

PHYS 622: Quantum Mechanics II
 Due date: Thursday, January 24, 2013

Problem Set #1
Basic Review of QM I

Sakurai and Napolitano problems:

3.16, 3.17 [3.15], 3.20 [3.18], 3.24 [3.20] (with $j_1=3/2$ and $j_2=1/2$)

The old (red) Sakurai (revised, 1st ed.) problems are listed in brackets. Problem 3.16 from Sakurai and Napolitano is the following:

Show that the orbital angular-momentum operator L commutes with both the operators p^2 and x^2 .

1. Operator functions

If $f(z)$ is function that can be written with the series expansion $f(z) = \sum_{n=0}^{+\infty} f_n z^n$ with f_n real

numbers, then for an operator A we define $f(A) = \sum_{n=0}^{+\infty} f_n A^n$.

Consider two operators A and B that commute with their commutator.

a) Show that $[A, f(B)] = [A, B]f'(B)$.

b) Show that $[\vec{R}, f(\vec{P})] = i\hbar \nabla_{\vec{P}} f$ and $[\vec{P}, g(\vec{R})] = -i\hbar \nabla_{\vec{R}} g$, where \vec{R} and \vec{P} are the position and momentum operators.

c) Show the Glauber relation: $e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]}$.

hint: define the function $F(t) = e^{At} e^{Bt}$ and show that it satisfies the differential equation

$\frac{dF(t)}{dt} = (A + B + t[A, B])F(t)$. Integrate the equation and use the $F(t=0)$ and $F(t=1)$.

2. Angular momentum operators

A general angular momentum operator $\vec{J} = J_x \hat{x} + J_y \hat{y} + J_z \hat{z}$ obeys the following

commutation relations $[J_i, J_j] = i\hbar \epsilon_{ijk} J_k$.

Show the following relations:

$$[J^2, J_x] = [J^2, J_y] = [J^2, J_z] = 0$$

$$[J_+, J_-] = 2\hbar J_z$$

$$[J_z, J_{\pm}] = \pm \hbar J_{\pm}$$

3. Specific angular momentum operators in their $|J, m_J\rangle$ basis.

For the cases of $J=0$, $J=1/2$, $J=1$, and $J=3/2$, write down the matrices for J_+ , J_- , J_x , J_y , J_z , and J^2 .