Quantum Transition Edge Sensor Theory Using a Double Well Potential Toy Model

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by

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Abstract

This paper develops a quantum sensing theory to identify small changes in energy similar to transition-edge sensors that identifies small changes in energy using thermal transitions. The theory is developed by creating a toy model of a double well potential to determine the force needed for a given Bose-Einstein Condensate (BEC) to gather in one well. Beginning with the simpler example of a single well potential, we then construct a model for a double well potential and analyze its energy as it depends on various parameters. The parameters of the potential are chosen so that the system remains stable near the point where the scattering length becomes positive. In the symmetric case, the energy is minimized when the population imbalance parameter satisfies $\eta = 0$, corresponding to an equal distribution of particles between the two wells.

Chapter 1

Introduction

It is easy to tell when there is an elephant in the room with you. It is more difficult to tell if there is an ant in the room with you. Without precise tools and clever planning, it is impossible to determine if there is an extra electron in a system, let alone know how many electrons there are supposed to be. However, a single electron can have a noticeable effect when interacting with objects on a similar scale.

Quantum sensing can be used to operate and improve atomic clocks and it can potentially be used to detect these electrons.

Atomic clocks work by isolating atoms, and measuring the frequency of atoms changing energy levels. That frequency is used as the beat of the clock, with several billion of these beats being equivalent to a second. For example, as any two Cesium-133 atoms are identical, atomic clocks are more consistent with all other atomic clocks than clocks relying on quartz crystal as their beat, which will never be fully identical. However, a single unwanted and nearby electron will shift the energy of such an atomic clock, and therefore its frequency, causing inaccuracies in the clock's timekeeping, which can then be detected.

One type of device that can measure these transitions are transition-edge sensors (TESs). They work by taking advantage of the phase transition between superconducting and normal-conducting states of a material. When a material transitions to

become a superconductor, its resistance sharply drops to near 0. When said material is held at its critical temperature, its resistance is sensitive to small temperature, and therefore energy, changes. Thus TESs can be very accurate detectors by measuring the resistance of those superconductors [1].

This paper develops a quantum sensing scheme based on Bose-Einstein Condensates and Ramsey interferometry. A Ramsey interferometer can measure changes in energy by measuring transition frequencies. We can look at the orange curve in Figure 1.1 and see that a change in energy will cause a change in the signal allowing us to measure it. However, for our purposes (e.g. detecting the presence of an electron), we do not need to measure the magnitude of change in energy, just the existence of one. So if that Ramsey signal was more digital, as seen in the blue line of Figure 1.1, then a small change in energy from the black dot (our initial position), will cause a significant change in signal, allowing it to be immediately apparent that an extra energy source or electron is present.

We can represent this digital signal abstractly as a double well potential. Think of one well as the "spin up" well and the other as the "spin down" well. A visual can be seen in Figure 1.2. When both wells are even, we see the spins are evenly distributed to minimize the energy, but when there is a small shift to make one well deeper/shallower, all will go into the deeper energy well.

Electrons have an electric field proportional to $1/r^2$, so atoms close to the electron will have a big shift, but particles farther away will have a smaller shift. This variation in shift is represented by the double well potential with one well getting deeper than the other.

We see in Figure 1.3 that the right-side well (well B) is less affected by the electron and so has a lower minimum energy. Therefore, the particles will tend to gather in well B. So, instead of needing a precise sensor, it is very easy to see that there was a

