

### Problem set #4

#### 1. Adding a divergence term to the Lagrangian density

Consider the following modification of the Lagrange density  $\mathcal{L}(\phi, \partial^\mu \phi)$ ,

$$\mathcal{L} \rightarrow \mathcal{L} + \partial_\mu G^\mu(\phi)$$

where  $G^\mu$  is any vector function of the field  $\phi(x^\mu)$ .

- Show that the Euler-Lagrange equations of motion are unchanged.
- Show that this extra term leaves variations of the action integral unchanged.

#### 2. Lagrangian density practice

Find the Euler-Lagrange equations of motion for the scalar field  $\phi(x^\mu)$  if the Lagrangian density is  $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2 \phi^2$ .

#### 3. Jackson problem 12.14

#### 4. A simple classical mechanics field theory in 1D

It is often possible to derive a field theory as the limit of a discrete system. Consider an infinite system of identical point masses  $m$ , separated by identical springs with spring constant  $k$  and equilibrium length  $a$ . Let  $\eta_i$  be the displacement from equilibrium of the  $i$ th point mass.

a) Derive the exact (particle) Lagrangian and Euler-Lagrange equations of motion for this classical system (non-relativistic).

b) Next consider the limit  $m, a \rightarrow 0, k \rightarrow \infty$ , but with  $\mu = m/a$  and  $Y = ka$  held fixed. Now, replace  $\eta_i$  with a smooth function  $\eta(x, t)$  and show that in this limit the Lagrangian may be written as the integral of a density

$$L = \int dx \frac{1}{2} \left[ \mu \left( \frac{\partial \eta}{\partial t} \right)^2 - Y \left( \frac{\partial \eta}{\partial x} \right)^2 \right]$$

and write down the corresponding Euler-Lagrange equation for the  $\eta(x, t)$  field.

#### 5. Particle tracking

A relativistic electron enters a strong uniform magnetic field (directed along the z-axis) at some angle  $\theta$ . No electric field is present.

- Derive an equation for the trajectory of the particle in time (in the lab frame of the magnet) from the Lorentz invariant form of the Lorentz force.
- What are the numerical parameters of the trajectory for a 12.0 GeV electron in a 1.00 Tesla magnetic field if  $\theta = \pi/2$ .