PHYS 610: Electricity & Magnetism I
Due date: Thursday, April 28, 2016

Problem set #8

1. Green's function inside a conducting spherical shell

Find the Green's function for calculating the electric potential $V(\vec{r})$ due to an arbitrary charge distribution inside a conducting spherical shell of radius *R*.

2. Free space Green's functions by eigenfunction expansion

Find the free-space Green's function for the following 3 situations:

a) dimension=3.

b) dimension=2.
Note: you will need (i) an integral representations of $J_0(x)$ and $K_0(x)$ and (ii) the regularization $1/k = \lim_{\delta \to 0} k/(k^2 + \delta^2)$.

c) dimension=1.
Note: you will need $sgn(x) = -1 + 2\int_{-\infty}^{x} \delta(y)dy$.

3. The almost-concentric spherical capacitor

A spherical conducting shell centered at the origin has radius R_1 and is maintained at potential V_1. A second spherical conducting shell is maintained at potential V_2 has radius $R_2 > R_1$ but is somewhat off-center by a distance $\delta << R_1$ in the $+\hat{z}$ direction.

a) Calculate the capacitance of the system for $\delta = 0$.

b) To lowest order is δ, show that surface charge density induced on the inner shell is given by

$$\sigma(\theta) = \epsilon_0 \frac{R_1 R_2 (V_2 - V_1)}{R_2 - R_1} \left(\frac{1}{R_1^2} - \frac{3\delta}{R_2^3 - R_1^3} \cos\theta \right)$$

Note: It may be helpful to first show that the boundary of the outer spherical shell can be approximated as $r_2 = R_2 + \delta \cos\theta$.

c) Compute the total charge Q on the inner shell and the capacitance of the off-center capacitor to lowest order in δ.

d) To lowest order in δ, show that the inner shell experiences a total force

$$\vec{F} = -\frac{Q^2}{4\pi\epsilon_0} \frac{\delta}{R_2^3 - R_1^3} \hat{z}$$

Note: the force is given by the charge density and the electric field in the vicinity of the charge.

e) Integrate the force in (d) to find the capacitance of the system to second order in δ.

4. A little more Jackson

Jackson 3.6

5. Green's reciprocity theorem

a) Consider a charge distribution $\rho_1(\vec{r})$ that produces a potential $V_1(\vec{r})$, and a separate charge distribution $\rho_2(\vec{r})$ that produces a potential $V_2(\vec{r})$. The charge distributions are entirely unrelated, and are not even present at the same time, i.e. these two electrostatic situations are different "problems" and are not present simultaneously. Prove Green's reciprocity theorem:

$$\int_{\substack{all \\ space}} \rho_1 V_2 \, d^3r = \int_{\substack{all \\ space}} \rho_2 V_1 \, d^3r$$

Note: you may find it useful to calculate $\int \vec{E}_1 \cdot \vec{E}_2 \; d^3r$ in two different ways.

b) Consider two spatially separated and distinct conductors, A and B. If you charge up conductor A with charge Q (B remains uncharged), then the resulting potential of conductor B is V_{AB}. Alternatively, if instead you charge up conductor B with charge Q (A remains uncharged), then the resulting potential of conductor A is V_{BA}. Use Green's reciprocity theorem to show that $V_{AB} = V_{BA}$.

Note: this result makes no assumptions about the position or shapes of conductors A and B.

c) Both plates of a very large parallel plate capacitor are grounded and separated by a distance d. A point charge q is placed between them at a distance x from plate 1. Use Green's reciprocity theorem to calculate the induced charge on each plate.

Hint: For the charge on plate 1, use the actual situation, while for the charge on plate 2, remove the q, and set one of the conductors at potential V_0.

6. Dielectric sphere with free charge

A dielectric sphere (with dielectric constant κ) of radius R is filled with a uniform free charge density ρ_c.

a) Find the polarization $\vec{P}(\vec{r})$.

b) Calculate the total bound charge of the sphere (volume and surface). Explain the result briefly.