

Today's Topics

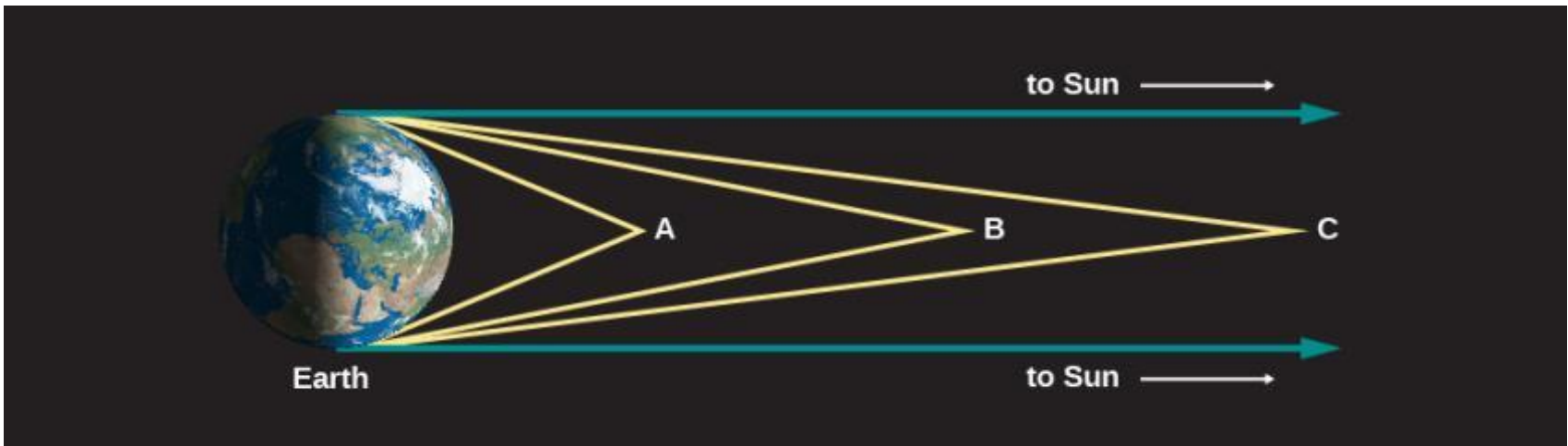
Monday, September 2, 2019 (Week 1, lecture 3) – Chapters 2 & 3.

1. Eratosthenes: radius of the Earth
2. Retrograde motion of the planets
3. Earth's axis tilt, seasons, precession
5. Kepler's Laws

Ancient Greek Physics

Determining the Radius of the Earth

Parallel light rays from the sun



[OpenStax: Astronomy]

Light Rays from Space. The more distant an object, the more nearly parallel the rays of light coming from it.

