PHYS 622: Quantum Mechanics II
Due date: Thursday, March 27, 2014

Problem Set #7

Sakurai and Napolitano problems
4.3 [4.3], 4.11 [4.11], 5.15 [5.15]

The old (red) Sakurai (revised, 1^{st} ed.) problems are listed in brackets.

1. Parity violation in the $nS_{1/2}$ ground state of an alkali atom

We consider an alkali atom in its $nS_{1/2}$ ground state. We will work in the $|n,l,j,m_j\rangle$ basis (i.e. nominally, we ignore hyperfine terms in the base Hamiltonian, while keeping the spin-orbit coupling term). We will treat the parity violating interaction term, $H_{Z0} = C_{weak}\vec{S}\cdot\left(\vec{P}\delta^3(\vec{R}) + \delta^3(\vec{R})\vec{P}\right)$, as a perturbation. C_{weak} is a very small constant.

a) Calculate the first order correction to the $nS_{1/2}$ eigenstates. In particular, show that the perturbation mixes in the $nP_{1/2}$ states but not the $nP_{3/2}$ states. Use the full Wigner-Eckart theorem (as applied to a (pseudo-) scalar operator) to show that higher order orbital angular momentum states do not contribute. Explain why the higher $n'P_{1/2}$ states are mixed in to a lesser degree than the $nP_{1/2}$ states.

b) Calculate the energy shift of the ground state to first and second order.

c) We now apply an electric field E along the z-axis. Use the parity violating eigenstates to compute the Stark shift to first order in perturbation theory.

2. Time reversal operator for spin-1/2

Read Sakurai & Napolitano pages 295-297 on time reversal of a spin-1/2 system (the "Digression on Symmetry Operations" on pages 287-289 may be helpful also).

Derive the following results for the operation of T on spin-1/2 kets:

- $T^2=-1$ and $T^4=1$ for action on spin-1/2 kets.
- $T|\uparrow\rangle = |\downarrow\rangle$ and $T|\downarrow\rangle = -|\uparrow\rangle$ with the appropriate phase convention
- $T^2|j=\textit{half-integer}\rangle = -|j=\textit{half-integer}\rangle$ and $T^2|j=\textit{integer}\rangle = |j=\textit{integer}\rangle$, where *j* refers to the angular momentum of the system subjected to the T operator.