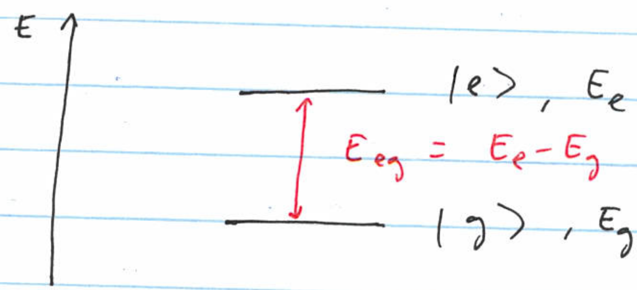
Tuesday, February 20, 2024

2-level atoms

(I) Definition: A 2-level atom is a quantum mechanical system with 2 distinct eigenstates & eigenenergies.



The basic Hamiltonian for the system is

$$H_0 = \begin{array}{c c} & \begin{array}{cc} |g\rangle & |e\rangle \end{array} \\ \begin{array}{c} \langle g| \\ \langle e| \end{array} & \begin{pmatrix} E_g & 0 \\ 0 & E_e \end{pmatrix} \end{array}$$

One can write an arbitrary state of the system as

$$|\psi\rangle = \alpha|g\rangle + \beta|e\rangle \equiv |\alpha||g\rangle + |\beta|e^{i\phi}|e\rangle$$

normalization requires

$$|\langle\psi|\psi\rangle|^2 = |\alpha|^2 + |\beta|^2 = 1$$

II Block Sphere picture

We want a geometric representation of the state of a 2-level system.

→ we could use a 2D vector space representation

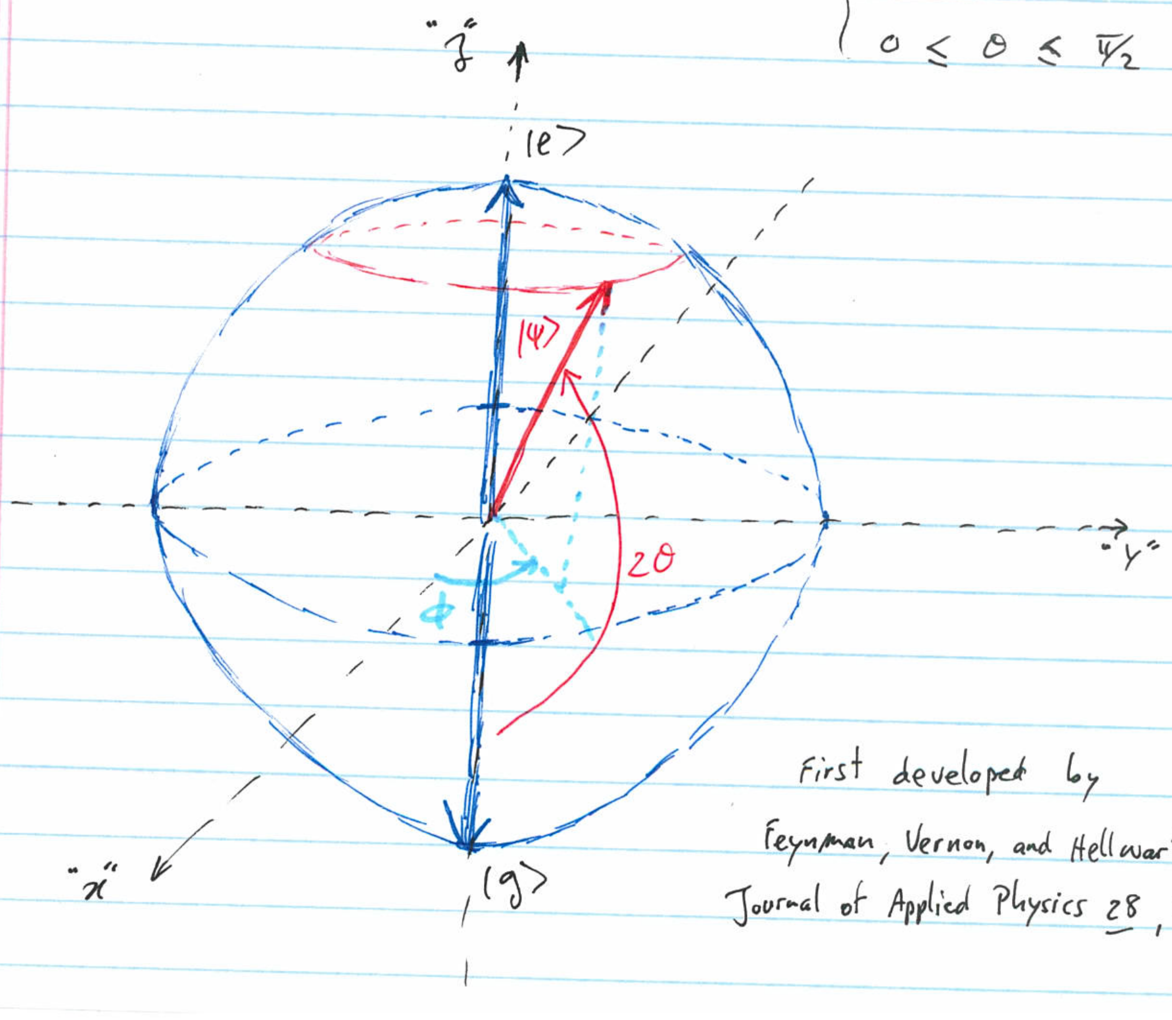
↳ but phase is hard to show then!

