Thursday, February 15, 2024

## $2^{nd}$ order auto-correlation function

**definition:** $$g^{(2)}(\tau) = \frac{\langle I(t) \cdot I(t+\tau) \rangle}{\langle I(t) \rangle^2}$$

time average

**Example:** Monochromatic light

$$E(t) = E_0 \, e^{i(kz - \omega t + \phi)}$$

$$\equiv E_0 \cos(kz - \omega t + \phi)$$

at $z = 0$, $I(t) = \varepsilon_0 c E_0^2 \cos^2(\omega t - \phi)$

$$\Rightarrow \underbrace{\langle I(t) \rangle_t}_{\text{average over many optical cycles}} = \frac{1}{2} \varepsilon_0 c E_0^2$$

$$\langle I(t) \cdot I(t+\tau) \rangle = (\varepsilon_0 c)^2 E_0^4 \underbrace{\langle \cos^2(\omega t - \phi) \cos^2(\omega(t+\tau) - \phi) \rangle}_{\left\langle \left\{ \frac{1}{2} \left[ \cos(2\omega t + \omega\tau - 2\phi) + \cos(\omega\tau) \right] \right\}^2 \right\rangle}$$

our photo detector is not fast enough to see changes at frequency $\omega$ or $2\omega$ (average over small $\tau$) (changes also average over $\frac{1}{\omega}$ and $\frac{1}{\omega^2}$) average over timescales

$$= \frac{1}{4} \left[ \cancelto{1/2}{\langle \cos^2(\;) \rangle} + \cancelto{1/2}{\langle \cos^2(\;) \rangle} + 2 \cancelto{0}{\langle \cos(\;) \cos(\;) \rangle} \right]$$

$$= \frac{1}{4} (\varepsilon_0 c)^2 E_0^4$$

thus $\boxed{g^{(2)}(\tau) = \dfrac{\frac{1}{4}(\varepsilon_0 c)^2 E_0^4}{\left[\frac{1}{2}(\varepsilon_0 c) E_0^2\right]^2} = 1}$ for all $\tau$

2- Chaotic light sources

Monochromatic light with random-time dependent phases generated by $N$ atoms (disregard polarization).

atom $j$ emits: $E_j(t) = E_0 e^{-i\omega t + i\phi_j(t)}$

$\phi(t)$ changes randomly in the range $[0, 2\pi]$ over a characteristic $t = \tau_c$ (coherence time)

for $N$ atoms: $E_{total}(t) = \sum_{j=1}^{N} E_j(t)$

$$= E_0 e^{-i\omega t} \sum_{j=1}^{N} e^{i\phi_j(t)}$$

$$\Rightarrow \underbrace{\langle I_{total}(t)\rangle_t}_{\text{average over many optical cycles}} = \frac{1}{2}\varepsilon_0 c E_0^2 \underbrace{\left\langle \left| e^{i\phi_1(t)} + e^{i\phi_2(t)} + \dots + e^{i\phi_N(t)} \right|^2 \right\rangle_{time}}_{\underbrace{\langle |e^{i\phi_1(t)}|^2\rangle}_{=1} + \dots + \underbrace{\langle e^{i\phi_m(t) - i\phi_n(t)}\rangle}_{=0}}$$

no correlations between atoms

$$\underbrace{\qquad\qquad}_{= N}$$

$$= \underbrace{\frac{1}{2}\varepsilon_0 c E_0^2}_{I_{single\ atom}} N$$

$$\Leftrightarrow \quad \langle I_{total} \rangle = \langle I_{\substack{single\\atom}} \rangle N$$

Instead of treating the general case, let's choose $\tau = 0$.

$$\lim_{\tau \to 0} \langle I(t) \cdot I(t+\tau) \rangle = \langle I(t) \cdot I(t) \rangle = \langle I(t)^2 \rangle$$

$$= \frac{1}{4} \varepsilon_0^2 c^2 E_0^4 \left\langle \left( \left| e^{i\phi_1(t)} + e^{i\phi_2(t)} + \dots + e^{i\phi_N(t)} \right|^2 \right)^2 \right\rangle_{time}$$

$$= \frac{1}{4} \varepsilon_0^2 c^2 E_0^4 \left\langle \left( \underbrace{\underbrace{|e^{i\phi_1}|^2}_{=1} + \dots + \underbrace{|e^{i\phi_N}|^2}_{=1}}_{N} + \sum_{m>n} \underbrace{\left( e^{i(\phi_m - \phi_n)} + e^{-i(\phi_m - \phi_n)} \right)}_{2\cos(\phi_m - \phi_n)} \right)^2 \right\rangle_{time}$$

$$= \underbrace{\frac{1}{4} \varepsilon_0^2 c^2 E_0^4}_{I^2_{single\ atom}} \left\langle N^2 + 4N \sum_{m>n} \cos(\phi_m - \phi_n) + 4 \left( \sum_{m>n} \cos(\phi_m - \phi_n) \right)^2 \right\rangle_{time}$$