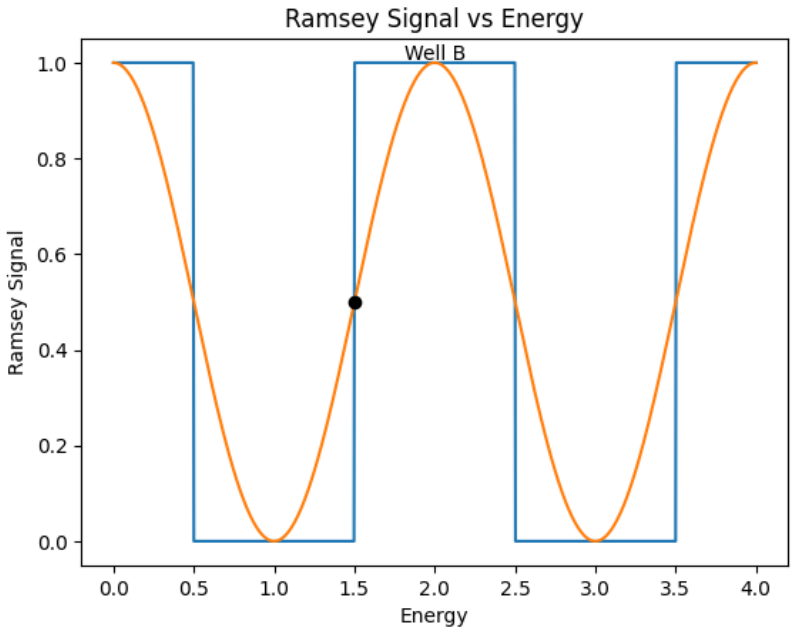

Figure 1.1: A representation of the change in probability of a particle being spin up depending on the energy. The orange line represents the typical curve of a Ramsey signal, while the blue line represents the digital curve of that signal. A Ramsey signal of 0 corresponds to Well A, or the "spin down" well, and a Ramsey signal of 1 corresponds to Well B, or the "spin up" well.

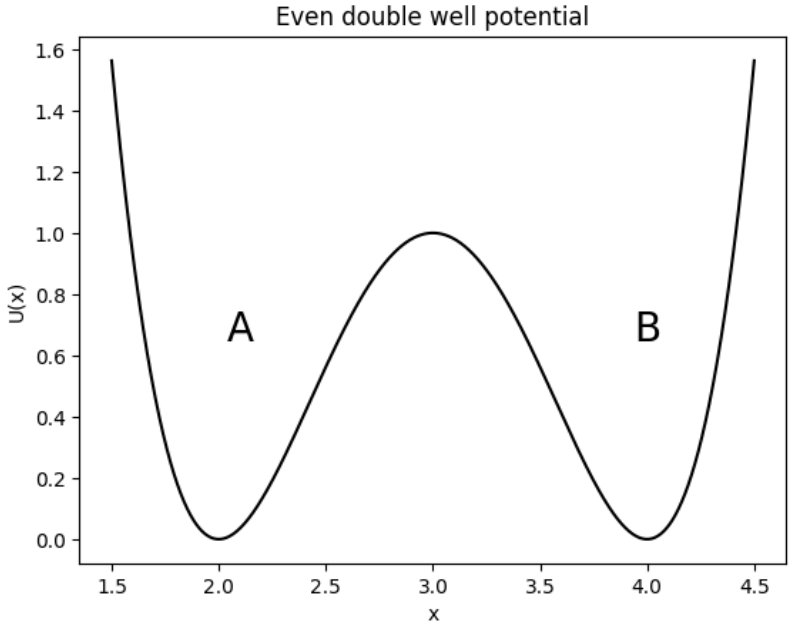

Figure 1.2: The collection of spin up and spin down particles represented as a double well potential. Well A represents the spin down particles and well B represents the spin up particles.

change in energy, by looking at the population imbalance in the two wells.

For this scheme to apply to Bose-Einstein condensates (BECs), we need to consider their properties a little more. BECs are made up of particles that either have repulsive or attractive interactions. If there are repulsive interactions, then they will not want to gather in the lower energy well as the repulsion will become too strong. If interactions are too attractive, they will want to gather in one well even when the wells are even. So the set up needs to be crafted, so that the particles are initially split evenly, but once a small force affects them, they will all fall into one well. Similarly to the transition-edge sensors that use temperature, we want to find the edge between the transition of spin up and down particles that will allow us to sense small changes in energy. In this paper, we will create a model that can be analyzed to find a small ϵ force that will recreate this scheme.

This thesis is structured in the following manner. Chapter 2 introduces the nec-

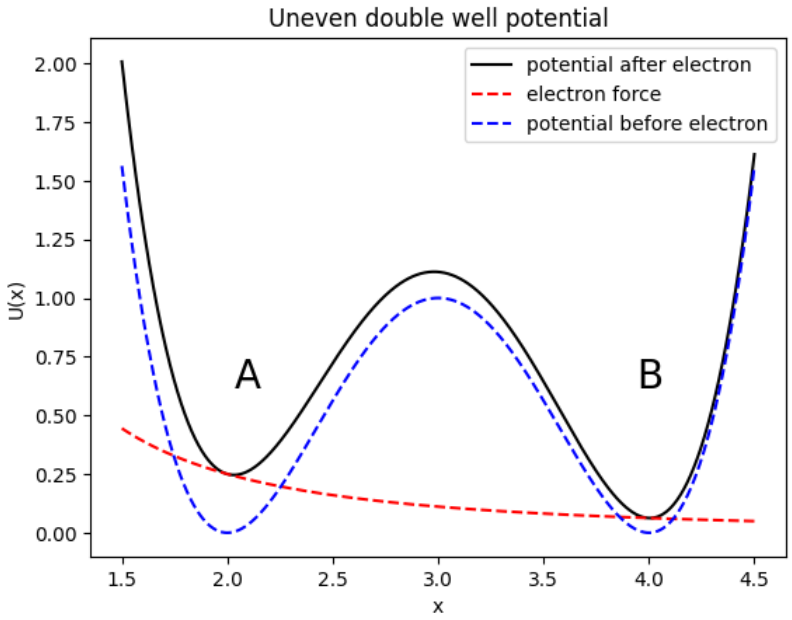

Figure 1.3: The double well potential after it comes into contact with an electron force. The wells have shifted in energy so that one well is shallower than the other. As the right well has a lower minimum energy, the BEC atoms will gather into that well.

