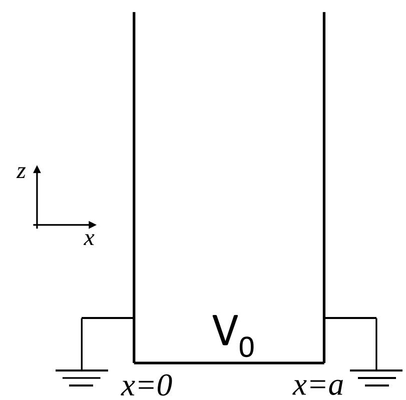
PHYS 610: Electricity & Magnetism I
Due date: Thursday, April 7, 2016

Problem set #7

1. Conducting slot
Consider an infinitely long and deep slot formed by two grounded conductor plates at $x=0$ and $x=a$, as shown in the figure below. A third conductor plate, at $z=0$, is held at a constant potential V_0.

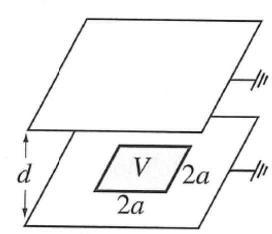


a) Find the potential inside the slot as a series solution.

b) Sum the series (hint: see Jackson section 2.10).

c) Determine the asymptotic behavior of the potential when $z>>a$.

2.
A Potential Patch by Separation of Variables The square region defined by $-a \le x \le a$ and $-a \le y \le a$ in the $z = 0$ plane is a conductor held at potential $\varphi = V$. The rest of the $z = 0$ plane is a conductor held at potential $\varphi = 0$. The plane $z = d$ is also a conductor held at zero potential.



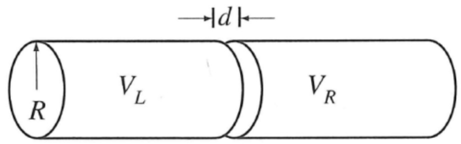
(a) Find the potential for $0 \le z \le d$ in the form of a Fourier integral.
(b) Find the total charge induced on the upper surface of the lower ($z = 0$) plate. The answer is very simple. Do not leave it in the form of an unevaluated integral or infinite series.
(c) Sketch field lines of $\mathbf{E}(\mathbf{r})$ between the plates.

3.

The Two-Cylinder Electron Lens Two semi-infinite, hollow cylinders of radius R are coaxial with the z-axis. Apart from an insulating ring of thickness $d \to 0$, the two cylinders abut one another at $z = 0$ and are held at potentials V_L and V_R. Find the potential everywhere inside both cylinders. You will need the integrals

$$\lambda \int_0^1 ds\, s\, J_0(\lambda s) = J_1(\lambda) \qquad \text{and} \qquad 2\int_0^1 ds\, s\, J_0(x_n s) J_0(x_m s) = J_1^2(x_n)\delta_{nm}.$$

The real numbers x_m satisfy $J_0(x_m) = 0$.



Plot the potential as a function r and z.

Note: Zangwill provides an alternate approach to this problem in "application 7.4".

4. Potential on a sphere

The potential on the surface of a sphere of radius R is given in spherical coordinates by

$$V(r = R, \theta, \phi) = V_0 \cos(3\theta)$$

a) Find the potential inside and outside the sphere (note: V_0 is a constant).

b) Find the surface charge density on the sphere (assume that all charge lies on the surface of the sphere).

5. Electric field inside a sphere

Calculate the volume charge density ρ and surface charge density σ which must be placed in and on a sphere of radius R (no other charges are present) such that the electric field within the sphere is given by

$$\vec{E} = -2V_0 \frac{xy}{R^3}\hat{x} + \frac{V_0}{R^3}(y^2 - x^2)\hat{y} - \frac{V_0}{R}\hat{z}$$

Express your answer in terms of trigonometric functions of the θ and ϕ spherical coordinates.

6. Planar boundary

Consider a conducting sheet in the *x*-*y* plane (i.e. *z=0*). The sheet is divided along the y-axis by a thin insulator, such that *x<0* and *x>0* portions of the sheet are at different potentials $-V_0$ and V_0, respectively (V_0 is a constant over each half sheet).

a) Use a spatial scaling argument to conclude that for *z>0* the potential $V(x, y, z)$ is independent of *r* and *y* (in cylindrical coordinates with a *y*-axis of symmetry).

b) Find the electrostatic potential $V(\phi)$ in the z>0 region, and make a quantitatively accurate sketch of the electric field lines.