Tuesday, February 27, 2024

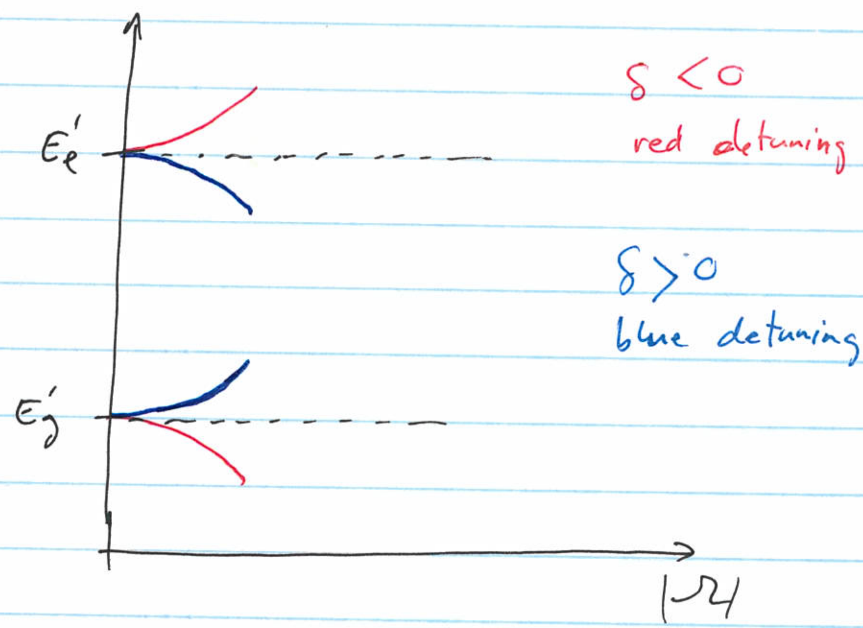
Last lecture, we showed for a ground state $|g\rangle$ atom in an EM field, #1
the $|g\rangle$ state experiences an energy shift: $\boxed{E_g' = E_g + \frac{\hbar\Omega^2}{4\delta}}$ for $|\delta| \gg |\Omega|$

by a similar analysis (with $C_g(0), C_e(0) = 1$),
we find that

$$|\psi(t)\rangle = \varepsilon|g\rangle + e^{-i\left(\omega_e - \frac{1}{4}\frac{\Omega^2}{\delta}\right)t}|e\rangle$$

$\hookrightarrow$ thus the excited state is shifted: $\boxed{E_e' = E_e - \frac{\hbar\Omega^2}{4\delta}}$ for $|\delta| \gg |\Omega|$

$\Rightarrow$ Energy of the excited state is shifted exactly opposite to the ground state shift.



For $\hbar\Omega = e\vec{E}_0 \cdot \vec{R} \cos(\omega_\ell t) \longrightarrow$ AC Stark shift

For $\hbar\Omega = -\vec{\mu} \cdot \vec{B}_0 \cos(\omega_\ell t) \longrightarrow$ AC Zeeman shift

For $\delta = 0 \Longrightarrow$ Autler-Townes splitting, Mollow triplet

Dressed Atom Theory – Another view

Motivation: Include the "quantum photon" description to our 2-level atom system.

A – Interstate Interaction – DC Hamiltonian (time independent)

Consider a 2-level atom with a generic DC interstate interaction W:

$$H = H_0 + W$$

$$= \begin{array}{c|cc} & |g\rangle & |e\rangle \\ \hline \langle g| & E_g & 0 \\ \langle e| & 0 & E_e \end{array} + \begin{array}{c|cc} & |g\rangle & |e\rangle \\ \hline \langle g| & 0 & W \\ \langle e| & W^* & 0 \end{array}$$

What are the new eigenstates & eigenenergies of H in terms of $|g\rangle$, $|e\rangle$, E_g, E_e, and W?

$$\det \begin{pmatrix} E_g - \lambda & W \\ W^* & E_e - \lambda \end{pmatrix} = 0$$

$$\Leftrightarrow \quad (E_g - \lambda)(E_e - \lambda) - |W|^2 = 0$$

If $E_g = -\frac{E_{eg}}{2}$ and $E_e = +\frac{E_{eg}}{2}$, then $\lambda^2 - \left(\frac{E_{eg}}{2}\right)^2 - |W|^2 = 0$

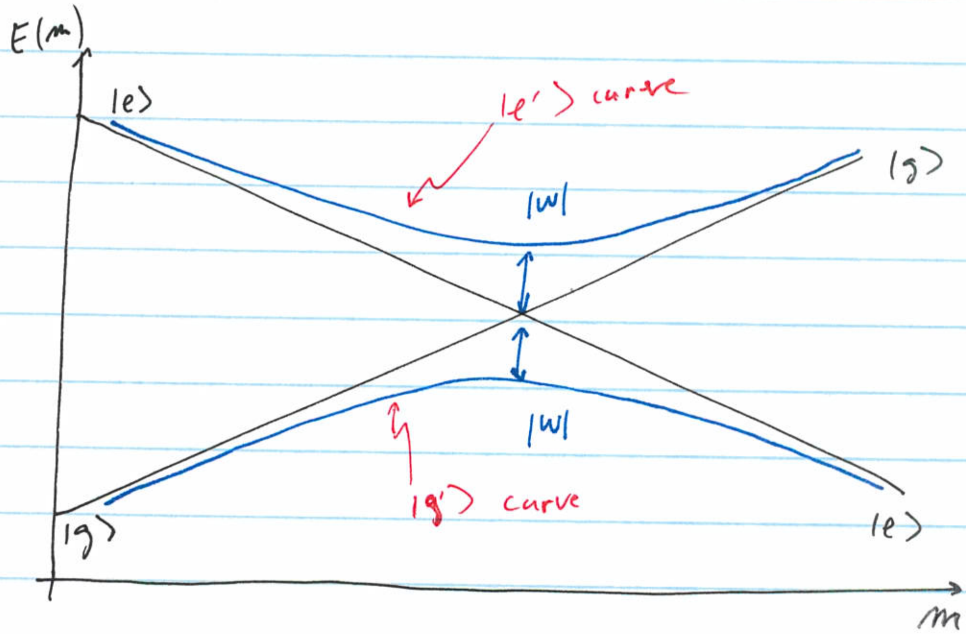
$$\Rightarrow \quad \lambda_\pm = \pm\sqrt{\left(\frac{E_{eg}}{2}\right)^2 + |W|^2}$$