essary background for the project. Chapter 3 introduces the math used to create the models in Chapter. The thesis concludes in chapter 5 and provides an outlook for future work.

Chapter 2

Theory

2.1 Fundamental Quantum Particles

Quantum particles are classified into two categories based on their intrinsic spin: fermions (half-integer spin) and bosons (integer spin). This spin determines the symmetry properties of their quantum wavefunctions.

Fermions have antisymmetric wavefunctions, meaning that when two identical fermions are exchanged, the wavefunction changes sign. This property leads to the Pauli exclusion principle: no two fermions can occupy the exact same quantum state. Common examples of fermions are electrons, protons, and neutrons, as well as atoms such as potassium-40 and lithium-6.

Bosons have symmetric wave functions, meaning that when two identical bosons are exchanged, the wavefunction remains unchanged. Bosons do not have an equivalent Pauli exclusion principle, and many bosons can occupy the same quantum state. Some common examples include photons and the Higgs boson, as well as atoms such as rubidium-87 and potassium-41.

A result of this is that no matter how much a system of fermions is cooled, they will move to less excited energy levels. But as they cannot occupy the exact same state, not all fermions will be able to move to the ground state.

Bosons do not have this limitation. When cooled below a critical temperature, a system of bosons can condense into the ground state, forming a type of matter called a Bose-Einstein condensate, where all of the bosons have the same wavefunction.

2.2 Creation of the Bose-Einstein Condensate

Bose-Einstein Condensates (BECs) were initially theorized by Satyendra Bose and Albert Einstein in 1924. It was 71 years before they were experimentally created by Eric Cornell and Carl Wieman in 1995 using rubidium-87 atoms[2].

Rubidium-87 is one of the most common isotopes used for creating BECs. Rubidium-87 is made up of 37 protons, 50 neutrons, and 37 electrons, all of which are fermion spin-1/2 particles. As 37+50+37=124, rubidium-87 has an integer spin, and so is bosonic.

BECs are characterized by their scattering length, denoted a, which describes interatomic interactions. A positive scattering length ($a > 0$) indicates repulsive interactions, where atoms push each other apart. A negative scattering length ($a < 0$) indicates attractive interactions, where atoms attract each other. Repulsive interactions typically produce more stable condensates, as attractive interactions can cause the BEC to collapse on itself if the density becomes too high. Rubidium-87 has $a = 100a_0$ where $a_0 = 0.53$Å, the Bohr radius.

Attractive interactions can enable interesting phenomena, which will be seen in our toy model in Chapter 4 which looks at BECs with both attractive and negative interactions.

2.2.1 Self-Trapping Effect

When a Bose-Einstein condensate is trapped in a double-well potential, atoms can tunnel between the two wells. The tunneling occurs because of the quantum mechan-

ical nature of the atoms, but the interatomic interaction in the BEC can influence the tunneling dynamics. When those interactions are weak, the atoms will evenly oscillate between the wells. When interactions become sufficiently strong, there will be nonlinear oscillations, and if the interaction is attractive the atoms will gather into one well, and self-trapping occurs[3].

Our model will be designed to be very near to having this self-trapping effect occur, and the effect of the electron will be enough to trigger it.

2.3 Gross-Pitaevskii Equation

The Gross-Pitaevskii equation describes the ground state of a quantum system of identical bosons, making it well-suited for describing BECs. It uses a mean-field approximation, so it is assumed that every atom experiences an average interaction with all other atoms, characterized by $U_0 = \frac{4\pi\hbar^2 a}{m}$, where $\hbar$ is the reduced Planck constant, a is the scattering length, and m is the mass of an individual atom. It holds for dilute gases where the scattering length a is much less than the mean interparticle spacing ($na^3 \ll 1$)[4].

