PHYS 622: Quantum Mechanics II
Due date: Thursday, March 28, 2013

Problem Set #6

Sakurai and Napolitano problems:
5.22 [5.22], 5.25 [5.25], 5.27[5.27], 5.39 [5.37]

The old (red) Sakurai (revised, 1st ed.) problems are listed in brackets.

1. Parity violation in the $nS_{1/2}$ ground state of an alkali atom

We consider an alkali atom in its $nS_{1/2}$ ground state. We will work in the $|n,l,j,m_j\rangle$ basis (i.e. nominally, we ignore hyperfine terms in the base Hamiltonian, while keeping the spin-orbit coupling term). We will treat the parity violating interaction term, $H_{Z0} = C_{weak}\vec{S}\cdot\left(\vec{P}\delta^3(\vec{R}) + \delta^3(\vec{R})\vec{P}\right)$, as a perturbation. C_{weak} is a very small constant.

a) Calculate the first order correction to the $nS_{1/2}$ eigenstates. In particular, show that the perturbation mixes in the $nP_{1/2}$ states but not the $nP_{3/2}$ states. Use the full Wigner-Eckart theorem (as applied to a (pseudo-) scalar operator) to show that higher order orbital angular momentum states do not contribute. Explain why the higher $n'P_{1/2}$ states are mixed in to a lesser degree than the $nP_{1/2}$ states.

b) Calculate the energy shift of the ground state to first and second order.

c) We now apply an electric field E along the z-axis. Use the parity violating eigenstates to compute the Stark shift to first order in perturbation theory.

2. AC Stark shift

In this problem, you will solve the Schrodinger Equation for a 2-level atom in a laser field without recourse to the rotating wave approximation (except at the end), at least for short times. As seen in class, the general form for the wavefunction of a 2-level atom is

$$\left|\psi(t)\right\rangle = c_g(t)e^{-i\omega_g t}\left|g\right\rangle + c_e(t)e^{-i\omega_e t}\left|e\right\rangle$$

We will assume that at t=0, $c_g\approx 1$ and $c_e\approx 0$ (i.e. the system is at the bottom of a Rabi flop). The interaction with the laser field is given by $\langle g|W|e\rangle = \langle e|W|g\rangle = \hbar\Omega\sin(\omega t)$ (there are no diagonal terms). You can assume that the laser field is in the vicinity of resonance (i.e. $\omega\approx\omega_{eg}$, though with a detuning $\delta=\omega-\omega_{eg}$ that is large enough that $\delta>>\Omega$). We want to derive the Stark energy shift of an atom in a time-varying (AC) electric field, which is difficult if the Hamiltonian does not explicitly conserve energy (i.e. it is time varying): we will do this by looking for the time oscillation frequency of $c_g(t)$.

a) Derive two first order coupled differential equations (exact) for $c_g(t)$ and $c_e(t)$ that are valid at all times (as seen in class).

b) Derive an expression for $c_e(t)$ (no remaining integrals) that is valid at short times by directly integrating one of the differential equations. You may assume that for short times $c_g(t)$ does not change too much.

c) Use the expression for $c_e(t)$ to derive an expression for $c_g(t)$ (no remaining integrals) that is valid at short times, but long enough to average time oscillating terms.

d) Infer the energy shift of the ground state. What happens if you apply the rotating wave approximation (in spirit, perhaps not the way we saw it in class)? Show that the energy shift is proportional to $1/\delta$, where $\delta=\omega-\omega_{eg}$ is the detuning of the laser from resonance. Is the shift linear or quadratic with driving electric field and how does it relate to the intensity of the laser light?