

PHYS 622: Quantum Mechanics II
Due date: Thursday, February 27, 2014

Problem Set #6

Sakurai and Napolitano problems:
5.28 [5.28], 5.30 [5.30]

The old (red) Sakurai (revised, 1st ed.) problems are listed in brackets.

1. AC Stark shift

In this problem, you will solve the Schrodinger Equation for a 2-level atom in a laser field without recourse to the rotating wave approximation (except at the end), at least for short times. As seen in class, the general form for the wavefunction of a 2-level atom is

$$|\psi(t)\rangle = c_g(t)e^{-i\omega_g t}|g\rangle + c_e(t)e^{-i\omega_e t}|e\rangle$$

We will assume that at $t=0$, $c_g \approx 1$ and $c_e \approx 0$ (i.e. the system is at the bottom of a Rabi flop). The interaction with the laser field is given by $\langle g|W|e\rangle = \langle e|W|g\rangle = \hbar\Omega \sin(\omega t)$ (there are no diagonal terms). You can assume that the laser field is in the vicinity of resonance (i.e. $\omega \approx \omega_{eg}$, though with a detuning $\delta = \omega - \omega_{eg}$ that is large enough that $\delta \gg \Omega$). We want to derive the Stark energy shift of an atom in a time-varying (AC) electric field, which is difficult if the Hamiltonian does not explicitly conserve energy (i.e. it is time varying): we will do this by looking for the time oscillation frequency of $c_g(t)$.

- Derive two first order coupled differential equations (exact) for $c_g(t)$ and $c_e(t)$ that are valid at all times (as seen in class).
- Derive an expression for $c_e(t)$ (no remaining integrals) that is valid at short times by directly integrating one of the differential equations. You may assume that for short times $c_g(t)$ does not change too much.
- Use the expression for $c_e(t)$ to derive an expression for $c_g(t)$ (no remaining integrals) that is valid at short times, but long enough to average time oscillating terms.
- Infer the energy shift of the ground state. What happens if you apply the rotating wave approximation (in spirit, perhaps not the way we saw it in class)? Show that the energy shift is proportional to $1/\delta$, where $\delta = \omega - \omega_{eg}$ is the detuning of the laser from resonance. Is the shift linear or quadratic with driving electric field and how does it relate to the intensity of the laser light?

(see other side)

2. The translating well

C1 (a) An infinitely deep quantum well of width L is moving with a constant speed \vec{v} along the x -axis as shown below. Find wave functions and corresponding energies of a particle of mass m in such a potential. Verify that your answer is a solution of the Schrodinger equation.

(b) Suppose that at time $t=0$ the potential well instantaneously comes to stop. Assuming that the particle was in the ground state of the moving potential well, write down the expression for the probability of finding it in the k^{th} state of the stationary well. *Do not evaluate the integral!*

(c) Assign a condition for the smallness of the well velocity, such that the particle most likely does not change its quantum state when after the well stops. Give some intuitive physical explanation for your answer.