The time-independent Gross-Pitaevskii equation is as follows

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r) + V(r)\psi(r) + U_0|\psi(r)|^2\psi(r) = \mu\psi(r) \tag{2.1}$$

where $\psi(r)$ is the wavefunction of the BEC, $V(r)$ is the external trapping potential, μ is the chemical potential, and $|\psi(r)|^2$ is the density. The 3 components on the left-hand side of Equation 2.1 correspond to the kinetic, potential, and interaction energy, respectively. We note that the BEC wavefunction is normalized such that

$$\int |\psi|^2 d^3\mathbf{r} = N$$

where N is the total number of atoms.

For a uniform Bose gas with no external potential, this simplifies to

$$\mu = U_0|\psi(r)|^2 = U_0 n \tag{2.2}$$

where n is the density of particles, $n = |\psi(r)|^2$. This shows the chemical potential is entirely dependent on the interaction strength.

The total energy of the condensate is given by [4]

$$E = \int d^3\mathbf{r} \left[\frac{\hbar^2}{2m} |\nabla\psi(r)|^2 + V(r)|\psi(r)|^2 + \frac{N(N-1)}{2N^2} U_0 |\psi(r)|^4 \right] \tag{2.3}$$

where the three components correspond to the kinetic, external potential, and interaction energy, once again.

Chapter 3

Methods

3.1 Single Well Solution

Before generating the double well potential model, we recreated a model of a Bose-Einstein Condensate in a single well potential using the variational method with the Gross-Pitaevskii equation.

Using the method shown in [4], we can use Equation 2.3 and omit the $1/N$ term to get the following equation for the energy:

$$E = \int dr[\frac{\hbar^2}{2m}|\nabla\psi(r)|^2 + V(r)|\psi(r)|^2 + \frac{1}{2}U_0|\psi(r)|^4] \tag{3.1}$$

where ψ is the wavefunction, V is the potential of the function, U_0 is the potential energy of the function, which corresponds to $U_0 = 4\pi\hbar^2 a/m$, where a is the scattering length, and m is the mass.

The equation for the harmonic potential of a single well potential is

$$V(x, y, z) = \frac{1}{2}m(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2) \tag{3.2}$$

where each ω_i is the oscillator frequency in the respective axis, and x, y, z is the position in that axis.

The wavefunction of a non-interacting Bose-Einstein Condensate in a single well potential is a Gaussian:

$$\psi(r) = \frac{N^{1/2}}{\pi^{3/4}(a_1 a_2 a_3)^{1/2}} e^{-x^2/2a_1^2} e^{-y^2/2a_2^2} e^{-z^2/2a_3^2} \tag{3.3}$$

where N is the number of particles and a_i is the oscillator lengths corresponding to $a_i^2 = \hbar/m\omega_i$.

As our model includes interatomic interactions, we can use the variational method, whereby we replace a_i with a generic parameter b_i that accounts for those interactions along with the oscillator length.

$$\psi(r) = \frac{N^{1/2}}{\pi^{3/4}(b_1 b_2 b_3)^{1/2}} e^{-x^2/2b_1^2} e^{-y^2/2b_2^2} e^{-z^2/2b_3^2} \tag{3.4}$$

As our equation for the energy of the wavefunction relies on different powers of the wavefunction, we raise Equation 3.4 to the power of 2 and 4 to get

$$|\psi(r)|^2 = \frac{N}{\pi^{3/2} b_1 b_2 b_3} e^{-x^2/b_1^2} e^{-y^2/b_2^2} e^{-z^2/b_3^2} \tag{3.5}$$

$$|\psi(r)|^4 = \frac{N^2}{\pi^3 (b_1 b_2 b_3)^2} e^{-2x^2/b_1^2} e^{-2y^2/b_2^2} e^{-2z^2/b_3^2} \tag{3.6}$$

We will also need to find the gradient squared, which is

$$|\nabla\psi(r)|^2 = \frac{N}{\pi^{3/2} b_1 b_2 b_3} e^{-x^2/b_1^2} e^{-y^2/b_2^2} e^{-z^2/b_3^2} \cdot \left(\frac{x^2}{b_1^4} + \frac{y^2}{b_2^4} + \frac{z^2}{b_3^4}\right) \tag{3.7}$$

To use Equation 3.1, we can set $dr = dxdydz$ and the bounds of the integral to be from $-\infty$ to ∞. This will allow us to use the definite integral of the gaussian integral so the integral is solvable.