In the new eigenbasis $\{|g'\rangle, |e'\rangle\}$:

$$H = \begin{array}{cc} & \begin{array}{cc} \quad |g'\rangle \quad & \quad\quad\quad |e'\rangle \end{array} \\ \begin{array}{c} \langle g'| \\ \\ \langle e'| \end{array} & \begin{bmatrix} -\sqrt{\left(\frac{E_{eg}}{2}\right)^2 + |W|^2} & 0 \\ & \\ 0 & +\sqrt{\left(\frac{E_{eg}}{2}\right)^2 + |W|^2} \end{bmatrix} \end{array}$$

with $|g'\rangle = |g\rangle + \varepsilon |e\rangle$ | The energy eigenstates

$|e'\rangle = |e\rangle - \varepsilon^* |g\rangle$ | are "repelled".



⇒ If 2 levels cross as a function of some parameter (i.e. m), then if there is any interaction, the energy states will repel.

⇒ avoided level crossing!!

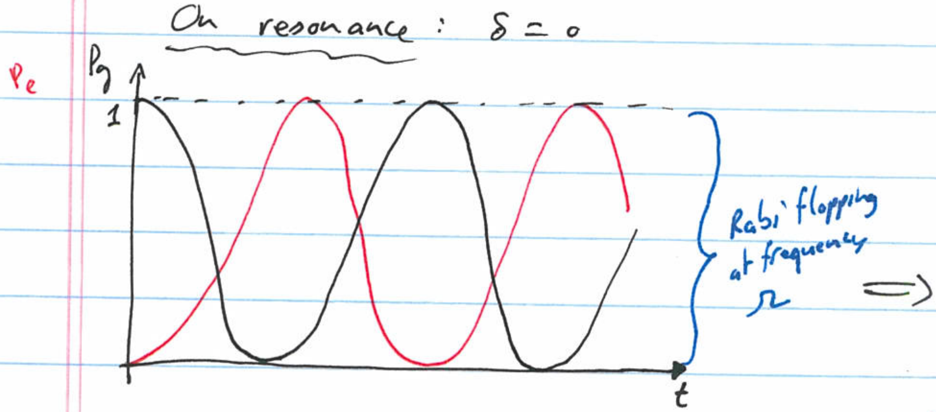
If you ramp $\underline{m}$ adiabatically, then $|g\rangle \to |e\rangle$

$|e\rangle \to |g\rangle$

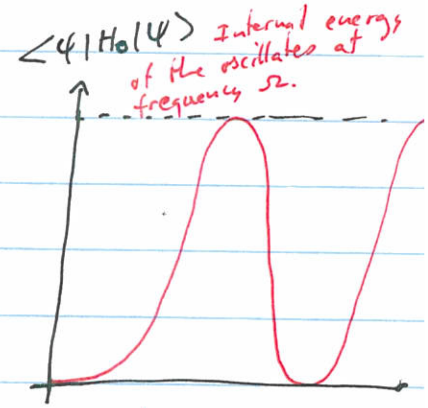
If you ramp $\underline{m}$ quickly (diabatically), then you can get a <u>Landau-Zener</u>

B – Basic Energetics of a 2-level atom in an EM field (laser)

On resonance : $\delta = 0$



$\Longrightarrow$



[omitted interaction energy between the EM field and the atom]

What if we include the energy in the EM field?

state 1: $|1\rangle = |g\rangle_a \, |N+1\rangle_{\omega_{eg}} \rightarrow E_1 = E_g + \hbar\omega_{eg}(N+1)$

state 2: $|2\rangle = |e\rangle_a \, |N\rangle_{\omega_{eg}} \rightarrow E_2 = \underbrace{E_e}_{E_g + \hbar\omega_{eg}} + \hbar\omega_{eg} N$

$= E_g + \hbar\omega_{eg}(N+1)$

so $E_1 = E_2 = E_0$, i.e. energy is conserved. (by construction)

$\boxed{\text{state } |1\rangle \text{ \& state } |2\rangle \text{ are the ``bare states'' for our dressed atom model.}}$

C – Hamiltonian for a 2-level dressed atom in an EM field

(i.e. how do we make a static system oscillate?)

1st guess: $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix}$

However, from the 1st lecture on 2-level atoms, we know that such a system will not display any oscillations, regardless of which basis we measure in! $(\Delta E = E_2 - E_1 = 0)$

2nd try: In some basis, we would like to write:

$$\text{"}H\text{"} = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 + \underbrace{\hbar\Omega} \end{pmatrix}$$

guarantees an oscillation frequency of Ω in some basis.

from section A, we know that

$$H = \begin{pmatrix} E_0 & \hbar\Omega/2 \\ \frac{\hbar\Omega^*}{2} & E_0 \end{pmatrix}$$ will work!

In this approach (dressed atom picture), Rabi flopping occurs because states $|1\rangle$ & $|2\rangle$ (actually, the new eigenstates) no longer have the same energy due to an off-diagonal photon + atom interaction.