A 2-level system is like a spin-$\frac{1}{2}$ system with

$$|\uparrow\rangle = |e\rangle \quad \text{and} \quad |\downarrow\rangle = |g\rangle$$

we note that we can write $|\psi\rangle = \cos\theta\,|g\rangle + e^{i\phi}\sin\theta\,|e\rangle$

$$\begin{cases} 0 \leq \phi < 2\pi \\ 0 \leq \theta \leq \pi/2 \end{cases}$$



First developed by

Feynman, Vernon, and Hellwarth

Journal of Applied Physics 28, 49 (1957)

III Time - dependence

What happens when you let an arbitrary state evolve in time?

Easy: $|\psi\rangle = \alpha|g\rangle + \beta|e\rangle \Rightarrow |\psi(t)\rangle = \alpha e^{-i\frac{E_g}{\hbar}t}|g\rangle + \beta e^{-i\frac{E_e t}{\hbar}}|e\rangle$

$$= \underbrace{e^{-i\frac{E_g}{\hbar}t}}_{\text{overall phase}}\left[\alpha|g\rangle + \beta \underbrace{e^{-i\frac{(E_e - E_g)t}{\hbar}}}_{\text{relative phase}}|e\rangle\right]$$

overall phase ↳ not important

relative phase ↳ totally important

$$P(|g\rangle)_{|\psi(t)\rangle} = |\langle g|\psi(t)\rangle|^2 = |\alpha|^2$$

$$P(|e\rangle)_{|\psi(t)\rangle} = |\langle e|\psi(t)\rangle|^2 = |\beta|^2$$

} no measureable time-dependence

[show phase precession on Bloch sphere]

