Tuesday, March 5, 2024

The Bloch Sphere Revisited

In the dressed atom picture for Ω = real, the Hamiltonian is given by:

$$H = \hbar \begin{array}{c|cc} & |1\rangle & |2\rangle \\ \hline \langle 1| & \delta/2 & \Omega/2 \\ \langle 2| & \Omega/2 & -\delta/2 \end{array}$$

(subtracted $\hbar\delta/2$ as an energy offset)

define: $|\psi(t)\rangle = C_g(t)\,|1\rangle + C_e(t)\,|2\rangle$

where $|1\rangle = \underbrace{|g\rangle_a |N+1\rangle_\gamma}_{\text{"ground state"}}$, $|2\rangle = \underbrace{|e\rangle_a |N\rangle_\gamma}_{\text{"excited state"}}$

Schrodinger Equation:

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$\Rightarrow \quad i\hbar \frac{d}{dt}\begin{pmatrix} C_g(t) \\ C_e(t) \end{pmatrix} = \hbar \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega/2 & -\delta/2 \end{pmatrix}\begin{pmatrix} C_g(t) \\ C_e(t) \end{pmatrix}$$

$$\Leftrightarrow \quad \begin{cases} i\dfrac{d}{dt}C_g(t) = \dfrac{\delta}{2}C_g(t) + \Omega/2\, C_e(t) \\[2ex] i\dfrac{d}{dt}C_e(t) = \dfrac{\Omega}{2}C_g(t) - \dfrac{\Omega}{2}C_e(t) \end{cases}$$

Feynman et al., Journal of Applied Physics 28, 49 (1957), discovered that with the substitution

$$\left.\begin{aligned} R_1 &= c_g c_e^* + c_g^* c_e \\ R_2 &= i(c_g c_e^* - c_g^* c_e) \\ R_3 &= |c_g|^2 - |c_e|^2 \end{aligned}\right| \Rightarrow \vec{R} = (R_1, R_2, R_3)$$

= "state vector"

the time evolution (i.e. Schrodinger equation) is described by

$$\boxed{\frac{d\vec{R}}{dt} = \vec{\Omega} \times \vec{R}}$$

where $\vec{\Omega} = (\Omega, 0, \delta)$ = "Rabi vector"

