Thursday, April 4, 2013

QM Motion in a periodic Potential (continued)

Bloch theorem:

The Hamiltonian eigenstates for a particle in a periodic potential are plane-wave-like, and have the form:

$$\langle x|\phi\rangle = \psi_k(x) = e^{ikx} \underbrace{u_k(x)}_{\text{determined by } V(x)}$$

, where $u_k(x)$ is a periodic function of period a

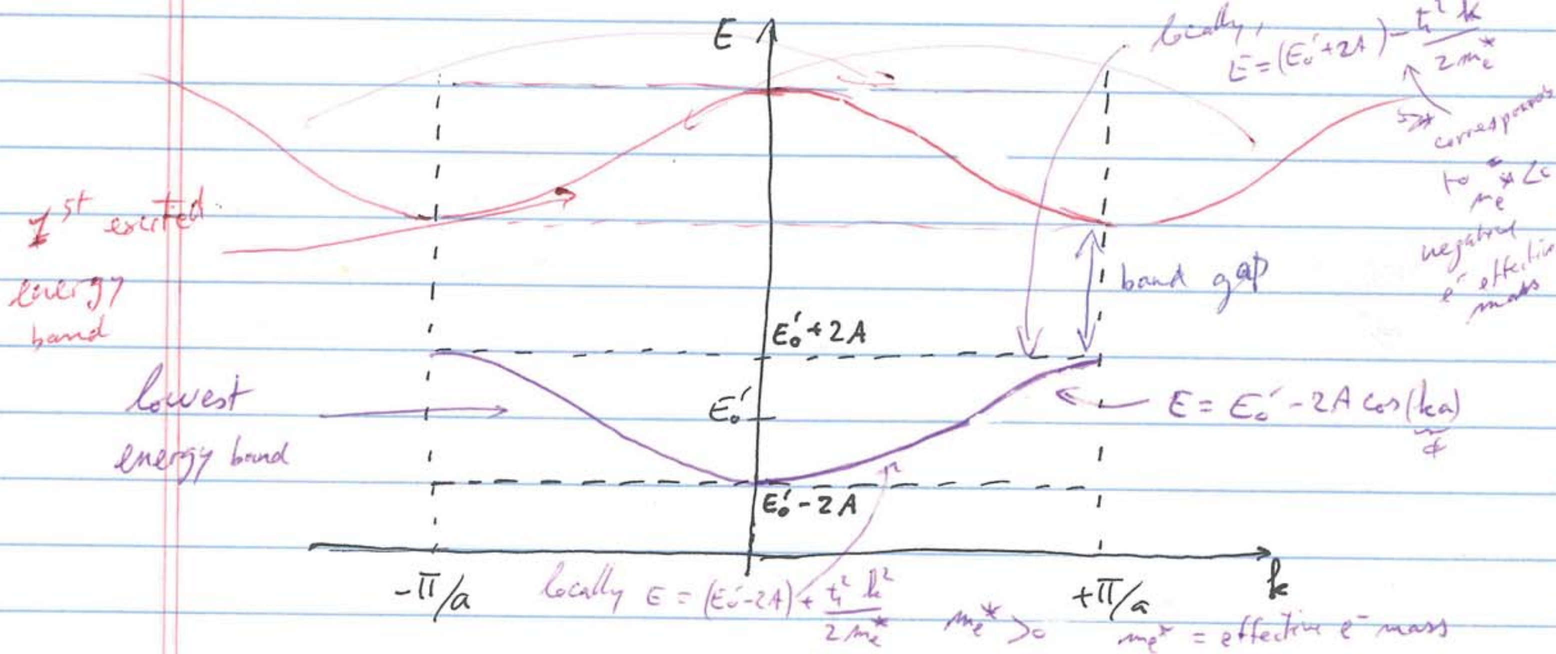
$$u_k(x+a) = u_k(x)$$

$\psi_k(x)$ is an eigenstate of H and $\tau(a)$.