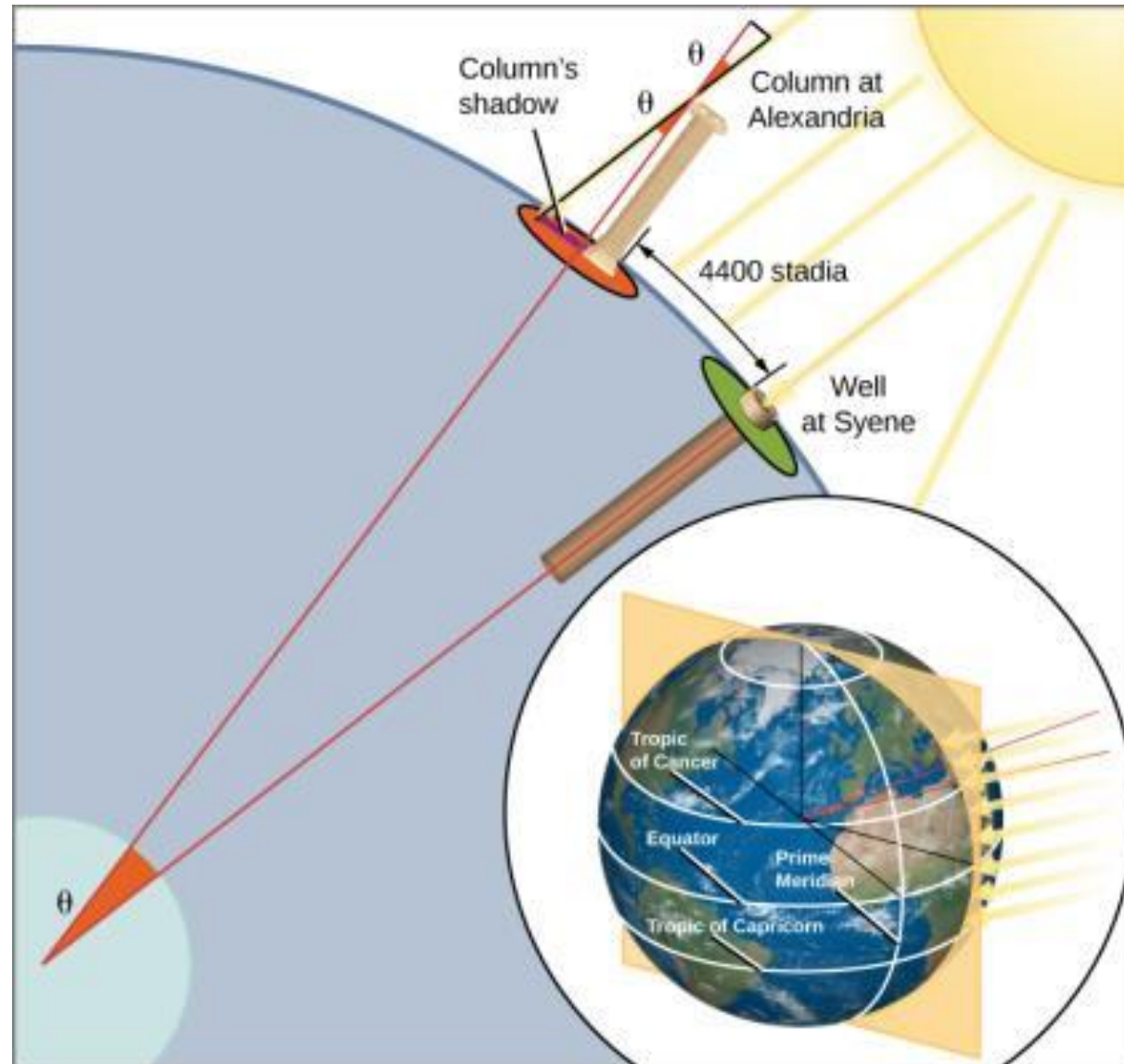
→ Light rays from Sun are quite parallel.

→ Light rays from stars