Tuesday, March 26, 2024

Resonance Fluorescence – Density matrix approach

coherent evolution: $$i\hbar \frac{d}{dt}\rho = [H, \rho]$$

density matrix

$$\text{with} \quad H = \begin{bmatrix} E_g & \hbar\Omega\cos(\omega_\ell t) \\ \hbar\Omega^*\cos(\omega_\ell t) & E_e \end{bmatrix} + \text{RWA}$$

$$\text{or} \quad H_{\text{dressed}} = \begin{pmatrix} \delta & \Omega/2 \\ \Omega^*/2 & 0 \end{pmatrix}$$

Add in decoherence / spontaneous emission (decay rate γ):

$$\left\{ \begin{aligned} \frac{d}{dt}\rho_{gg} &= -\frac{i}{\hbar}[H,\rho]_{gg} + \gamma\rho_{ee} \\ \frac{d}{dt}\rho_{ee} &= -\frac{i}{\hbar}[H,\rho]_{ee} - \gamma\rho_{ee} \\ \frac{d}{dt}\rho_{eg} &= -\frac{i}{\hbar}[H,\rho]_{eg} - \frac{1}{2}\gamma\rho_{eg} \\ \frac{d}{dt}\rho_{ge} &= -\frac{i}{\hbar}[H,\rho]_{ge} - \frac{1}{2}\gamma\rho_{ge} \end{aligned} \right.$$

optical Bloch equations

$\Rightarrow$ Solve for

$$\underbrace{\underset{\overset{\uparrow}{P_g}}{\rho_{gg}}, \underset{\overset{\uparrow}{P_e}}{\rho_{ee}}, \rho_{eg}, \rho_{ge}}_{\text{all continuous quantities}}$$

(wavefunction collapse is already included

For generalization to multilevel atoms and multiple lasers see
Marquardt, Robinso
Hollberg
JOSA B, $\underline{13}$, 1384 (1996)

we are interested in the steady state scattering rate:

$$\text{set } \frac{d}{dt}\rho = 0 = -\frac{i}{\hbar}[H,\rho]_{ij} \pm \gamma \rho_{ij} \left(\frac{1}{2}\right)$$

choose appropriately

$$\text{scattering rate} = \gamma_{\text{scattering}} = \gamma P_e = \gamma \rho_{ee}$$

P_e: probability to be in the excited state.

$$\boxed{\begin{aligned} P_g &= |c_g|^2 = \rho_{gg} \\ P_e &= |c_e|^2 = \rho_{ee} \end{aligned}}$$

some work

$$= \boxed{\frac{s_0}{1 + s_0 + \left(\frac{2\delta}{\gamma}\right)^2} \frac{\gamma}{2} = \gamma_{\text{scattering}}}$$

where

$$s_0 = \text{saturation parameter} = \frac{I}{I_{sat}} = \frac{2|\Omega|^2}{\gamma^2}$$

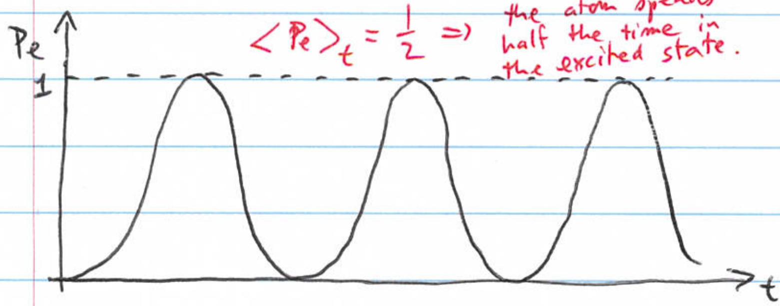
$$I_{sat} = \frac{\pi h c}{3\lambda^2 \tau} = 1.6 \text{ mW/cm}^2 \text{ for } {}^{87}\text{Rb } (D_2 \text{ line})$$

= intensity required to reach half of maximum upper state population in steady state (i.e. $\frac{1}{4}$)

On resonance: $\delta = 0$

Case $I \gg I_{sat}$, i.e. $s_0 \to +\infty$, then $\gamma_{\text{scattering}} = \frac{\gamma}{2} = \underbrace{19 \times 10^6 \text{ photons/s}}_{\text{rubidium}}$

$\langle P_e \rangle_t = \frac{1}{2} \Rightarrow$ the atom spends half the time in the excited state.