Using all of the equations above, we can find the energy to be equal to

$$E(\psi) = N \sum_i \hbar\omega_i \left(\frac{a_i^2}{4b_i^2} + \frac{b_i^2}{4a_i^2}\right) + \frac{N^2 U_0}{2(2\pi)^{3/2}(b_1 b_2 b_3)} \tag{3.8}$$

In the case where the potential is isotropic, this simplifies down to

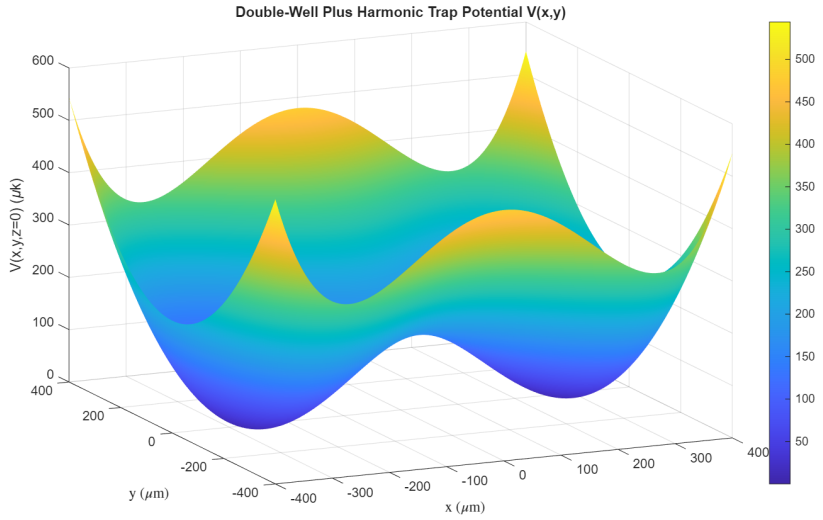

Figure 3.1: A graph of the double well trap potential used for our model.

$$E(\psi) = 3N\hbar\omega(\frac{\alpha^2}{4b^2} + \frac{b^2}{4\alpha^2}) + \frac{N^2U_0}{2(2\pi)^{3/2}b^3} \tag{3.9}$$

as $a_i = \alpha$ for all i and thus $b_i = b$ for all i.

The energy per particle vs b graph shown in Figure 4.1 uses the Equation 3.9.

3.2 Extension to Double Well Potential

Now if we change the potential into a double well potential, then this energy equation will change.

We will use Section B of [5] for the double-well potential function which will be the following:

$$V(r) = B(x^2 - \frac{d}{2B})^2 + \frac{1}{2}m\omega^2(y^2 + z^2) \tag{3.10}$$

A visualization of the double well potential is shown in Figure 3.1. It is a double-well potential in the x-direction, and a simple harmonic potential in the y, z direction. $\frac{d}{2B}$ represents the space between the two wells, and B is an amplitude for the well.

We will choose $B = \frac{1}{2}mw^2 \cdot \frac{1}{a_{\text{osc}^2}}$ and $d = 2Ba_{\text{osc}}^2$ with $a_{\text{osc}} = \sqrt{\frac{\hbar}{m\omega}}$ and m is the mass, and ω is the oscillator frequency. We will assume for the double well model that each axis has the same oscillator frequency.

Our new wavefunction using the variational method will be the sum of two Gaussians, one in each well, and we will replace the oscillator lengths with parameters b_1, b_2, b_3. Along with including an L parameter to represent how many particles are in the left well, with a fixed total number of particles N. We will also denote $q_0^2 = d/2B$, which indicates the position of the two wells at $x = \pm q_0$.

$$\begin{aligned}\psi(x,y,z) =& \frac{L^{1/2}}{\pi^{3/4}(b_1b_2b_3)^{1/2}} e^{-(x-q_0)^2/b_1^2} e^{-y^2/2b_2^2} e^{-z^2/2b_3^2} \\ &+ \frac{(N-L)^{1/2}}{\pi^{3/4}(b_1b_2b_3)^{1/2}} e^{-(x+q_0)^2/b_1^2} e^{-y^2/2b_2^2} e^{-z^2/2b_3^2}\end{aligned}$$

We can simplify this to

$$\psi(x,y,z) = \frac{e^{-(x+q_0)^2} e^{-y^2/2b_2^2} e^{-z^2/2b_3^2}}{\pi^{3/4}(b_1b_2b_3)^{1/2}} (L^{1/2} e^{2xq_0/b_1^2} + (N-L)^{1/2}) \tag{3.11}$$

We will assume both Gaussians have the same oscillator lengths, and that they are all isotropic, so $b_1 = b_2 = b_3$.

$$\psi(x,y,z) = \frac{e^{-\left((x+q_0)^2+y^2+z^2\right)/2b^2}}{\pi^{3/4}b^{3/2}} \left(L^{1/2} e^{2xq_0/b^2} + \sqrt{N-L}\right) \tag{3.12}$$

Using the same strategy as for the single well, we can solve for the energy using Equation 3.1 (see Chapter 4, section 4.2).

