PHYS 622: Quantum Mechanics II
Due date: Thursday, February 13, 2014

Problem Set #4

Sakurai and Napolitano problems:
5.7 [5.7] and 5.18 [5.18]

The old (red) Sakurai (revised, 1st ed.) problems are listed in brackets.

1. The $2S_{1/2}$ and $2P_{1/2}$ levels of hydrogen … with Stark shifts
Last week you considered the following Hamiltonian for the hydrogen atom

$$H = H_0 - \frac{P^4}{8m_e^3c^2} + \frac{1}{2}\frac{e^2}{m_e^2c^2}\frac{1}{R^3}\vec{L}\cdot\vec{S} \quad \text{with} \quad H_0 = \frac{P^2}{2m_e} - \frac{e^2}{R}$$

Recall in part a) and b) (last week): You identified a suitable basis which diagonalizes this Hamiltonian and computed the 1st order correction to the eigen-energies of H_0 corresponding to the $2S_{1/2}$ and $2P_{1/2}$ states.

Part c. We now add an electric field E directed along the z-axis. Compute the Stark shifts of the n=2, J=1/2 levels to lowest non-zero order using perturbation theory.

2. Zeeman effect in hydrogen in the strong field limit
Calculate the Zeeman shift for the $1S_{1/2}$ ground states of hydrogen in the limit of a strong magnetic field (i.e. $\mu_B B >> A$) – you should still neglect the quadratic Zeeman term.

The Hamiltonian is

$$H = H_0 + H_{LS} + H_{Zeeman} + H_{HF}$$

with $H_0 = \frac{P^2}{2m_e} - \frac{e^2}{R}$, $H_{LS} = \frac{1}{2}\frac{e^2}{m_e^2c^2}\frac{1}{R^3}\vec{L}\cdot\vec{S}$, and $H_{Zeeman} = \left(\frac{-q_e}{2m_e}\left(\vec{L}+2\vec{S}\right) - \frac{g_I|q_e|}{2m_p}\vec{I}\right)\cdot\vec{B}$.

You will treat the hyperfine Hamiltonian $W = H_{HF} = \frac{A}{\hbar^2}\vec{I}\cdot\vec{S}$ using 1st order perturbation theory, but otherwise you will not make any other approximations. You should also make sure to employ the Wigner-Eckart projection theorem.