are very parallel.

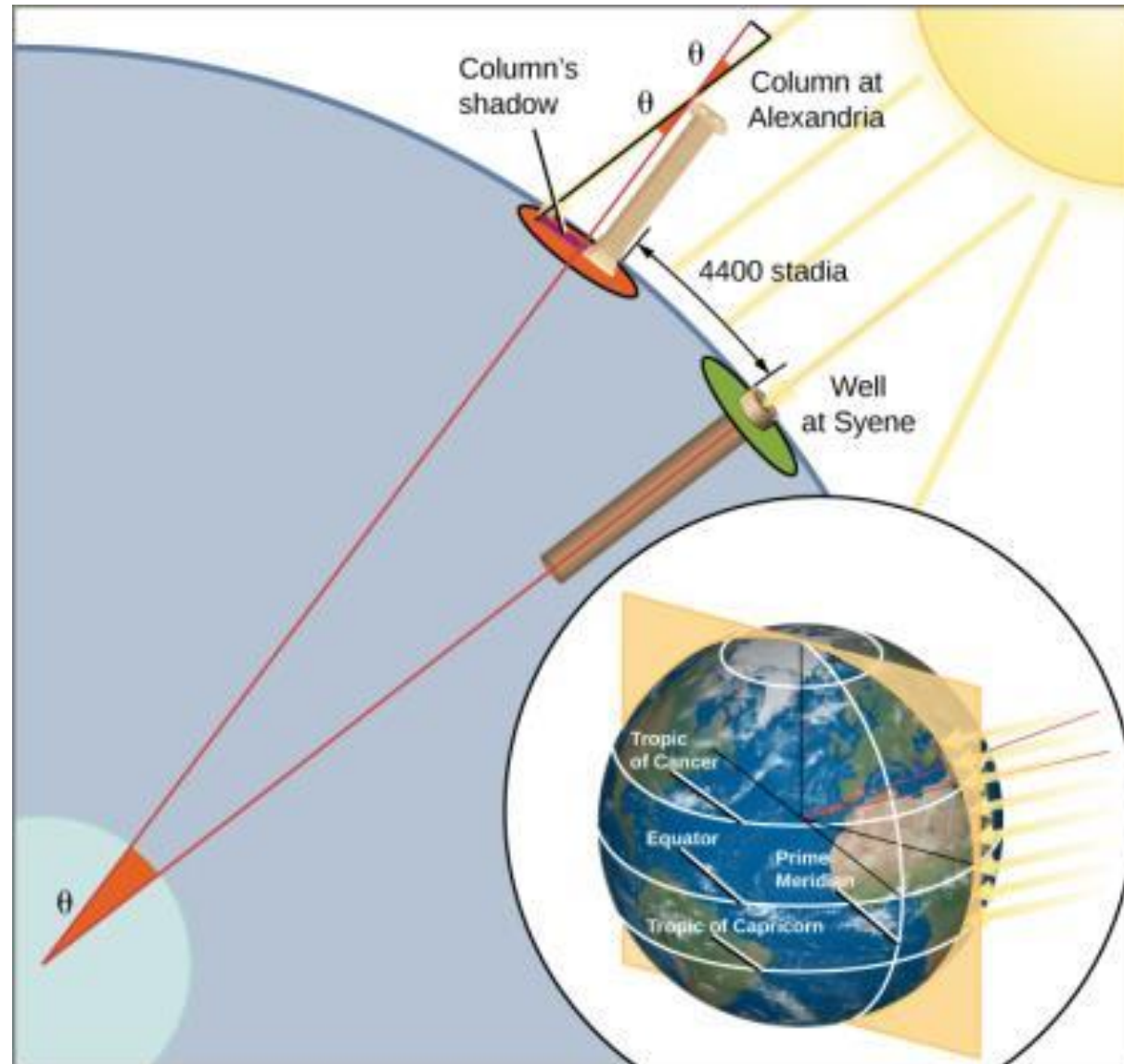
How Eratosthenes Measured the Size of Earth