Chapter 4

Results

In this chapter, we will use the equations from Chapter 3 to graph the energy with the parameter b, η, where b is a parameter for the width of the Gaussian wavefunction and η is a ratio of how many particles are in the left and right well.

4.1 Single Well Model

If we want to create the desired digital effect, we need our system set up so that the system with no extra electron or energy source has a local minimum, but only barely so. This way, even with a slight change in the ground state energies of the two wells, the BEC will stay stable.

From Figure 4.1, we can see that when the ratio Na/a_{osc} is too negative or too positive, the energy curve has no local maximum. When the ratio is sufficiently negative, corresponding to strong attractive interactions and a negative scattering length, the slope of the curve is always negative. This means the Bose-Einstein condensate will start to collapse.

When the ratio is positive, implying that the Bose-Einstein Condensate has repulsive interactions (the scattering length is positive), the interactions prevent the particles from gathering together. If the repulsive interactions are too strong, the energy curve is shaped similarly to a parabola, so the system will resist changes in

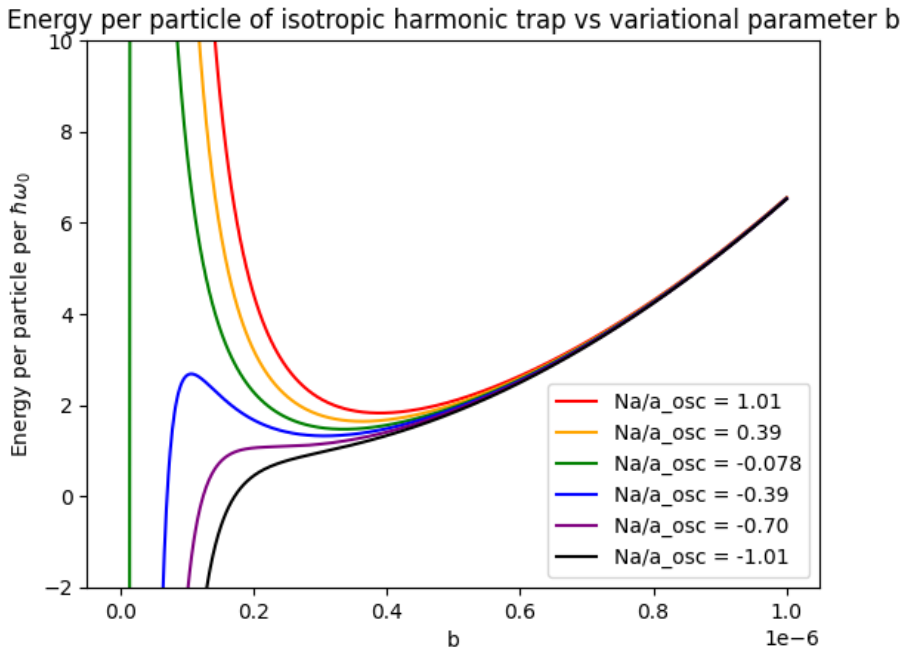

Figure 4.1: The graph shows how the energy per particle changes depending on b, a parameter for the width of the Gaussian wavefunction. The different lines correspond to wavefunctions that have different rations of N, a, and a_{osc}. The BEC will take on the value of b that minimizes the energy $E/\hbar\omega_0$.

energy and will be minimally affected by the extra energy source.

Therefore, we need to carefully tune the ratio to be negative but smaller than the critical point where the BEC starts to collapse. In the single well case, it happens at $Na/a_{\text{osc}} = -.67$. Above this critical value, the BEC is stable. Below this value, caused by increasing N, decreasing a_{osc} (tightening the trap), or making a more negative, the BEC will collapse in on itself.

4.2 Double Well Equation

We will create a model of the double well potential with different parameters according to the energy equation we solved below. The full solution to Equation 2.3 will be complicated as it has many moving parts, so we will split it into 3 components, $E_{\text{kin}}, E_{\text{pot}}$, and E_{int}. They are named for the type of energy they originate from: kinetic, potential, and interaction. So Equation 3.1 can be written as

$$E_{\text{total}} = E_{\text{kin}} + E_{\text{pot}} + E_{\text{int}} \tag{4.1}$$

