PHYS 610: Electricity & Magnetism I
Due date: Thursday, April 12, 2018

Problem set #9

1. Free space Green's functions by eigenfunction expansion
Find the free-space Green's function by eigenfunction expansion for the following 3 situations:

a) dimension=3.

b) dimension=2.
Note: you will need (i) an integral representations of $J_0(x)$ and $K_0(x)$ and (ii) the regularization $1/k = \lim_{\delta \to 0} k/(k^2 + \delta^2)$.

c) dimension=1.
Note: you will need $sgn(x) = -1 + 2\int_{-\infty}^{x} \delta(y)dy$.

2. Potential on a sphere
The potential on the surface of a sphere of radius R is given in spherical coordinates by

$$V(r = R, \theta, \phi) = V_0 \cos(3\theta)$$

a) Find the potential inside and outside the sphere (note: V_0 is a constant).

b) Find the surface charge density on the sphere (assume that all charge lies on the surface of the sphere).

3. Electric field inside a sphere
Calculate the volume charge density ρ and surface charge density σ which must be placed in and on a sphere of radius R (no other charges are present) such that the electric field within the sphere is given by

$$\vec{E} = -2V_0 \frac{xy}{R^3}\hat{x} + \frac{V_0}{R^3}(y^2 - x^2)\hat{y} - \frac{V_0}{R}\hat{z}$$

Express your answer in terms of trigonometric functions of the θ and ϕ spherical coordinates.

4. Planar boundary
Consider a conducting sheet in the *x*-*y* plane (i.e. *z=0*). The sheet is divided along the y-axis by a thin insulator, such that *x<0* and *x>0* portions of the sheet are at different potentials $-V_0$ and V_0, respectively (V_0 is a constant over each half sheet).

a) Use a spatial scaling argument to conclude that for *z>0* the potential $V(x, y, z)$ is independent of *r* and *y* (in cylindrical coordinates with a *y*-axis of symmetry).

b) Find the electrostatic potential $V(\phi)$ in the z>0 region, and make a quantitatively accurate sketch of the electric field lines.

5. A little more Jackson
Jackson 3.6