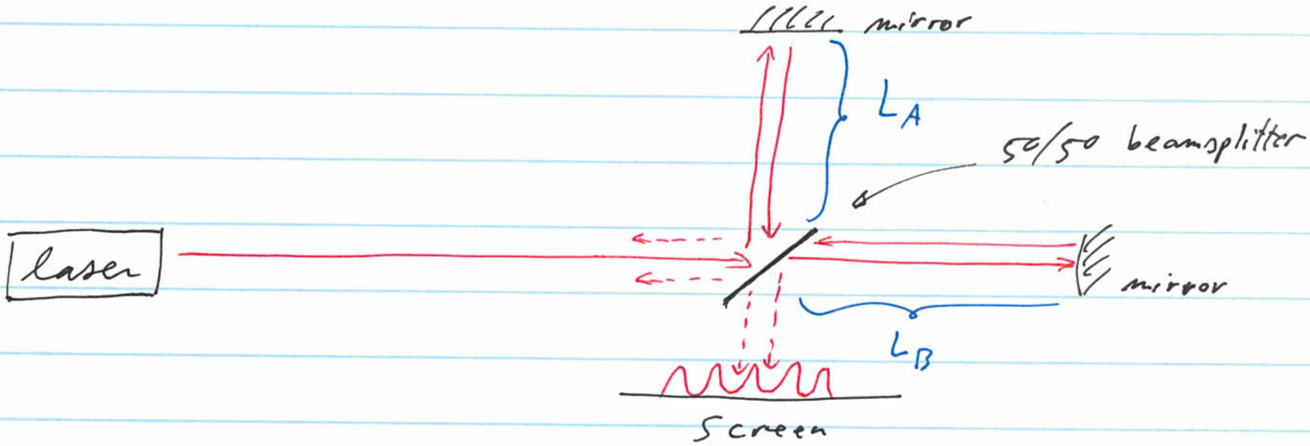
Thursday, February 8, 2024

Review of the Michelson interferometer experiment



$$\Delta\phi = \phi_A - \phi_B = 2\pi \left[2(L_A - L_B) \bmod \lambda \right]$$

$$\begin{cases} \Delta\phi = 0 \Rightarrow \text{constructive interference} \\ \Delta\phi = \pi \Rightarrow \text{destructive interference} \end{cases}$$

Experimental Fact 1: For $\underbrace{L_A \simeq L_B}_{\text{i.e. roughly equal arms}}$, the fringe visibility is near 100%

Experimental Fact 2: For $\underbrace{|L_A - L_B| \gg c}_{\text{i.e unbalanced arms}}$, the fringe visibility goes to zero.

Two Explanations

1 - Bandwidth

Consider 2 optical frequencies f_1 & f_2 (λ_1, λ_2)

$$\Delta\phi_1 = 2\pi \Delta L/\lambda_1 = 2\pi \Delta L f_1/c$$
$$\Delta\phi_2 = 2\pi \Delta L/\lambda_2 = 2\pi \Delta L f_2/c$$

$$\lambda f = c$$
$$\frac{1}{\lambda} = f/c$$

If $\Delta\phi_1$ & $\Delta\phi_2$ differ by π, then the interference fringes of f_1 & f_2 anti-overlap $\rightarrow$ overall interference will washout

$$\Delta\phi = \Delta\phi_1 - \Delta\phi_2 = 2\pi \frac{\Delta L}{c} (f_1 - f_2)$$

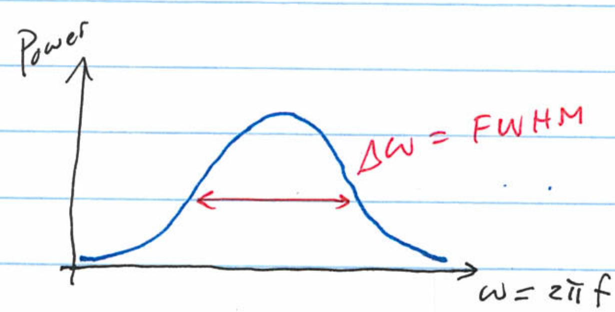
<u>Example:</u> $\Delta\phi = \pi = 2\pi \frac{\Delta L}{c} \Delta f$

$$\Delta f = 1 \text{ GHz} \Rightarrow \Delta L = 0.15 \text{ m} = 15 \text{ cm}$$

$$\Delta f = 1 \text{ MHz} \Rightarrow \Delta L = 150 \text{ m}$$

$$\Delta f = \underbrace{1000 \text{ GHz}}_{\simeq 1 \text{ nm} \quad @ \, 780 \text{ nm}} = 1 \text{ THz} \Rightarrow \Delta L = 0.15 \text{ mm} = 150 \,\mu\text{m}$$

Coherence Length $= L_c =$ characteristic difference in path length over ~~which~~ which interference washes out.

