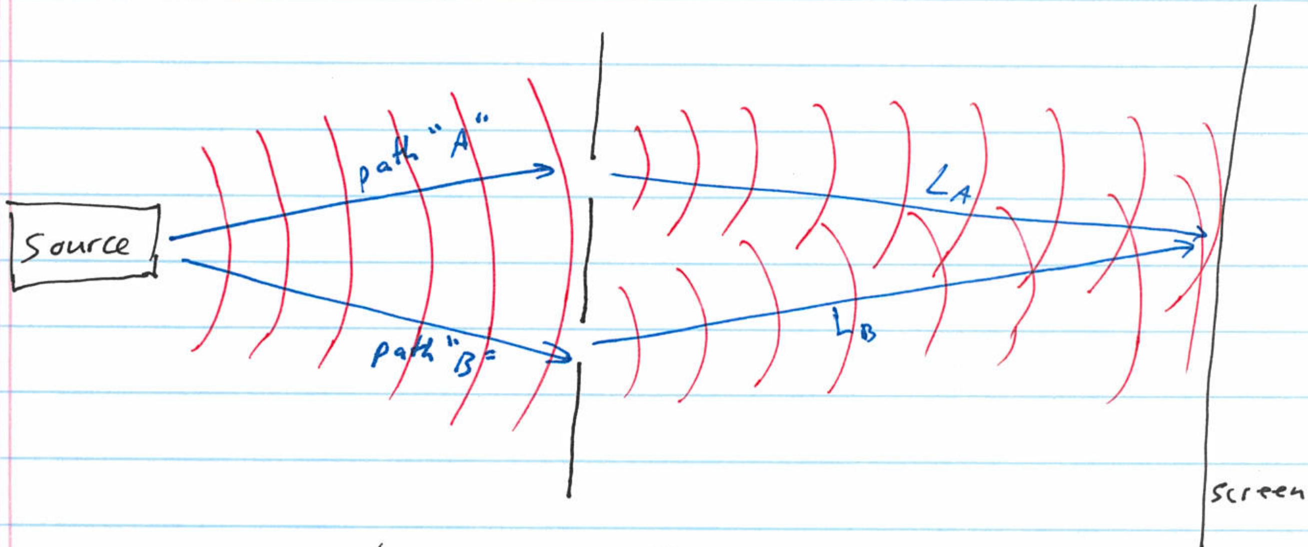
coherent: $\phi_1 = \phi_2 = \ldots = \phi_n$

incoherent: $\phi_1 \neq \phi_2 = \ldots \neq \phi_n$

(II) Spatial Coherence

Different points of an optical wavefront share a definite, fixed phase relationship.

Young's Double slit experiment probes spatial coherence:



phase accumulated along path $A = \phi_A = 2\pi \dfrac{L_A}{\lambda} = 2\pi [L_A \text{ mod.}$

phase accumulated along path $B = \phi_B = 2\pi \dfrac{L_B}{\lambda} = 2\pi [L_B \text{ mod } \lambda]$

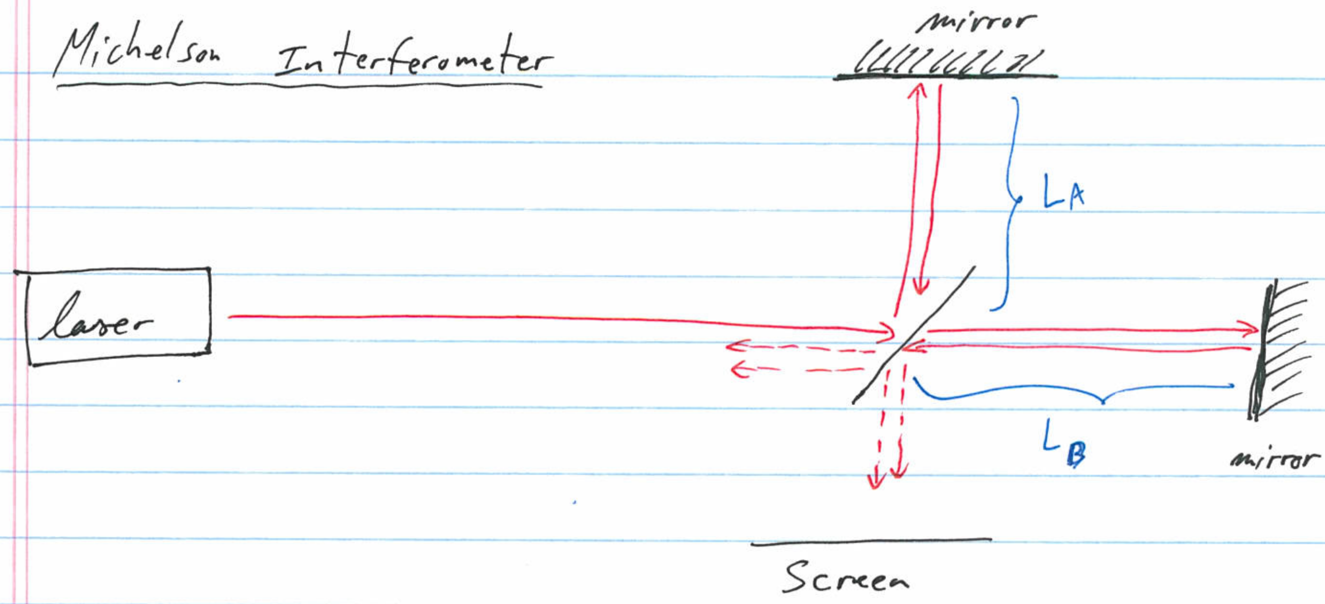
$$\text{If } \begin{cases} \phi_A - \phi_B \simeq 0 \pm n2\pi & \Rightarrow \text{ constructive interference} \\ \phi_A - \phi_B \simeq \pi \pm n2\pi & \Rightarrow \text{ destructive interference} \end{cases}$$

Q: How did scientists do Young's double slit experiment without a laser?
[Thomas Young: 1773-1829]

If ϕ_A or ϕ_B has some phase jitter on it (maybe due to a local air density fluctuation), then $\phi_A - \phi_B$ will vary in time.
↳ interference can wash out

Spatial coherence is straightforward to achieve/observe with laser, radio frequency, and thermal sources over short distances (mm → m).

Michelson Interferometer



In class demonstration : 1- show interference pattern

2- Touch one of the mirrors to vary phase in one arm.

Question: How did Michelson perform the interferometer experiment without a laser?

$$\Delta\phi = \phi_A - \phi_B = 2\pi\left[2(L_A \bmod \lambda) - 2(L_B \bmod \lambda)\right]$$

$$= 2\pi\left[\,2(L_A - L_B) \bmod \lambda\right]$$

In class demonstration : 3- Show coherence length