Physics 482-01 and 690-01: Quantum Optics & Atomics
Due date: Tuesday, February 27, 2024.

Problem Set #3

1. *Wiener-Khintchine Theorem*

In class, we derived a formula for the Normalized Power Spectral Density, $F(\omega)$, in terms of the first order correlation function, $g^{(1)}(\tau)$. Derive an expression for $g^{(1)}(\tau)$ in terms of $F(\omega)$.

2. *Level crossings*

Consider a 2-level atom with ground and excited states $|g\rangle$ and $|e\rangle$, respectively, and energies, E_g and E_e, respectively. These energies depend on a parameter *m* such that

$$E_g(m) = E_g(0) + \alpha \cdot m \text{ and } E_e(m) = E_e(0) - \alpha \cdot m$$

We modify the basic Hamiltonian of the system, H_0, by adding a generic interaction with Hamiltonian:

$$H_{\text{int}} = \begin{bmatrix} 0 & W \\ W^* & 0 \end{bmatrix}$$

a) Calculate the new eigenenergies of the system with the interaction present, as a function of m.

b) Plot the energy of the system as function of *m*, with and without the interaction present.

c) Calculate the new eigenstates of the system with the interaction present, and show that one can go continuously from modified state $|g\rangle$ to modified state $|e\rangle$, and vice versa, by adiabatically varying *m*. What is the condition for adiabaticity? Make a quantitative (and logical) argument. What happens if you ramp *m* much faster than the adiabatic condition (support your answer with a quantitative argument)?

Extra graduate student problem

3. *Gaussian lineshape*

Consider a gas of identical atoms that emit light at a resonant frequency of ω_0 (when at rest). The atoms in the gas have a spread of velocities due to their finite temperature, T, which leads to a spread in the resonant frequency of the atoms due to the Doppler effect. The probability for an atom to emit light at a frequency ω close to ω_0 in a given direction is then given by

$$P(\omega) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(\omega - \omega_0)^2}{\sigma^2} \right) \quad \text{with} \quad \sigma = \omega_0 \sqrt{\frac{kT}{mc^2}} \,,$$

where m is the mass of the atom, c is the speed of light, and k is Boltzmann's constant.

We will assume that the atom is excited by some mechanism, and we look at the radiated light in a given direction. The emitted electric field in this direction is then given by

$$E(t) = E_0 \sum_{i=1}^{N} \exp(-i\omega_i t + i\phi_i)$$

where the sum is over the N atoms in the gas and the ϕ_i are random stationary phases.

a. Show that $\left\langle E^*(t) E(t+\tau) \right\rangle = E_0^2 \sum_{i=1}^{N} \exp(-i\omega_i \tau)$.

b. Show that $g^{(1)}(\tau) = \exp\left(-i\omega_0 \tau - \frac{1}{2} \sigma^2 \tau^2 \right)$.