Eratosthenes (276-194 BC) observed that:

1. A Sun's ray at **Syene** comes straight down whereas a ray at **Alexandria** makes an **angle of 7° with the vertical**.



How Eratosthenes Measured the Size of Earth

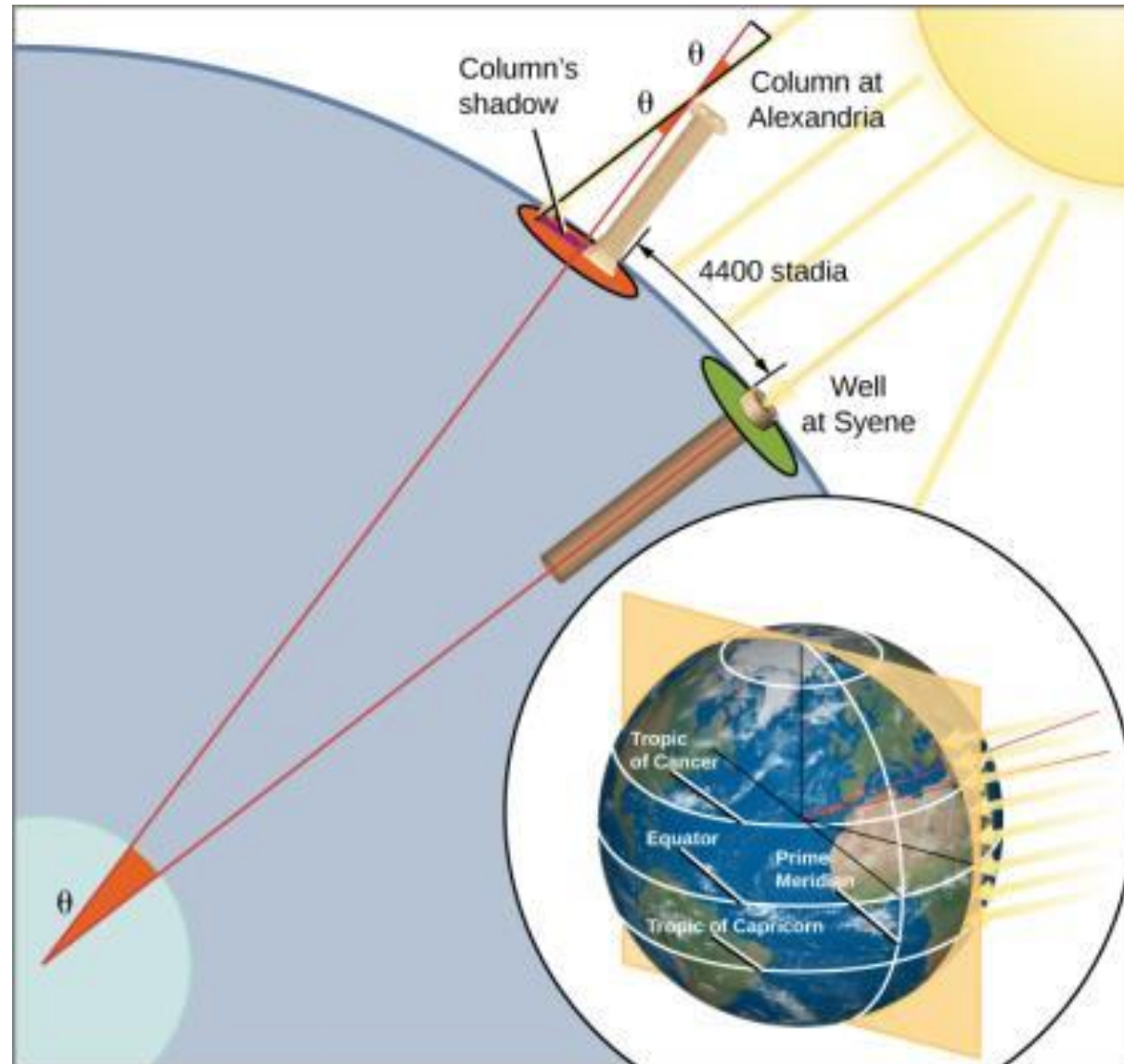


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2. At Alexandria, Earth's surface has curved away from Syene by 7° of 360° , or **$\sim 1/50$ of a full circle**.

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2. At Alexandria, Earth's surface has curved away from Syene by 7° of 360° , or **$\sim 1/50$ of a full circle**.
3. The distance between the two cities, i.e. 5000 stadia, must be **$1/50$ the circumference** of Earth.

How Eratosthenes Measured the Size of Earth

Circumference of Earth = 50×5000 stadia
= 250,000 stadia *(1 stadia ~ 180 m)*
 \approx 45,000 km

Actual circumference of Earth = 40,000 km

How Eratosthenes Measured the Size of Earth

$$\begin{aligned}\text{Circumference of Earth} &= 50 \times 5000 \text{ stadia} \\ &= 250,000 \text{ stadia} && (1 \text{ stadia} \sim 180 \text{ m}) \\ &\approx 45,000 \text{ km} \\ &\rightarrow \text{Radius} = 45,000 / 2\pi \approx 7,200 \text{ km}\end{aligned}$$

$$\begin{aligned}\text{Actual circumference of Earth} &= 40,000 \text{ km} \\ &\rightarrow \text{Radius} = 40,000 / 2\pi \approx 6,400 \text{ km}\end{aligned}$$

Retrograde Motion of Mars

- If you map out the motion of the planet Mars against the background stars (celestial sphere), it follows a “line.”
- But, roughly every 2 years Mars appears to go **backwards**.

Retrograde Motion of Mars

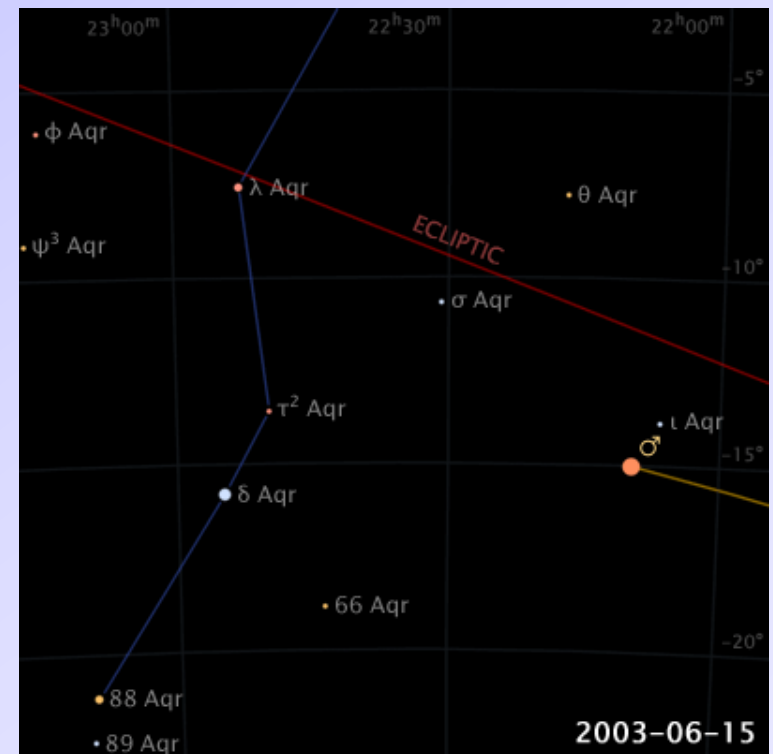
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2003 Retrograde



