PHYS 610: Electricity & Magnetism I
Due date: Thursday, February 4, 2016

Problem set #2

1. Adding rapidities
Prove that collinear rapidities are additive, i.e. if A has a rapidity η relative to B, and B has rapidity ξ relative to C, then A has rapidity η+ξ relative to C.

2. Velocity transformation
Consider a particle moving with velocity $\vec{u} = (u_x, u_y, u_z)$ in frame S. Frame S' moves with velocity $\vec{v} = v\hat{z}$ in from S. Show that the velocity $\vec{u}' = (u'_x, u'_y, u'_z)$ of the particle as measured in frame S' is given by the following expressions:

$$u'_x = \frac{dx'}{dt'} = \frac{u_x}{\gamma(1 - vu_z/c^2)}$$

$$u'_y = \frac{dy'}{dt'} = \frac{u_y}{\gamma(1 - vu_z/c^2)}$$

$$u'_z = \frac{dz'}{dt'} = \frac{u_z - v}{(1 - vu_z/c^2)}$$

Note that the velocity components perpendicular to the frame motion are transformed (as opposed to the Lorentz transformation of the coordinates of the particle). What is the physics for this difference in behavior?

3. Relativistic acceleration
Jackson, problem 11.6.

4. Lorenz gauge
Show that you can always find a gauge function $\lambda(\vec{r}, t)$ such that the Lorenz gauge condition is satisfied (you may assume that a wave equation with an arbitrary source term is solvable).

5. Relativistic Optics
An astronaut in vacuum uses a laser to produce an electromagnetic plane wave with electric amplitude E_0' polarized in the y'-direction travelling in the positive x'-direction in an inertial reference frame S'. The astronaut travels with velocity v along the +z-axis in the S inertial frame.

a) Write down the electric and magnetic fields for this propagating plane wave in the S' inertial frame – you are free to pick the phase convention. Draw a diagram of the set-up that shows the S and S' inertial coordinate systems and the E-M wave in the S' frame

b) Compute the angle θ that the emitted plane wave makes with the +z-axis in the inertial frame S using the Lorentz transformation and the 4-wavevector K^{μ}, which is given by

$K^{\mu} = \begin{pmatrix} \omega / c \\ k_x \\ k_y \\ k_z \end{pmatrix}$, where k is the wavevector and ω is the frequency of the plane wave.

Describe the qualitative behavior of the plane wave angle in the S frame in the relativistic limit.

c) Using $F^{\mu\nu}$ and the Lorentz transformation compute expressions for the electric and magnetic field amplitude components of the plane wave in the S inertial frame from those you established in part a) in the S' frame. Compute the angle that the propagation direction in S makes with the +z-axis. Compare your result with that obtained in part b).
Note for part c): You do ***not*** need to specify the temporal and spatial dependence of the electric and magnetic fields of the plane wave in the S frame.

6. Tensor inner ("scalar") product

a) Compute the scalar invariants $F^{\mu\nu}F_{\mu\nu}$, $G^{\mu\nu}G_{\mu\nu}$, $F^{\mu\nu}G_{\mu\nu}$ in terms of $\vec{E}$ and $\vec{B}$.

b) Is it possible to have an electromagnetic field that appears as a purely electric field in one frame and as a purely magnetic field in another inertial frame?