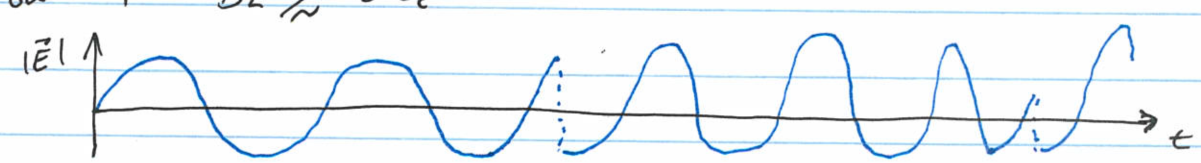


$$\boxed{L_c \simeq \frac{c}{\Delta W}}$$

(depends on the exact nature of the source)

2_ Dephasing time or coherence time

If the phase of the laser jumps (or wanders) (resets itself) on a characteristic time scale τ_c, then the interference washes out for $\Delta L \gtrsim c\tau_c$



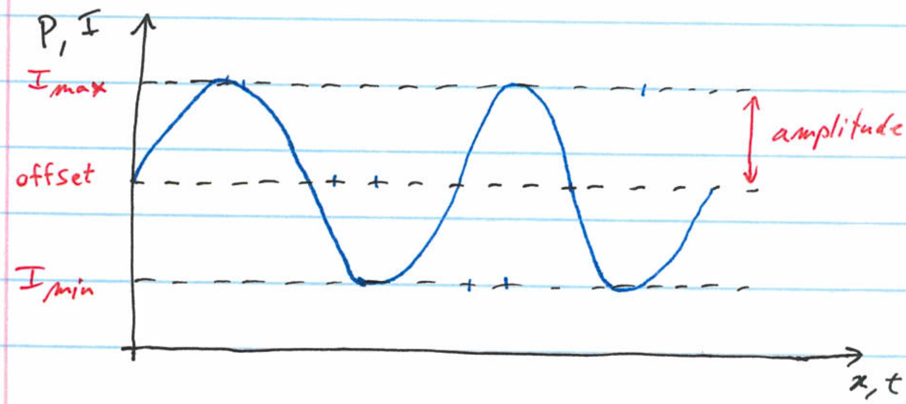
Which explanation is correct?

How could you design an experiment to distinguish the two?

In fact, both explanations are equivalent, since in Fourier space (frequency space), phase jumps indicate that the frequency is not that well determined: In other words, the source has a certain bandwidth.

$$\boxed{L_c = c\,\tau_c}$$

Fringe visibility and $g^{(1)}(\tau)$



$$\text{Fringe visibility} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\text{Amplitude of fringe}}{\text{offset of fringe}}$$

Fringe visibility = degree of first order coherence

$$= \begin{cases} = 0 & \rightarrow \text{no coherence} \\ = 1 & \rightarrow \text{perfect coherence} \end{cases}$$

1^{st} order correlation function:

treat $\vec{E}$-field as a scalar for simplicity

Consider the electric field due to light: $E(t) = E_0 e^{-i\omega t + i\phi(t)}$

(fixed position)

Definition: $$g^{(1)}(\tau) = \frac{\langle E^*(t)\, E(t+\tau)\rangle_{time}}{\langle E^*(t)\, E(t)\rangle_{time}}$$

$$= \frac{\langle \cancel{E_0} e^{+i(\cancel{\omega t} - \phi(t))} \cancel{E_0} e^{-i(\omega(\cancel{t}+\tau) - \phi(t+\tau))}\rangle}{\langle \cancel{E_0}\, e^{\cancel{+i(\omega t - \phi(t))}} \cancel{E_0} e^{\cancel{-i(\omega t - \phi(t))}}\rangle}$$

$$= \underbrace{e^{-i\omega\tau}} \langle e^{i(\phi(t+\tau) - \phi(t))}\rangle_t$$

Often, this term is left out since it oscillates too quickly to be seen

Theorem: $|g^{(1)}(\tau)|$ = fringe visibility [for a 50/50 interferometer]