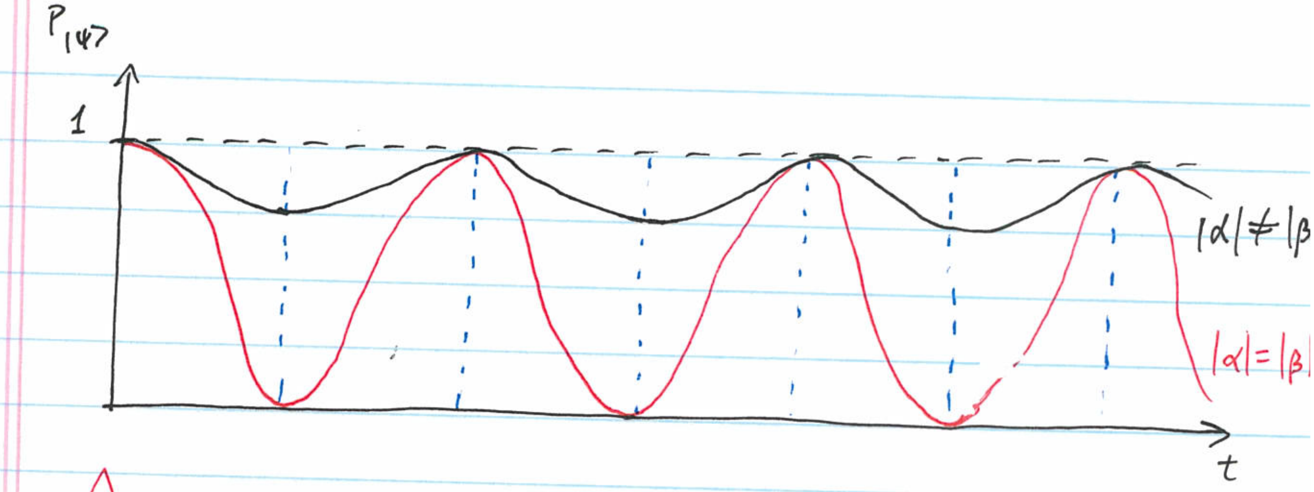
more interesting

$$P(|\psi\rangle)_{|\psi(t)\rangle} = |\langle\psi|\psi(t)\rangle|^2$$

$$= \left|\left(\alpha^*\langle g| + \beta^*\langle e|\right)\left(\alpha e^{-i\frac{E_g t}{\hbar}}|g\rangle + \beta e^{-i\frac{E_e t}{\hbar}}|e\rangle\right)\right|^2$$

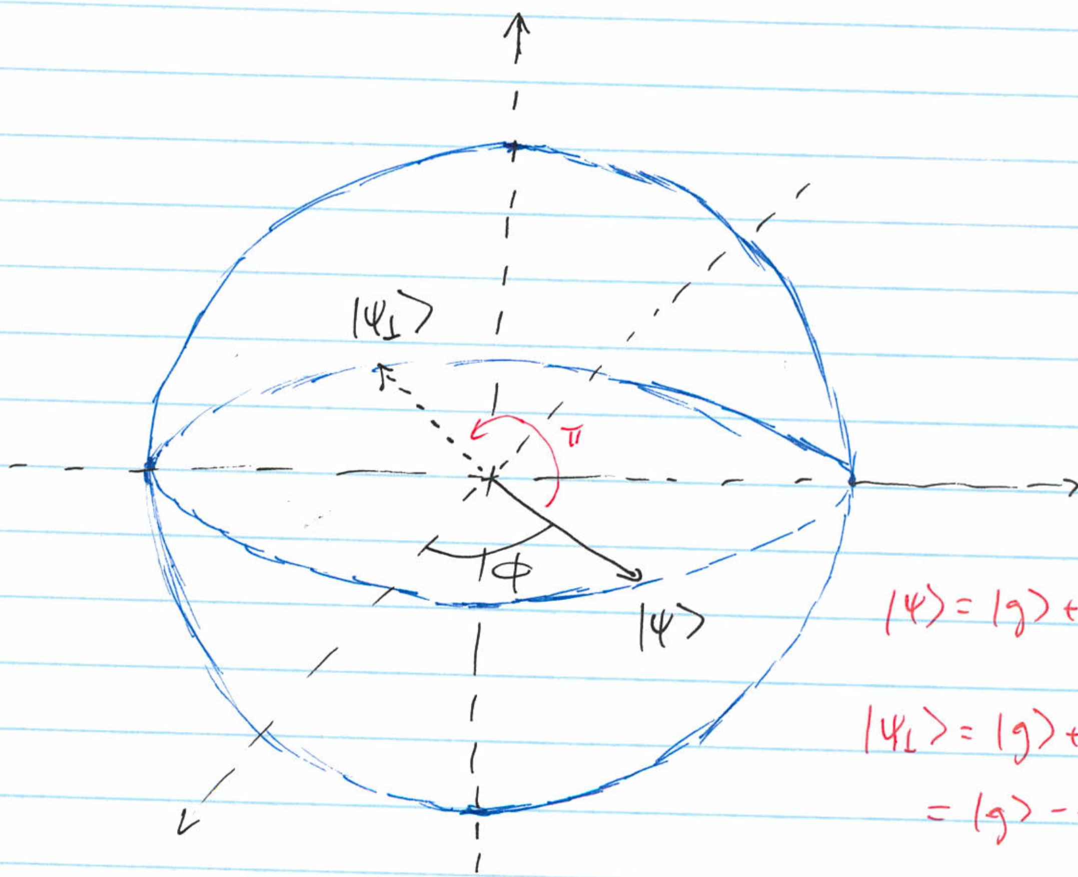
⋮ (some algebra)