Retrograde Motion of Mars

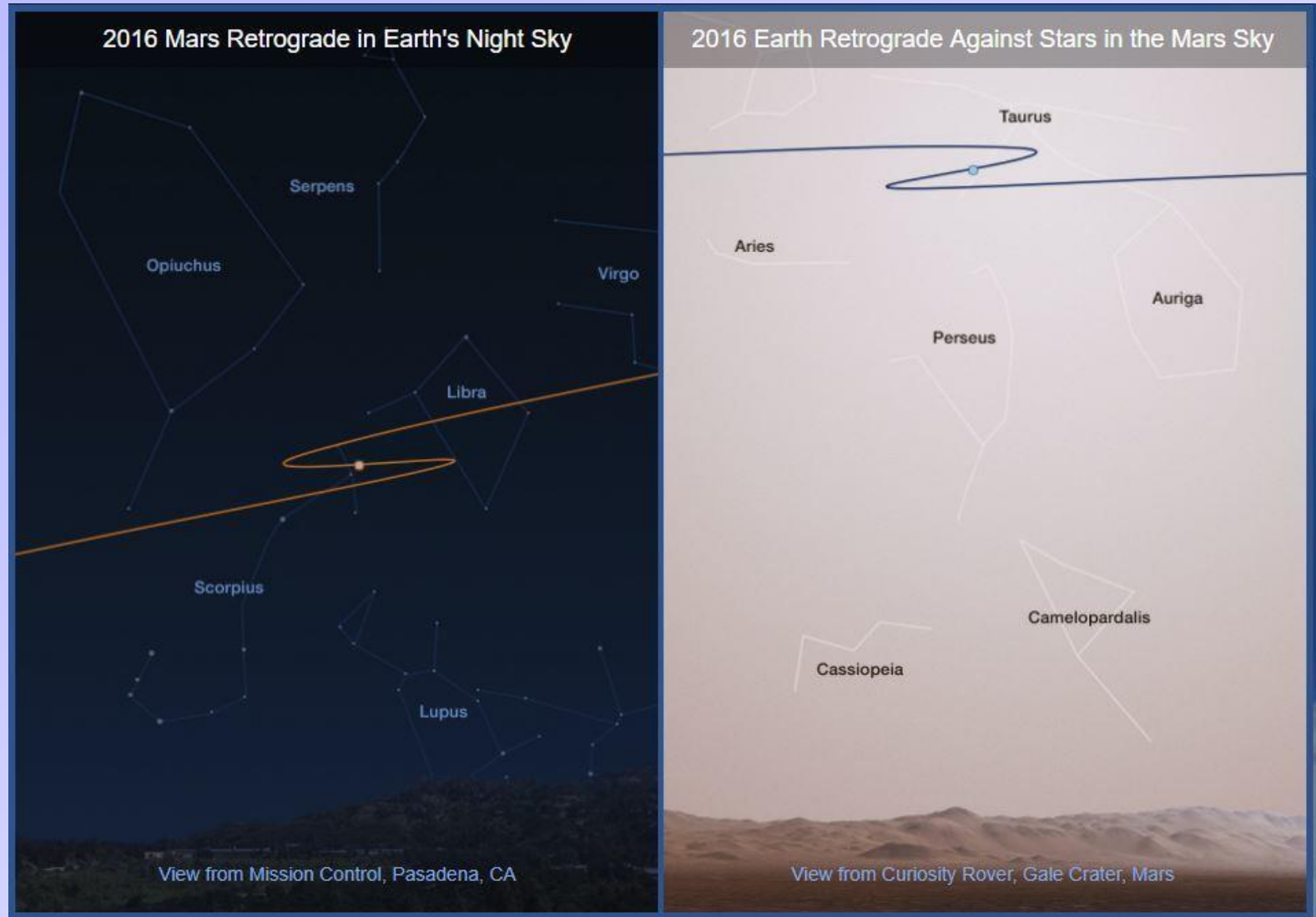
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[source: NASA, <https://mars.nasa.gov/all-about-mars/night-sky/retrograde/>]

