Tuesday, February 13, 2024

1st order correlation function review

def: $$g^{(1)}(\tau) = \frac{\langle E^*(t)\, E(t+\tau)\rangle_{time}}{\langle E^*(t)\, E(t)\rangle_{time}}$$

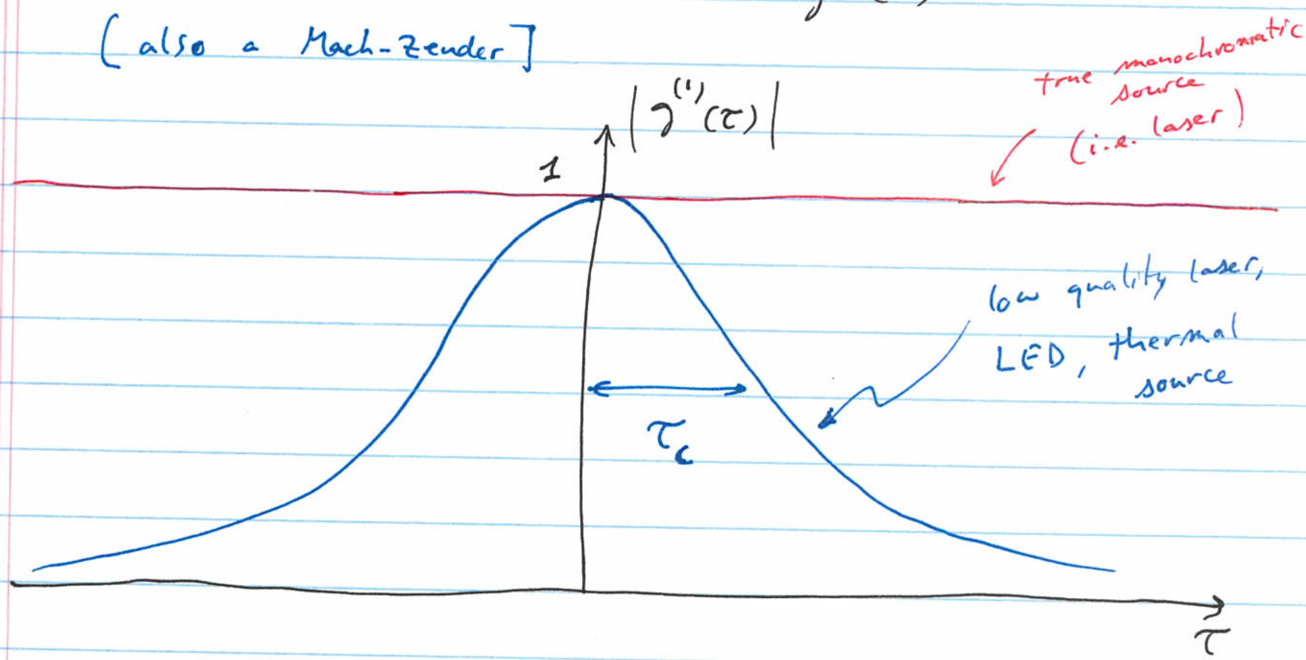
Theorem: $|g^{(1)}(\tau)|$ = fringe visibility [for a 50/50 beamsplitter]

Interferometer output $= I = \langle |\underbrace{E(t)}_{\text{path A}} + \underbrace{E(t+\tau)}_{\text{path B}}|^2\rangle_t$

$$\vdots$$

$$= 2 I_0 \left(1 + |g^{(1)}(\tau)| \cos(\omega\tau - \underset{\uparrow \text{ phase}}{\alpha})\right)$$

A Michelson interferometer measures $g^{(1)}(\tau)$
[also a Mach-Zender]

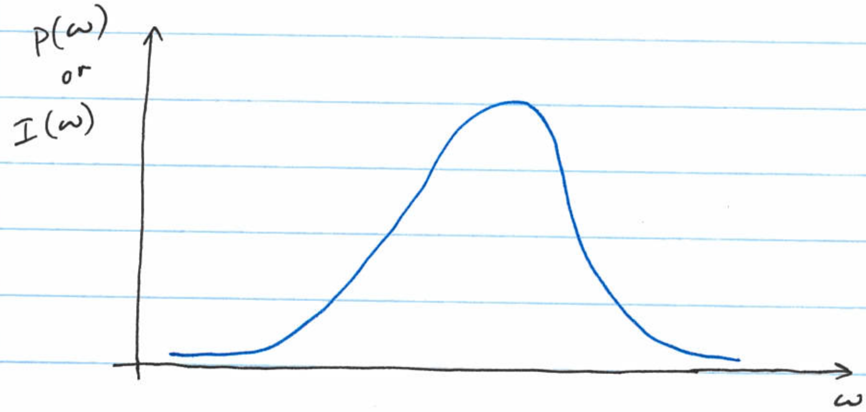


Definition: $$\tau_c = \int_{-\infty}^{+\infty} |g^{(1)}(\tau)|^2\, d\tau = \text{coherence time}$$

or $\frac{\Delta L}{c} = \tau$
path length difference

The Wiener – Khintchine Theorem

Consider a light field with the following spectrum $P(\omega)$



objective: we would like to find a relationship between "$P(\omega)$" and $g^{(1)}(\tau)$.