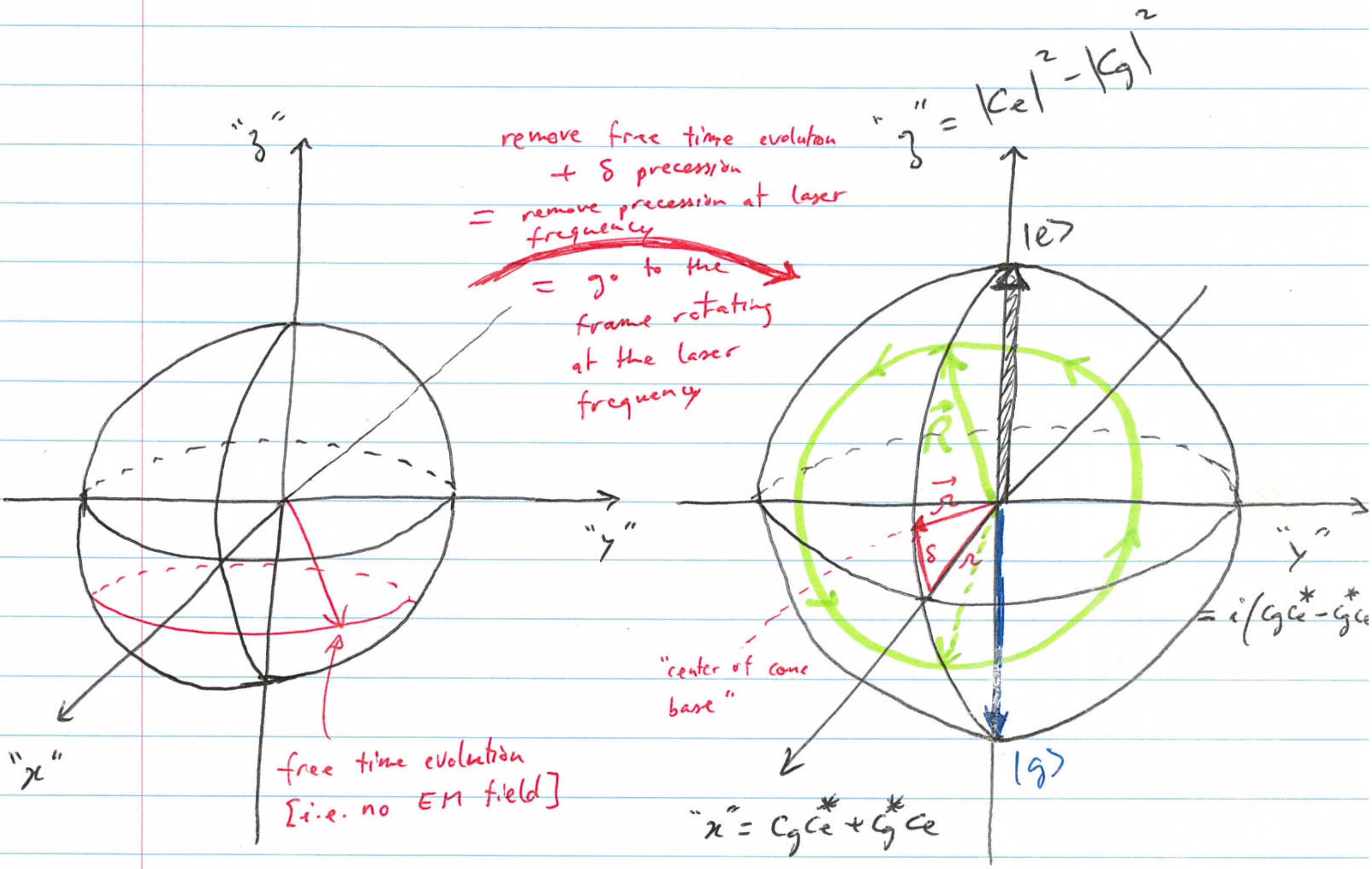
note: $|\vec{\Omega}|^2 = \Omega' = \sqrt{\Omega^2 + \delta^2}$ = generalized Rabi freq.

→ equation of motion for circular motion of $\vec{R}$, where $\vec{\Omega}$ is the "angular velocity."

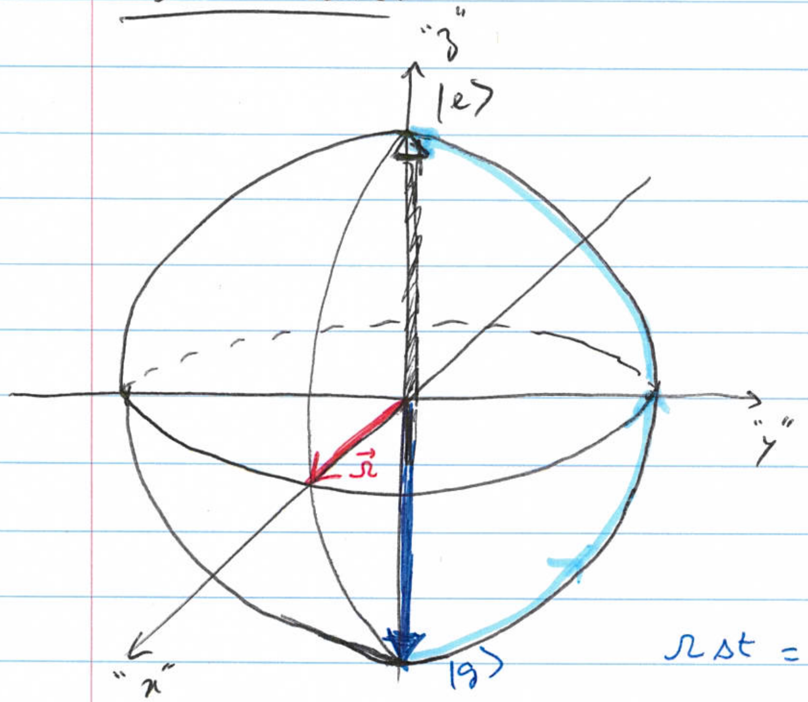
note: $|\vec{R}|^2 = R_1^2 + R_2^2 + R_3^2 = 1 \Rightarrow \vec{R}$ "lives" on a sphere.

