PHYS 622: Quantum Mechanics II
Due date: Thursday, January 23, 2014

Problem Set #1
Basic Review of QM I

Sakurai and Napolitano problems:
3.16, 3.17 [3.15], 3.20 [3.18], 3.24 [3.20] (with j_1=3/2 and j_2=1/2)

The old (red) Sakurai (revised, 1st ed.) problems are listed in brackets. Problem 3.16 from Sakurai and Napolitano is the following:
*Show that the orbital angular-momentum operator **L** commutes with both the operators $\mathbf{p}^2$ and $\mathbf{x}^2$.*

1. Operator functions

If *f(z)* is function that can be written with the series expansion $f(z) = \sum_{n=0}^{+\infty} f_n z^n$ with f_n real numbers, then for an operator A we define $f(A) = \sum_{n=0}^{+\infty} f_n A^n$.

Consider two operators A and B that commute with their commutator.
a) Show that $[A, f(B)] = [A,B]f'(B)$.
b) Show that $[\vec{R}, f(\vec{P})] = i\hbar\nabla_{\vec{P}} f$ and $[\vec{P}, g(\vec{R})] = -i\hbar\nabla_{\vec{R}} g$, where $\vec{R}$ and $\vec{P}$ are the position and momentum operators.
c) Show the Glauber relation: $e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]}$.
hint: define the function $F(t) = e^{At}e^{Bt}$ and show that it satisfies the differential equation $\frac{dF(t)}{dt} = (A + B + t[A,B])F(t)$. Integrate the equation and use the F(t=0) and F(t=1).

2. Angular momentum operators

A general angular momentum operator $\vec{J} = J_x\hat{x} + J_y\hat{y} + J_z\hat{z}$ obeys the following commutation relations $[J_i, J_j] = i\hbar\varepsilon_{ijk}J_k$.
Show the following relations:

$$[J^2, J_y] = [J^2, J_y] = [J^2, J_z] = 0$$

$$[J_+, J_-] = 2\hbar J_z$$

$$[J_z, J_\pm] = \pm\hbar J_\pm$$

3. Specific angular momentum operators in their $|J, m_J\rangle$ basis.

For the cases of J=0, J=1/2, J=1, and J=3/2, write down the matrices for J_+, J_-, J_x, J_y, J_z, and J^2.