Corollary: $g^{(1)}(\tau)$ = degree of 1^{st} order coherence

proof: Interferometer output $= I = \langle |E(t) + E(t+\tau)|^2\rangle_t$

$I_0 = E_0 E_0^*$

path A path B

$$\Rightarrow I = \langle |E(t)|^2\rangle + \langle |E(t+\tau)|^2\rangle + \langle E(t) E^*(t+\tau)\rangle + \langle E^*(t) E(t+\tau)\rangle$$

$$= I_0 + I_0 + I_0 \langle e^{-i\cancel{\omega t} + i\phi(t)}\, e^{+i\omega(\cancel{t}+\tau) - i\phi(t+\tau)}\rangle$$

$$+ I_0 \langle e^{i\cancel{\omega t} - i\phi(t)}\, e^{-i\omega(\cancel{t}+\tau) + i\phi(t+\tau)}\rangle$$

$$= I_0 \Big(2 + e^{i\omega\tau} \overbrace{\left\langle e^{-i(\phi(t+\tau) - \phi(t))} \right\rangle_t}^{Ae^{-i\alpha}}$$

$$+ e^{-i\omega\tau} \underbrace{\left\langle e^{i(\phi(t+\tau) - \phi(t))} \right\rangle_t}_{Ae^{i\alpha}}$$

$|g^{(1)}(\tau)| \rightarrow A$

$$= 2 I_0 \left(1 + |g^{(1)}(\tau)| \cos(\omega\tau - \alpha) \right)$$

Note 1: $|g^{(1)}(\tau)| = 1 \Rightarrow$ coherent light

$|g^{(1)}(\tau)| = 0 \Rightarrow$ completely incoherent light

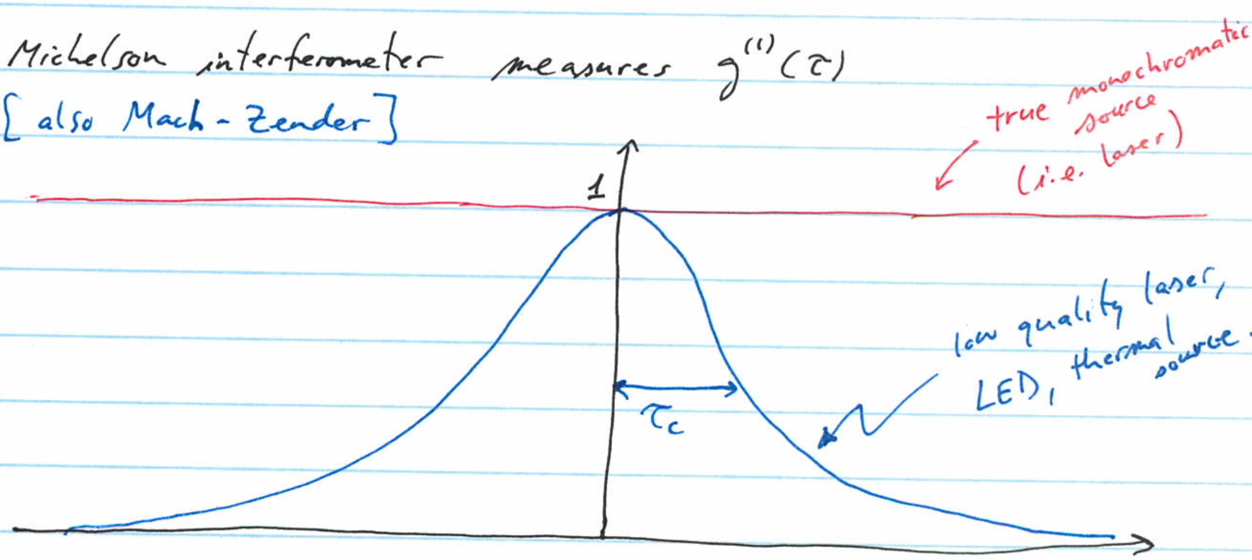
Note 2: $g^{(1)}(0) = 1$ always

↳ white light interferometer requires $\tau \simeq 0$

↳ $\Delta L \simeq 0$

A Michelson interferometer measures $g^{(1)}(\tau)$

[also Mach-Zender]



definition: $\tau_c = \int_{-\infty}^{+\infty} |g^{(1)}(\tau)|^2 \, d\tau$ = coherence time