

Problem set #9

1. Free space Green's functions by eigenfunction expansion

Find the free-space Green's function by eigenfunction expansion for the following 3 situations:

a) dimension=3.

b) dimension=2.

Note: you will need (i) an integral representations of $J_0(x)$ and $K_0(x)$ and (ii) the regularization $1/k = \lim_{\delta \rightarrow 0} k/(k^2 + \delta^2)$.

c) dimension=1.

Note: you will need $\text{sgn}(x) = -1 + 2 \int_{-\infty}^x \delta(y) dy$.

2. Potential on a sphere

The potential on the surface of a sphere of radius R is given in spherical coordinates by

$$V(r = R, \theta, \phi) = V_0 \cos(3\theta)$$

a) Find the potential inside and outside the sphere (note: V_0 is a constant).

b) Find the surface charge density on the sphere (assume that all charge lies on the surface of the sphere).

3. Electric field inside a sphere

Calculate the volume charge density ρ and surface charge density σ which must be placed in and on a sphere of radius R (no other charges are present) such that the electric field within the sphere is given by

$$\vec{E} = -2V_0 \frac{xy}{R^3} \hat{x} + \frac{V_0}{R^3} (y^2 - x^2) \hat{y} - \frac{V_0}{R} \hat{z}$$

Express your answer in terms of trigonometric functions of the θ and ϕ spherical coordinates.

4. Planar boundary

Consider a conducting sheet in the x - y plane (i.e. $z=0$). The sheet is divided along the y -axis by a thin insulator, such that $x < 0$ and $x > 0$ portions of the sheet are at different potentials $-V_0$ and V_0 , respectively (V_0 is a constant over each half sheet).

a) Use a spatial scaling argument to conclude that for $z > 0$ the potential $V(x, y, z)$ is independent of r and y (in cylindrical coordinates with a y -axis of symmetry).

b) Find the electrostatic potential $V(\phi)$ in the $z > 0$ region, and make a quantitatively accurate sketch of the electric field lines.

5. A little more Jackson

Jackson 3.6