The amplitude of the electric field distribution is given by

$$\underbrace{E_{\Delta T}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{T}^{T+\Delta T} E(t)\, e^{i\omega t}\, dt}_{\text{Fourier transform}} \qquad \text{where } \Delta T \gg \tau_c$$

In reverse:

$$E(t) = \frac{1}{\sqrt{2\pi}} \int_{\omega}^{\omega+\Delta\omega} E_{\Delta T}(\omega)\, e^{-i\omega t}\, d\omega$$

with $\Delta\omega \gg$ than the spectral width

The power spectral density is defined as

multiply by "$\varepsilon_0 c$" to get power/intensity

average over t

$$P(\omega) = f(\omega) = \frac{1}{\Delta T} \left| E_{\Delta T}(\omega) \right|^2$$

$$= \frac{1}{\Delta T} \left(\frac{1}{\sqrt{2\pi}}\right)^2 \int_{\Delta T} dt \int_{\Delta T} dt'\, E^*(t)\, E(t')\, e^{\overbrace{i\omega(t'-t)}^{e^{i\omega\tau}}}$$

substitution $\begin{cases} \tau = t' - t \\ d\tau = dt' \end{cases}$

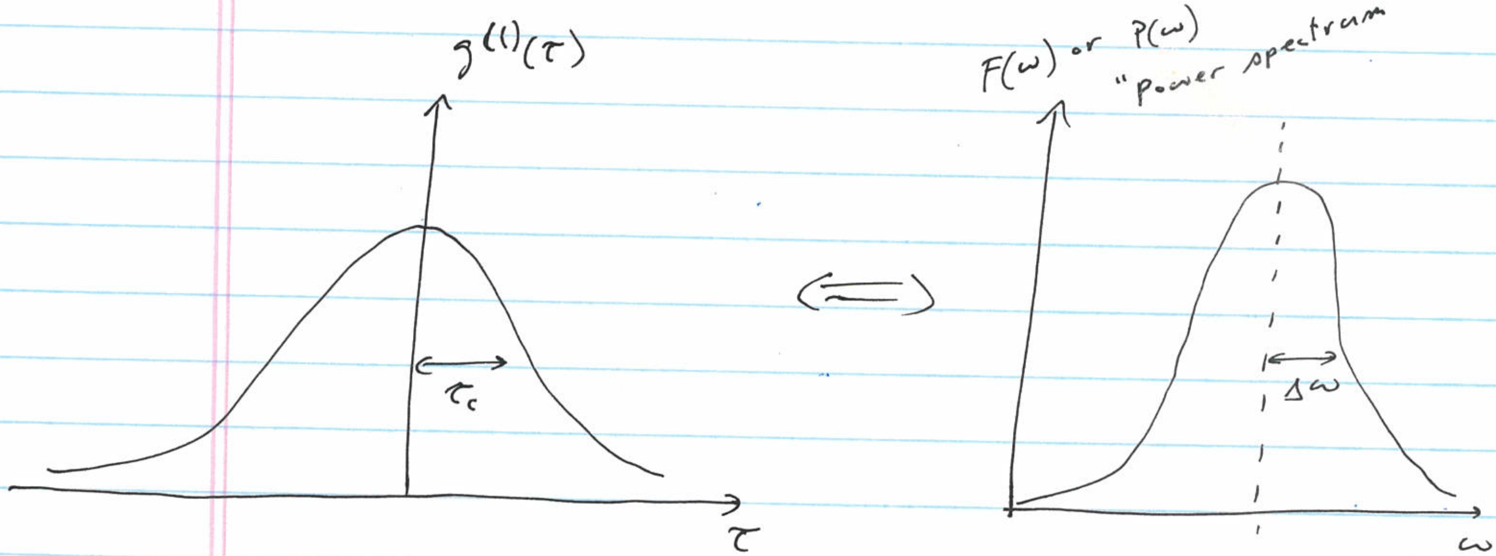
$$= \frac{1}{2\pi} \int_{\Delta T} d\tau \, \langle E^*(t)\, E(t+\tau) \rangle_t \, e^{i\omega\tau}$$

define Normalization: $N = \int_{-\infty}^{+\infty} d\omega \, f(\omega) = \langle E^*(t)\, E(t) \rangle$

Thus $$\boxed{F(\omega) = \frac{f(\omega)}{N} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} g^{(1)}(\tau)\, e^{i\omega\tau}\, d\tau}$$

normalized power spectral density

Wiener-Khintchine Theorem



$$\tau \sim \frac{1}{\Delta\omega}$$

$g^{(1)}(\tau)$ measures the spectral width of the light.

A Michelson interferometer measures $|g^{(1)}(\tau)|$ and the spectral width of the light.