When we substitute in the trial wavefunction of Equation 3.11 and 3.12 into Equation 3.1, then we get the following expressions for the terms of Equation 4.1:

$$E_{\text{kin}} = \frac{\hbar^2}{2m}\left[\frac{N}{2}\cdot\frac{1}{b^2}\left(1+2e^{-\frac{q_0^2}{b^2}}\right)+\frac{\sqrt{L}\sqrt{N-L}}{b^2}\left(2+(2q_0^2-b^2)\frac{e^{-\frac{q_0^2}{b^2}}}{b^2}\right)\right] \tag{4.2}$$

$$E_{\text{pot}} = \frac{AN(2q_0^2+b^2)}{2}+\frac{\sqrt{L}\sqrt{N}\,e^{-\frac{q_0^2}{b^2}}(Ab^2-d)}{2}-\frac{dN}{2}+\frac{m\omega^2}{4}\left[2b^2\left(N+e^{-\frac{q_0^2}{b^2}}\right)\right] \tag{4.3}$$

$$E_{\text{int}} = \frac{U_0}{2^{5/2}\pi^{3/2}b^3}\left[L^2+(N-L)^2+3L(N-L)e^{-\frac{2q_0^2}{b^2}}+2^{3/2}\sqrt{L}\sqrt{N-L}\,e^{-\frac{3q_0^2}{2b^2}}N\right] \tag{4.4}$$

This complicated equation becomes considerably simpler when we consider the case where $N = L$ and $N_{\text{right}} = 0$.

$$E_{\text{kin}} = \frac{\hbar^2 N}{4mb^2}\left(1 + 2e^{-\frac{q_0^2}{b^2}}\right) \tag{4.5}$$

$$E_{\text{pot}} = \frac{AN(2q_0^2 + b^2)}{2} + \frac{N}{2}e^{-\frac{q_0^2}{b^2}}(Ab^2 - d) - \frac{dN}{2} + \frac{m\omega^2}{2}\,b^2\left(N + e^{-\frac{q_0^2}{b^2}}\right) \tag{4.6}$$

$$E_{\text{int}} = \frac{N^2 U_0}{2^{5/2}\pi^{3/2}b^3} \tag{4.7}$$

This equation has several similarities to the single well potential energy equation, as expected since this is the case where all the particles are in one well.

In Figure 4.2, we see the total energy vs b graph with b ranging from 0 to $.2\mu m$. In Figure 4.3, we see that same graph but with b ranging from 0 to $300\mu m$.

We can also look at how the energy changes as L changes. We will define $\eta = \frac{L - N_{\text{right}}}{N} = \frac{2L - N}{N}$. So we will create a graph showing the energy for $-1 < \eta < 1$, as seen in Figure 4.4. The figure show lines for several different b values.

4.2.1 Limitations of Model

The model only works when we have condensates with small attractive interactions, which our lab's primary boson source, Rubidium-87, does not have. Condensates with attractive interactions tend to be less stable than those with repulsive interactions as well However, potassium-41 and potassium-39 have a number of Feshbach resonances that can be used to tune the scattering length from $a > 0$ to $a < 0$.

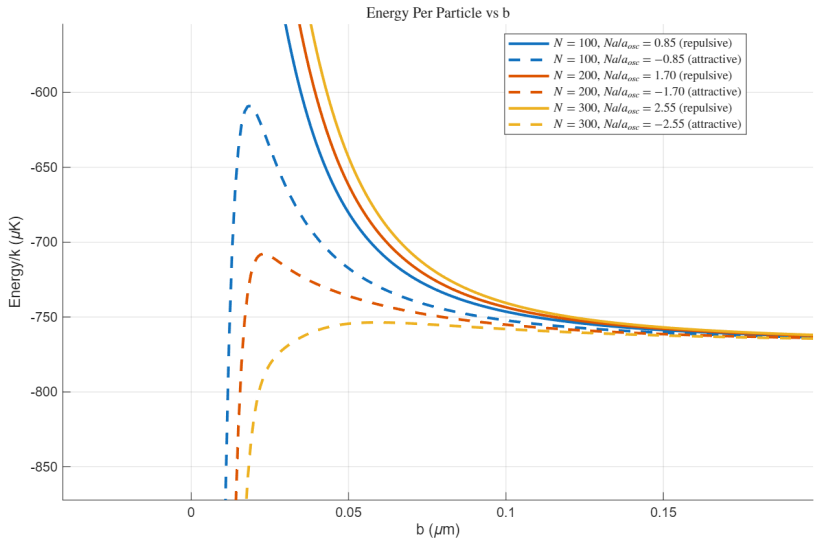

Figure 4.2: A graph showing energy values for different b, a parameter determined by the width of a Gaussian trial wavefunction. When a is positive, meaning there are repulsive interactions, the energy approaches ∞ as b approaches 0. When a is negative and Na/a_{osc} is small enough, the energy will have a local maximum before the energy approaches $-\infty$. In this case, that signifies the BEC collapsing. The dotted lines have scattering lengths of $a = -100a_0$ while the filled in lines have scattering lengths of $100a_0$ where a_0 is the Bohr radius. The graph has $\eta = 0$, as both wells are equally populated.