In strong contradiction with classical physics, but in good agreement with experiment

Case $I \ll I_{sat}$, i.e. $s_0 \to 0$, then $\underbrace{\gamma_{scattering} \simeq s_0 \frac{\gamma}{2} \propto \text{Incident Intensity}}_{\text{good agreement with classical physics}}$

off-Resonance

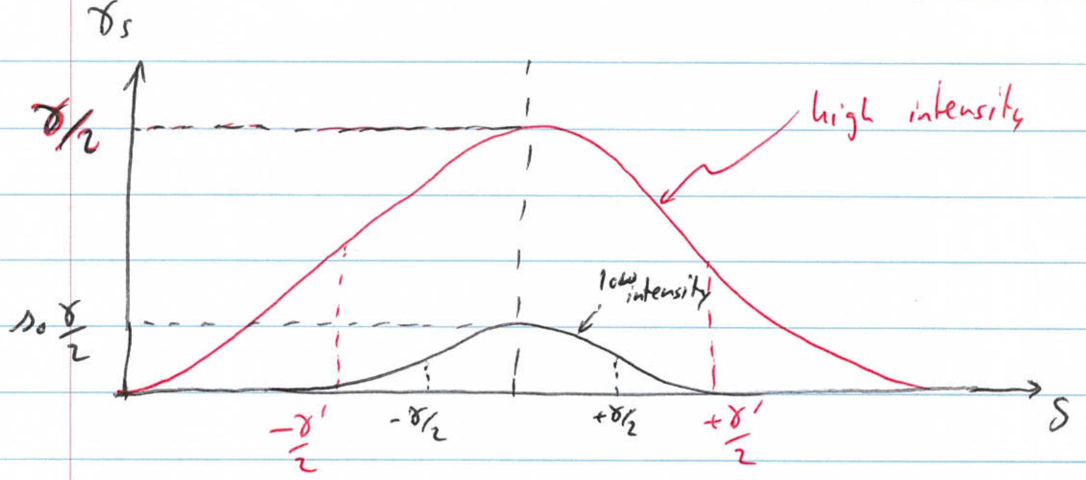
Linewidth for $s_0 \to 0$: $\gamma_s \simeq \frac{1}{1+\left(2\frac{\delta}{\gamma}\right)^2} s_0 \frac{\gamma}{2}$

$$\Rightarrow \text{FWHM} = \gamma$$

Linewidth for $s_0 \to +\infty$ "power broadening"

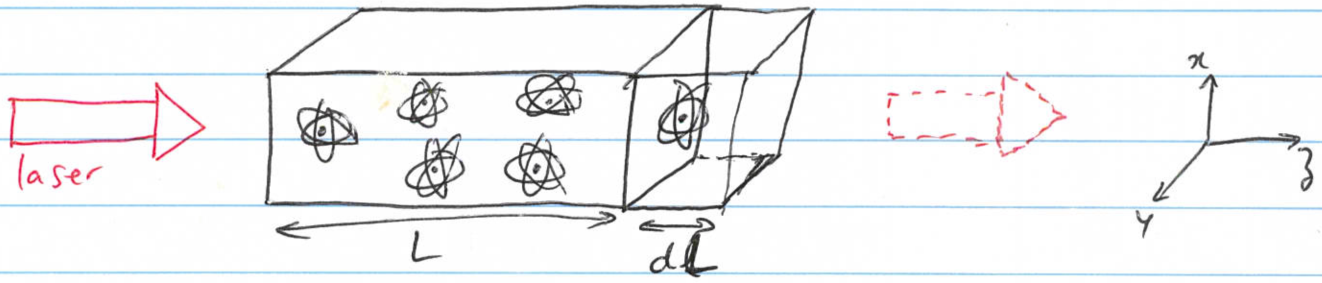
$$\gamma_s = \frac{s_0}{1+s_0} \frac{1}{1+\left(\frac{2\delta}{\gamma\sqrt{1+s_0}}\right)^2} \frac{\gamma}{2} \simeq \frac{1}{1+\left(\frac{2\delta}{\gamma'}\right)^2} \frac{\gamma}{2}$$

with $\gamma' = \gamma\sqrt{1+s_0} = \text{FWHM}$



Beer's law : Absorption in an atomic vapor

How is a laser attenuated as it travels through a gas of resonant atoms?



$$\text{gas density} = n = \frac{N_{\text{atoms}}}{\text{Volume}}$$

$$\text{change in intensity} = dI = \frac{d\,\text{Power}}{\text{Area}} = -\frac{\boxed{\hbar\omega_\ell \gamma_s}}{\cancel{A}}\,\underbrace{n\cancel{A}dL}_{\text{\# atoms in ``dL'' volume}}$$

A = area of laser

photons per atom

$$\text{thus}\quad \frac{dI}{dL} = -\hbar\omega_\ell \gamma_s n$$

In the weak intensity limit: $\gamma_s = \frac{I}{I_{sat}} \cdot \frac{\gamma}{2}$ (on resonance)

$$\frac{dI}{dL} = -\hbar\omega_\ell \frac{\gamma}{2} \frac{n}{I_{sat}} I = -\boxed{\frac{3\lambda^2}{2\pi}}\, n I \quad \text{(on resonance)}$$

$$= -\sigma_{eg} n I$$

σ_{eg} = cross section on resonance

$$\Rightarrow \boxed{I(z) = I_o \, e^{-\sigma_{eg} n z}}$$

Beer's Law

__note:__ Optical depth $= \sigma_{eg} n z$

$\hookrightarrow I = I_o e^{-\text{optical depth}}$

__Application__ : absorption imaging of cold atoms.