Tight binding model

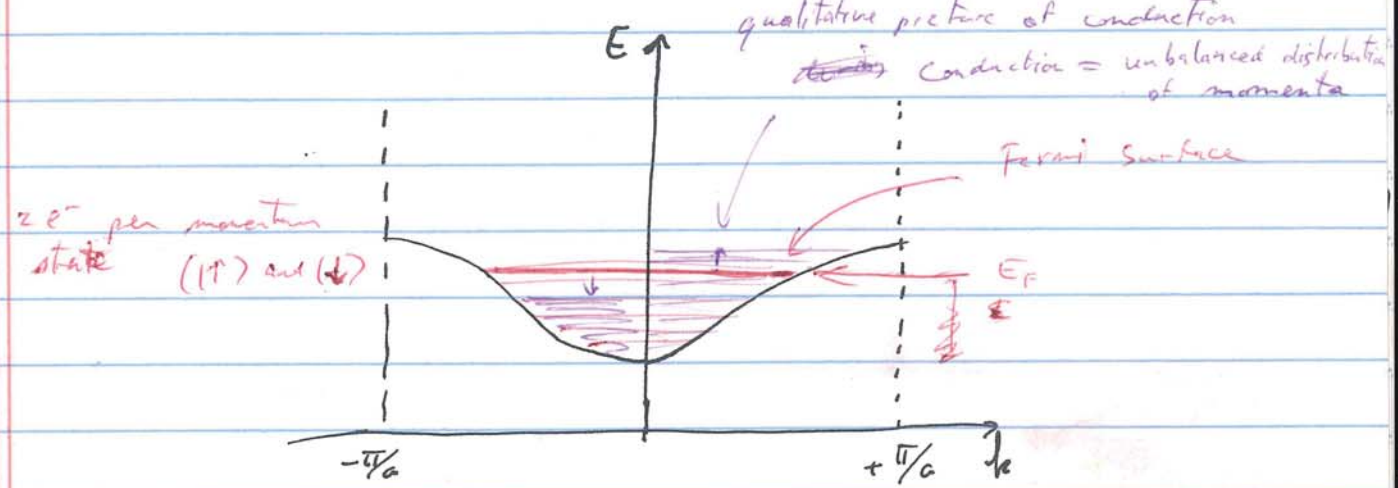
We found that for our model $ka = \phi$

$$H|\psi_k\rangle = H|\phi\rangle = \left(E_0' - 2A\cos\phi\right)|\phi\rangle$$
$$= \left(E_0' - 2A\cos(ka)\right)|\psi_k\rangle$$



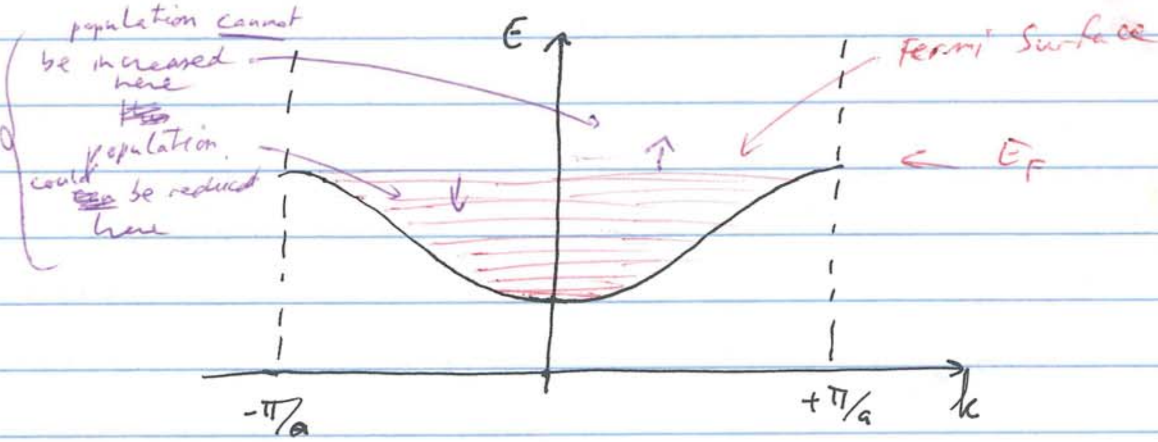
<u>Metals</u> have ~~sufficient~~ ~~an~~ a partially filled ~~the~~ valence energy band.