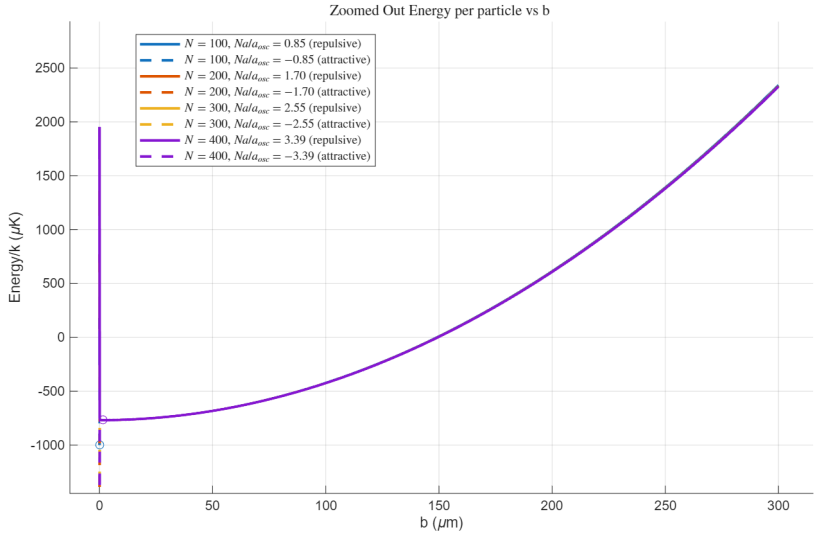

Figure 4.3: A graph showing the energy vs particle so that we can see the change in energy over a wide range of b values. There are three different N values that vary the dimensionless constant Na/a_{osc}. For every line, there is an oscillator frequency $\omega = 300 * 2\pi$. In all cases, after the energy reaches its minimum, the energy increases parabolically.

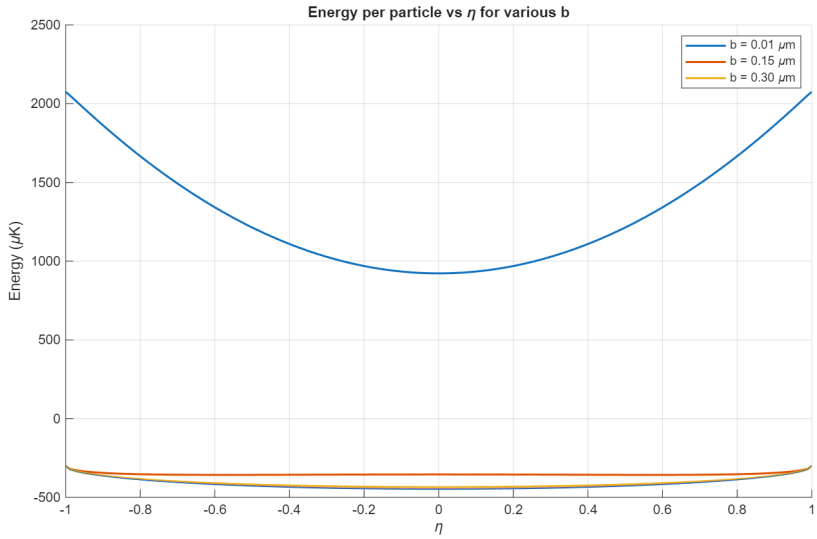

Figure 4.4: A graph showing energy vs η for various b between .1-6μm. For every line, we see that there is a minimum at $\eta = 0$. As b gets higher, the curvature gets flatter and the minimum energy decreases. The b for the 3 most visible lines are given in the legend.

Chapter 5

Conclusions

In order to improve the ease of determining whether a system is fully isolated from any other energy sources, we developed a toy model of a Bose-Einstein Condensate in a double well potential. We developed the solution for the single well potential, and used that as the base for the double well potential. For the double well case, we created graphs showing the energy of particles in the BEC varying b and η. The graphs showcased energies for different sizes of condensates ($N = 100, 200, 300$) and scattering lengths $a = \pm 100 a_0$ with a_0 representing the Bohr radius.

5.1 Plans for Future Work

In the future, we can continue to analyze Equations 4.2-4.4 to find a critical value for the system to collapse. It is not fully dependent on Na/a_{osc} as different starting parameters can tolerate lower Na/a_{osc} before collapse.

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