$\frac{d\vec{R}}{dt} = \cancel{\ } cst \Rightarrow$ constant circular motion

Bloch Sphere Picture



δ = 0 case:



Δt = duration of EM pulse

π-pulse: Δt chosen such that $|g\rangle \rightarrow |e\rangle$

π/2-pulse: Δt chosen such that $|g\rangle \rightarrow \frac{1}{\sqrt{2}}|g\rangle - \frac{i}{\sqrt{2}}|e\rangle$

$\Omega \Delta t$ = pulse area → determines "rotation angle".

see Feb. 22 lecture : equ. 6a & 6b for $\delta = 0$

$$|\psi(t)\rangle = c_g(t)\,|g\rangle + c_e(t)\,|e\rangle \qquad \text{(in rotating frame)}$$

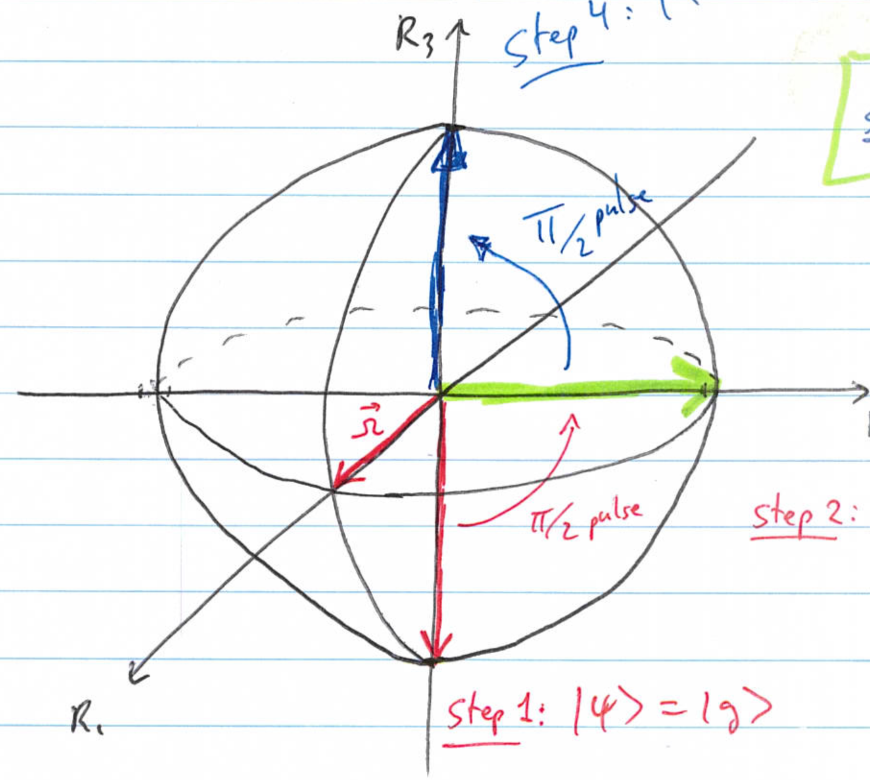
$$= \cos\left(\frac{\Omega t}{2}\right)|g\rangle - i\sin\left(\frac{\Omega t}{2}\right)|e\rangle$$

pick $\frac{\Omega t}{2} = \begin{cases} \pi \\ \text{or} \\ \pi/2 \end{cases}$

Application: Atomic clocks / Ramsey interferometer

Invented by Norman Ramsey (Nobel 1989)
(using the "method of separated oscillatory fields" developed in 1949)