$$= \frac{1}{2}\left[1 + \left(|\alpha|^2 - |\beta|^2\right)^2\right] + \frac{1}{2}\left[1 - \left(|\alpha|^2 - |\beta|^2\right)^2\right]\cos\left(\underbrace{\omega_{eg}}_{\frac{E_{eg}}{\hbar}} t\right)$$



So there is time dependence if you look in another basis.

Bloch Sphere picture



$$|\psi\rangle = |g\rangle + e^{i\phi}|e\rangle$$

$$|\psi_\perp\rangle = |g\rangle + e^{i(\phi-\pi)}|e\rangle$$
$$= |g\rangle - e^{i\phi}|e\rangle$$

if $|\psi\rangle = \alpha|g\rangle + \beta|e\rangle$

then $|\psi_\perp\rangle = \beta^*|g\rangle - \alpha^*|e\rangle$

2-level atom in an EM field

$\underbrace{\text{2-level atom}}_{\text{QM}}$ in an $\underbrace{\text{EM field}}_{\text{Classical EM}}$

(I) Hamiltonian

The interaction between an <u>electric field</u> and the electric dipole moment of an atom is given by

$$H_{int} = -\vec{d}\cdot\vec{E} = e\vec{R}\cdot\vec{E}_0 \underbrace{\cos(\omega_\ell t)}_{\cos(kz - \omega_\ell t) \text{ with } z=0}$$

(see Griffith equ. 4.6)

(Assumption: single e^- orbiting a nucleus)

In the 2-level atom basis $\{|g\rangle, |e\rangle\}$, we have

$$H_{int} = e\vec{E}_0 \cos(\omega_\ell t) \begin{bmatrix} \langle g|\vec{R}|g\rangle & \langle g|\vec{R}|e\rangle \\ \langle e|\vec{R}|g\rangle & \langle e|\vec{R}|e\rangle \end{bmatrix}$$

parity operator

<u>Recall</u>: $[P, H_{atom}] = 0 \Rightarrow |g\rangle$ & $|e\rangle$ have a definite parity.

$$\langle g|\vec{R}|g\rangle = \underset{\text{even}}{\langle g|} \underset{\text{odd}}{(X,Y,Z)} \underset{\text{even}}{|g\rangle} = \int \underset{\text{even}}{\psi_g^*} \underset{\text{odd}}{(x,y,z)} \underset{\text{even}}{\psi_g}\, d^3r$$

$$= 0$$

similarly, $\underset{\text{odd}}{\langle e|} \underset{\text{odd}}{\vec{R}} \underset{\text{odd}}{|e\rangle} = 0$

"2-level atom" atom = alkali atom (1 valence e^-) $nS \longleftrightarrow nP$ transition

$|g\rangle \longleftrightarrow |e\rangle$

In AMO, we define

$$\Omega = \frac{e\langle g|\vec{R}|e\rangle \cdot \vec{E}_0}{\hbar} = \frac{\langle g|H_{int}|e\rangle}{\hbar} = \text{Rabi frequency}$$

So we have:

$$H = \underbrace{\begin{pmatrix} E_g & 0 \\ 0 & E_e \end{pmatrix}}_{H_0} + \underbrace{\hbar \begin{pmatrix} 0 & \Omega \\ \Omega^* & 0 \end{pmatrix} \cos(\omega_\ell t)}_{H_{int}}$$