PHYS 610: Electricity & Magnetism I
Due date: Thursday, January 26, 2017

Problem set #1

1.

Measuring B Let $\mathbf{F}_1$ and $\mathbf{F}_2$ be the instantaneous forces that act on a particle with charge q when it moves through a magnetic field $\mathbf{B}(\mathbf{r})$ with velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, respectively. Without choosing a coordinate system, show that $\mathbf{B}(\mathbf{r})$ can be determined from the observables $\boldsymbol{v}_1 \times \mathbf{F}_1$ and $\boldsymbol{v}_2 \times \mathbf{F}_2$ if $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are appropriately oriented.

2.

The Coulomb and Biot-Savart Laws The electric and magnetic fields for time-independent distributions of charge and current which go to zero at infinity are

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int d^3r'\,\rho(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} \qquad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r'\mathbf{j}(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}.$$

(a) Calculate $\nabla \cdot \mathbf{E}$ and $\nabla \times \mathbf{E}$.
(b) Calculate $\nabla \cdot \mathbf{B}$ and $\nabla \times \mathbf{B}$. The curl calculation exploits the continuity equation for this situation.

3.

Prelude to Electromagnetic Angular Momentum A particle with charge q is confined to the x-y plane and sits at rest somewhere away from the origin until $t = 0$. At that moment, a magnetic field $\mathbf{B}(x, y) = \Phi\delta(x)\delta(y)\hat{\mathbf{z}}$ turns on with a value of Φ which increases at a constant rate from zero. During the subsequent motion of the particle, show that the quantity $\mathbf{L} + q\boldsymbol{\Phi}/2\pi$ is a constant of the motion where $\mathbf{L}$ is the mechanical angular momentum of the particle with respect to the origin and $\boldsymbol{\Phi} = \Phi\hat{\mathbf{z}}$.

4.

Rotation of Free Fields in Vacuum Let θ be a parameter and define "new" electric and magnetic field vectors as linear combinations of the usual electric and magnetic field vectors:

$$\begin{aligned}\mathbf{E}' &= \mathbf{E}\cos\theta + c\mathbf{B}\sin\theta \\ c\mathbf{B}' &= -\mathbf{E}\sin\theta + c\mathbf{B}\cos\theta.\end{aligned}$$

(a) Show that $\mathbf{E}'$ and $\mathbf{B}'$ satisfy the Maxwell equations without sources ($\rho = \mathbf{j} = 0$) if $\mathbf{E}$ and $\mathbf{B}$ satisfy these equations.
(b) Discuss the implications of this result for source-free solutions of the Maxwell equations where $\mathbf{E} \perp \mathbf{B}$ everywhere in space.

5.

Lorenz Gauge Forever

(a) Suppose φ_L and $\mathbf{A}_L$ satisfy the Lorenz gauge constraint. What equation must Λ satisfy to ensure that $\mathbf{A}' = \mathbf{A}_L - \nabla\Lambda$ and $\varphi' = \varphi_L + \dot{\Lambda}$ are Lorenz gauge potentials also?
(b) What equation must Λ satisfy to ensure that φ' satisfies the same inhomogeneous wave equation as φ_L? Show that the same equation for Λ' also ensures that $\mathbf{A}'$ satisfies the same inhomogeneous wave equation as $\mathbf{A}_L$.