Suppose $\delta = 0$



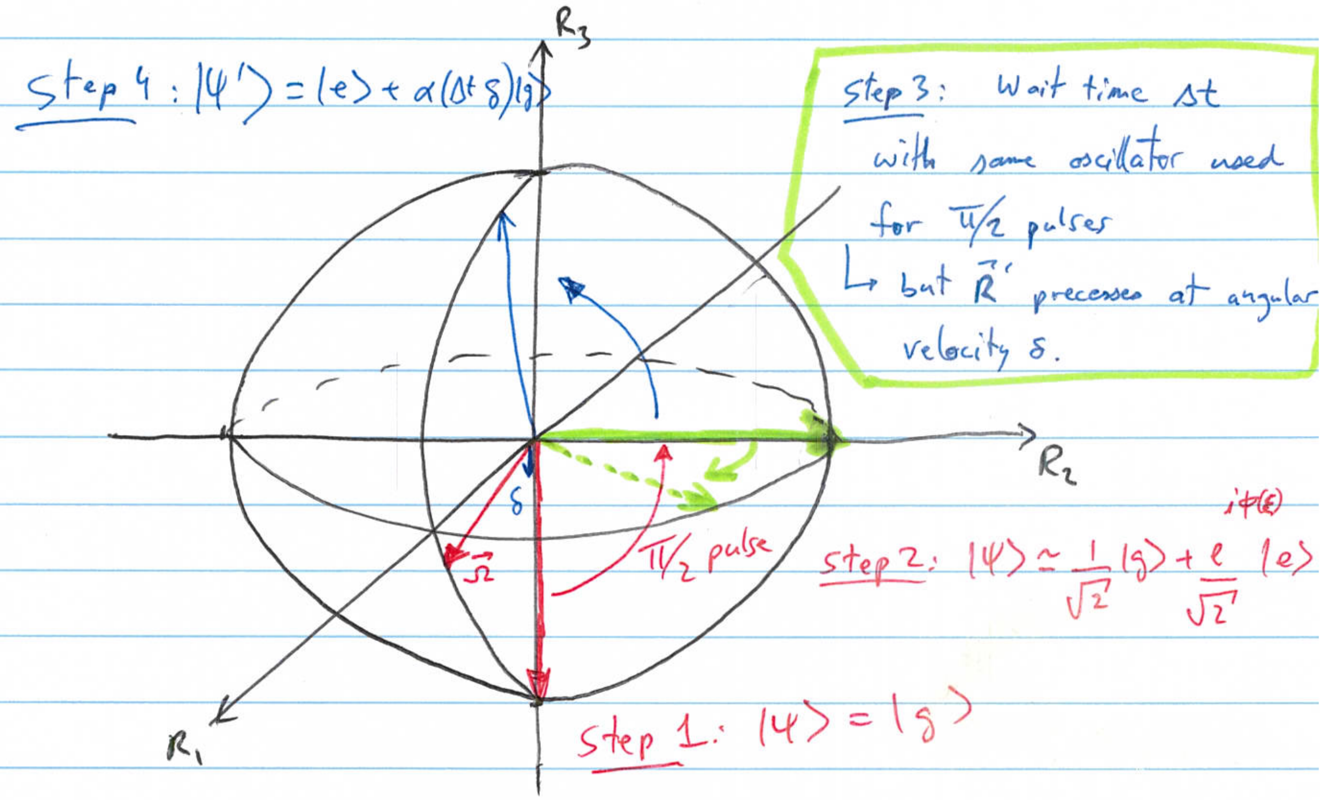
step 4: $|\psi'\rangle = |e\rangle$

step 3: wait time Δt
counted with same oscillator used for $\pi/2$ pulse

step 2: $|\psi\rangle = \frac{1}{\sqrt{2}}|g\rangle - \frac{i}{\sqrt{2}}|e\rangle$
$= \frac{1}{\sqrt{2}}|g\rangle + \frac{e^{-i\phi}}{\sqrt{2}}|e\rangle$
$\phi = \pi/2$

step 1: $|\psi\rangle = |g\rangle$

Suppose $\delta = \pm \varepsilon$



by looking at the population ratio $\frac{|c_e'|^2}{|c_g'|^2}$ you can correct δ!!