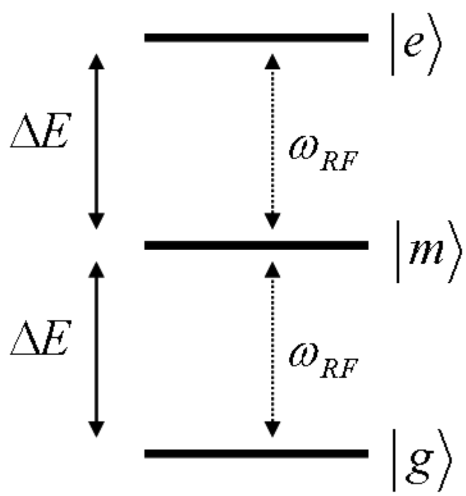
Physics 404 and 690-03: Intro to AMO Physics
Due Date: Tuesday, March 26, 2024.

Problem Set #4: Dressed Atom Theory

Problem #1: 3-Level Rabi Flopping
Solve the degenerate 3-level Rabi flopping problem (similar to the 5-level Rabi flopping movie and data that I showed in class, except easier). Consider a 3-level atom with states $|g\rangle$, $|m\rangle$, and $|e\rangle$ with the following energy level structure:



An RF magnetic field with frequency ω_{RF} is applied to the atom resulting in a (resonant) 2-level Rabi frequency of Ω for both the $|g\rangle \leftrightarrow |m\rangle$ and $|m\rangle \leftrightarrow |e\rangle$. The $|g\rangle \leftrightarrow |e\rangle$ transitions are forbidden.

1) Assuming that you can treat the system as two 2-level atoms with a shared level, show/explain in detail that in the dressed atom picture the Hamiltonian is given by (Ω is real, and $\delta = \omega_{RF} - \Delta E/\hbar$ is the detuning of the driving RF magnetic field)

$$H = \hbar \begin{bmatrix} 2\delta & \Omega/2 & 0 \\ \Omega/2 & \delta & \Omega/2 \\ 0 & \Omega/2 & 0 \end{bmatrix}$$

Also, specify the basis for the Hamiltonian.

2) Find the eigen-energies and eigen-states of the Hamiltonian for δ=0.

3) Consider an initial state of the system where the atom is in the $|g\rangle$ state at t=0. Derive expressions for the probabilities to be in states $|g\rangle$, $|m\rangle$, and $|e\rangle$ as a function of time for δ=0, and plot these probabilities as a function of time.

Extra Graduate Student Problem

Problem #2: Deriving the 2-Level Dressed Atom Hamiltonian

In this problem, you will derive the dressed atom Hamiltonian without resorting to dressed atom theory or its "atom + photon" basis, i.e. you will only use the $\{|g\rangle, |e\rangle\}$ basis (for the ground and excited states of a 2-level atom). You will work in the Schrodinger picture.

The standard 2-level atom Hamiltonian with energies E_g and E_e for the ground and excited atomic states is given by

$$H_{atom} = \begin{bmatrix} E_g & 0 \\ 0 & E_e \end{bmatrix}$$

The interaction Hamiltonian for the interaction of the atom with an oscillating electromagnetic field at frequency ω_l is given by (here Ω is the Rabi frequency for the interaction)

$$H_{int} = \hbar \begin{bmatrix} 0 & \Omega \\ \Omega^* & 0 \end{bmatrix} \cos \omega_l t$$

We will write the time dependence $c_g(t)$ and $c_e(t)$ of the $|g\rangle$ and $|e\rangle$ amplitudes, respectively, of the atomic wavefunction as (here $\omega_{g,e} = E_{g,e}/\hbar$)

$$|\psi(t)\rangle = c_g(t) e^{-i\omega_g t}|g\rangle + c_e(t) e^{-i\omega_e t}|e\rangle \qquad (1)$$

(a) Write down the Schrodinger equation for $|\psi(t)\rangle$.

(b) Apply the Rotating Wave Approximation and show that you obtain the following equations for $c_g(t)$ and $c_e(t)$ (here $\delta = \omega_l - \omega_{eg}$, with $\omega_{eg} = \omega_e - \omega_g$):

$$i\hbar \frac{d}{dt} c_g(t) = c_e(t) \frac{\hbar\Omega}{2} e^{+i\delta t} \qquad (2)$$

$$i\hbar \frac{d}{dt} c_e(t) = c_g(t) \frac{\hbar\Omega^*}{2} e^{-i\delta t} \qquad (3)$$

(c) Next you will go to the "rotating frame" by introducing the rotating frame amplitudes $\tilde{c}_g(t)$ and $\tilde{c}_e(t)$. The rotating frame transformation is defined as

$$\tilde{c}_g(t) = c_g(t)$$
$$\tilde{c}_e(t) = c_e(t) e^{+i\delta t}$$

Using the rotating frame amplitudes, show that equations (2) and (3) can be written in the form of a Schrodinger-like equation based on the dressed atom Hamiltonian:

$$i\hbar \frac{d}{dt} \begin{pmatrix} \tilde{c}_g(t) \\ \tilde{c}_e(t) \end{pmatrix} = \hbar \begin{bmatrix} 0 & \Omega/2 \\ \Omega^*/2 & -\delta \end{bmatrix} \begin{pmatrix} \tilde{c}_g(t) \\ \tilde{c}_e(t) \end{pmatrix}$$

(d) Write down the time dependence of the atomic wavefunction $|\psi(t)\rangle$ from equation 1 in terms of the rotating frame amplitudes $\tilde{c}_g(t)$ and $\tilde{c}_e(t)$, i.e. instead of $c_g(t)$ and $c_e(t)$.

Note: The Bloch sphere picture and dynamics describe $\tilde{c}_g(t)$ and $\tilde{c}_e(t)$.

Also $P_g = |\langle g|\psi(t)\rangle|^2 = |c_g(t)|^2 = |\tilde{c}_g(t)|^2$ & $P_e = |\langle e|\psi(t)\rangle|^2 = |c_e(t)|^2 = |\tilde{c}_e(t)|^2$.