Pauli Principle

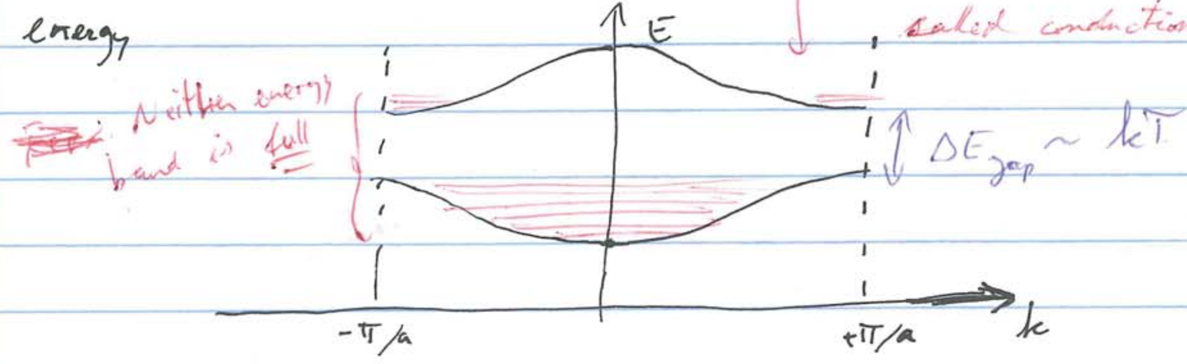


<u>Insulators</u> have a completely full valence energy band

e⁻'s cannot be redistributed to create an momentum unbalanced distribution



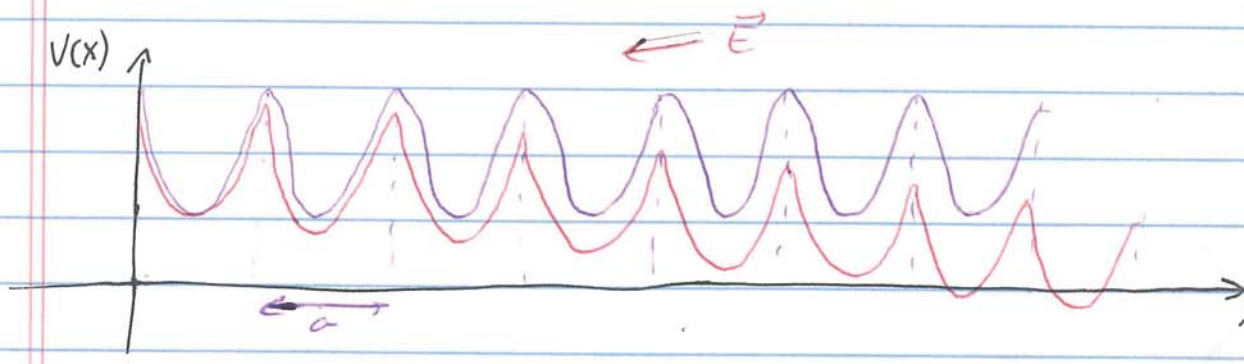
<u>Semiconductors</u> have a completely full valence energy band that thermally populates the <u>first excited band</u> due the ~~small~~ band gap ΔE being ~~...~~ on the order of the thermal energy



Bloch oscillations

Consider a periodic potential for an e^- subject to an E-field

$$H = \frac{p^2}{2m_e} + V_{\substack{\text{periodic}\\ \text{lattice}}}(x) - q_e E x$$



This is a model for conduction of e^-'s in a crystal lattice.

- Classically, if we permit e^-'s to tunnel, then we expect e^- motion to the right $(+x)$.

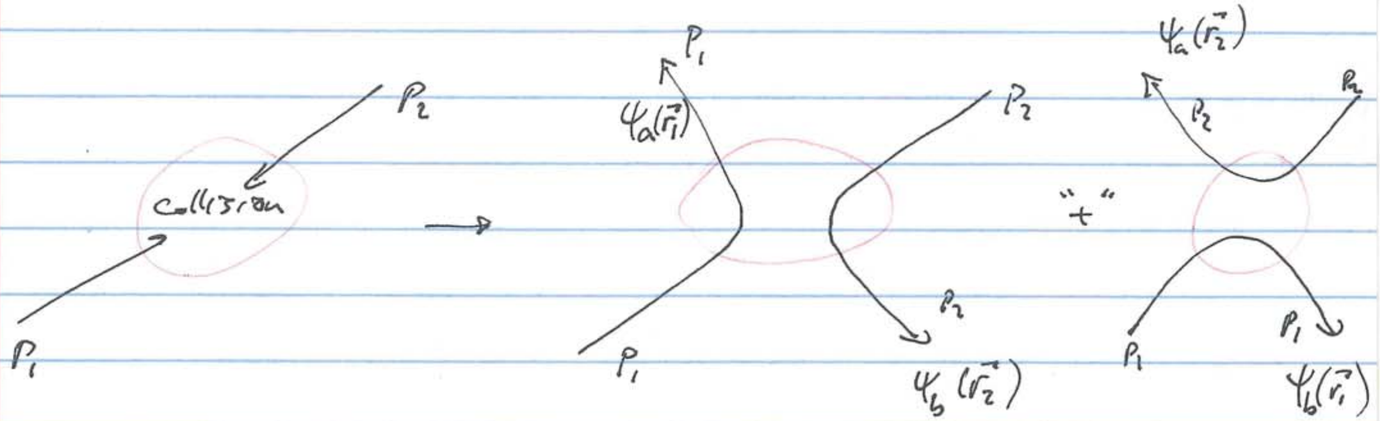
- Quantum Mechanically, we find that the reflection of e^-'s off the lattice barriers as the move right leads to a constructive interference phenomenon, where the ~~the~~ e^-'s oscillate back and forth (at frequency $\omega_{Bloch} = \frac{a\, q_e |E|}{\hbar}$ ← Force) without making progress to the right

