PHYS 610: Electricity & Magnetism I
Due date: Thursday, February 8, 2018

**Problem set #3**

**1. Transverse E-field transformation – alternate derivation**
Use the Lorentz transformation properties of the 4-potential and the 4-divergence/gradient operator to directly prove that

$$\vec{E}'_\perp = \gamma(\vec{E} + \vec{v} \times \vec{B})_\perp$$

**2. Magnetic dipole in motion**
a) A magnetic dipole moves with constant velocity $\vec{v}$ as view in the lab frame. Find the vector potential $\vec{A}$ and scalar potential V in the lab frame.
**Hint:** It may be useful to employ the vector $\vec{R} = \vec{r} - \vec{v}t$.

b) Go to the non-relativistic limit ($v \ll c$) and show that the moving magnetic dipole possesses both a magnetic dipole moment and an electric dipole moment.

**3.** Jackson problem 11.19

**4. Relativistic charged particle in a constant electric field**
A point charge *q* with mass *m* moves in a uniform electric field $\vec{E} = E\hat{z}$. The initial energy (kinetic energy plus rest energy), relativistic momentum, and velocity are $U_0$, $p_0$, and $v_0\hat{y}$, respectively. Find $\vec{r}(t)$ and show that eliminating *t* gives the particle trajectory:

$$z = \frac{U_0}{qE}\cosh\left(\frac{qEy}{cp_0}\right)$$

Check the non-relativistic limit.

Note: Neglect any momentum that is stored in the EM field or potential of the charge, i.e. do not use a Lagrangian.

**5. Lagrangian and Hamiltonian of a non-relativistic charged particle**
While the electromagnetic force on a charged particle cannot be derived exclusively from a scalar potential (as required for standard Lagrangian mechanics), in this problem you will show that there exists a Lagrangian functional that produces the correct equations of motion for a charged particle in an arbitrary electromagnetic field.

**a)** Write down the Lorentz force law for charged particle in an electric field $\vec{E}(\vec{r},t)$ and magnetic field $\vec{B}(\vec{r},t)$. Write down the Lorentz force law in terms of the electric potential $V(\vec{r},t)$ and the vector potential $\vec{A}(\vec{r},t)$.

**b)** Show that the Lagrangian functional ***L*** below produces the Lorentz force law for a particle of charge ***q*** and mass ***m*** when it is used in conjunction with the Lagrange equation of motion (i.e. the equations that follow from the principle of least action):

$$L = \frac{1}{2} m\dot{\vec{r}}^{\,2} + q\dot{\vec{r}} \cdot \vec{A}(\vec{r},t) - qV(\vec{r},t)$$

**c)** Derive an expression for the canonical momentum $\vec{p}$, and show that the Hamiltonian ***H*** for the charged particle can be written as

$$H = \frac{1}{2m}\left(\vec{p} - q\vec{A}(\vec{r},t)\right)^2 + qV(\vec{r},t).$$