sum gives a factor of $\frac{N(N-1)}{2}$

$$= I^2_{\substack{single\\atom}} \left\{ \underbrace{\langle N^2 \rangle_t}_{N^2} + \underbrace{4N \sum_{m>n} \underbrace{\langle \cos(\phi_m - \phi_n) \rangle_t}_{=0}}_{=0} + 4 \sum_{m>n} \underbrace{\langle \cos^2(\phi_m - \phi_n) \rangle}_{1/2} \right.$$

sum also averages to zero

$$\left. + \underbrace{4 \sum_{\substack{m>n \\ l>p \\ (m,n) \neq (l,p)}} \underbrace{\langle \cos(\phi_m - \phi_n) \cos(\phi_l - \phi_p) \rangle}_{\underbrace{\langle \cos(\phi_m - \phi_n) \rangle}_{=0} \underbrace{\langle \cos(\phi_l - \phi_p) \rangle}_{=0}}}_{=0} \right\}$$

sum also averages to zero

$$= I^2_{\substack{single\\atom}} \left\{ N^2 + \underbrace{\cancel{4} \frac{N(N-1)}{\cancel{2}} \frac{1}{\cancel{2}}}_{N^2 - N} \right\} = I^2_{\substack{single\\atom}} (2N^2 - N)$$

$$= \underbrace{I^2_{\substack{single\\atom}} N^2}_{\langle I_{total}(t) \rangle^2} \left( 2 - \frac{1}{N} \right) = \langle I_{total}(t) \rangle^2 \left( 2 - \frac{1}{N} \right)$$

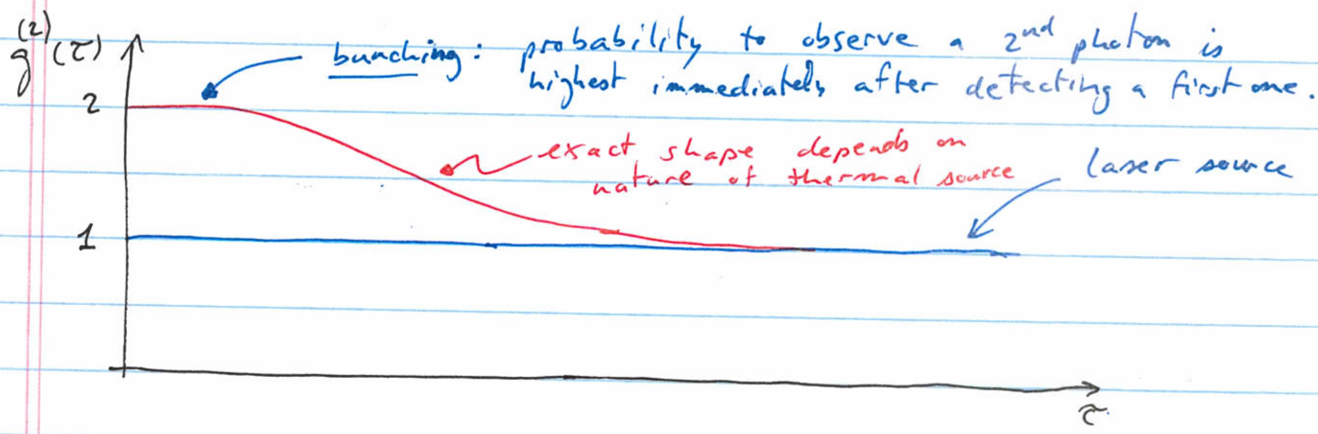
for $N \to +\infty$, we have $\langle I_{total}^2(t)\rangle = 2\langle I(t)\rangle^2$

thus $\boxed{g^{(2)}(\tau=0) = 2}$ for the random time-dependent phase chaotic light

for $\tau \gg \tau_c$, we expect that all correlations are lost, so that $\underbrace{\langle I(t)\cdot I(t+\tau)\rangle = \langle I(t)\rangle\langle I(t+\tau)\rangle}_{\tau \gg \tau_c}$

$$\Rightarrow \boxed{g^{(2)}(\tau \gg \tau_c) = 1}$$

The same results are true for chaotic thermal light in which $\phi_j(t) = cst_j$, but there is a spread of atomic resonance frequencies.




For chaotic light (thermal light), one can show that

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2$$

For an ~~a~~ arbitrary classical light source, the following properties hold

- $g^{(2)}(\tau) = g^{(2)}(-\tau)$
- $1 \leq g^{(2)}(0) \leq \infty$ and $0 \leq g^{(2)}(\tau) \leq \infty$ for $\tau \neq$
- $g^{(2)}(\tau) \leq g^{(2)}(0)$ ← not true for "quantum light"