This conduction paradox is resolved because in real metals the e^-'s scatter of each other, phonons, and impurities. These scattering processes dephase the e reflections enough to washout the interference effects. → Bloch oscillations are very difficult to observe with e^-'s in crystal, but easy with ~~[illegible]~~ ultracold atoms in an optical lattice.

~~Statistics~~ Identical Particles : Quantum Statistics

Two identical particles

Consider a collision between 2 identical particles



the final two-particle state is

$$\psi_f(\vec{r}_1, \vec{r}_2) = \psi_a(\vec{r}_1)\psi_b(\vec{r}_2) + e^{i\phi}\psi_b(\vec{r}_1)\psi_a(\vec{r}_2)$$

If you exchange particles 1 & 2, then

$$1 \rightleftarrows 2 : \psi_f(\vec{r}_2, \vec{r}_1) = \psi_a(\vec{r}_2)\psi_b(\vec{r}_1) + e^{i\phi}\psi_b(\vec{r}_2)\psi_a(\vec{r}_1)$$

define the permutation operator P: interchanging two identical particles should not really change the wavefunction

$$P\,\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1) \overset{?}{=} e^{i\delta}\psi(\vec{r}_1, \vec{r}_2)$$

However, we require $P^2\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_1, \vec{r}_2) \overset{?}{=} e^{2i\delta}\psi_f(\vec{r}_1, \vec{r}_2)$

$$\Rightarrow e^{i2\delta} = 1 \Rightarrow \delta = 0 \text{ or } \pi$$

$\Rightarrow$ eigenvalues of P are ± 1 → symmetric / antisymmetric

~~Bosons~~ Permutation Symmetry

Identical <u>Bosons</u> are particles whose wavefunction is ~~in which remains~~ <u>symmetric</u> under exchange of <u>any two</u> particles.

<u>2 particles</u>: $\psi(\vec{r}_1,\vec{r}_2) = +\psi(\vec{r}_2,\vec{r}_1) = \frac{1}{\sqrt{2}}\left[\psi_a(\vec{r}_1)\psi_b(\vec{r}_2)+\psi_b(\vec{r}_1)\psi_a(\vec{r}_2)\right]$

$P\psi(\vec{r}_1,\vec{r}_2) = +\psi(\vec{r}_2,\vec{r}_1)$

<u>N particles</u>: $\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_i,\ldots,\vec{r}_j,\ldots,\vec{r}_N)$
$= +\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_j,\ldots,\vec{r}_i,\ldots,\vec{r}_N)$

<u>special case</u>: if only one state is available, the $\psi(\vec{r}_1,\ldots,\vec{r}_N) = \psi_a(\vec{r}_1)\ldots\psi_a(\vec{r}_N)$
(i.e. BEC)

Identical <u>Fermions</u> are particles whose wavefunction is <u>anti-symmetric</u> under exchange of <u>any</u> two particles.

<u>2 particles</u>: $\psi(\vec{r}_1,\vec{r}_2) = -\psi(\vec{r}_2,\vec{r}_1) = \frac{1}{\sqrt{2}}\left[\psi_a(\vec{r}_1)\psi_b(\vec{r}_2)-\psi_b(\vec{r}_1)\psi_a(\vec{r}_2)\right]$

$P\psi(\vec{r}_1,\vec{r}_2) = -\psi(\vec{r}_2,\vec{r}_1)$

<u>N particles</u>: $\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_i,\ldots,\vec{r}_j,\ldots,\vec{r}_N)$
$= -\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_j,\ldots,\vec{r}_i,\ldots,\vec{r}_N)$

~~special case~~ Note:
- You cannot construct an anti-symmetric wavefunction with only a single state
- Each identical fermion must be in a different state, otherwise antisymmetry ~~is~~ is not possible.
  - ↳ <u>Pauli exclusion principle</u>
    - ↳ responsible for chemical differences ~~most of chemistry~~ between elements