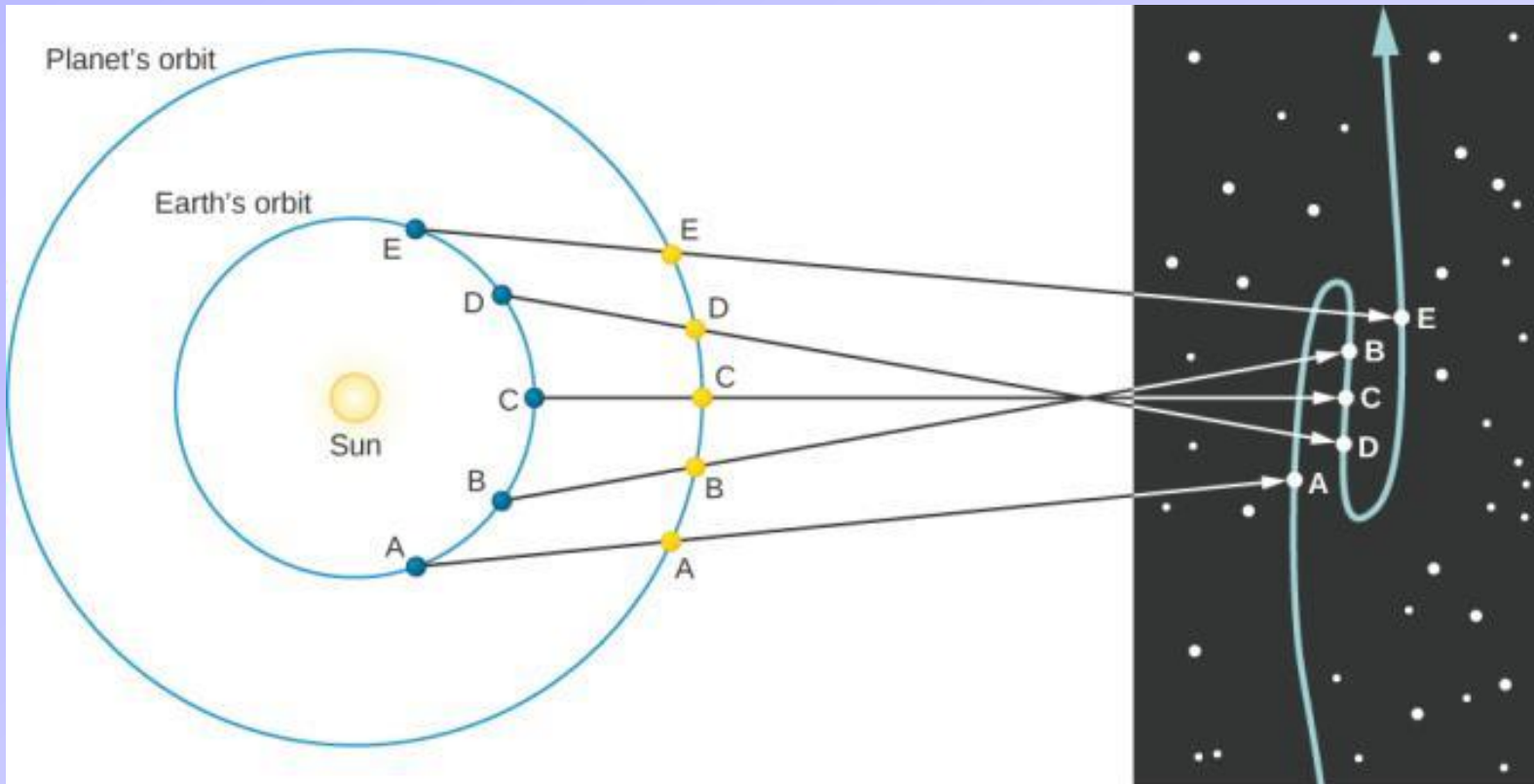
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Retrograde Motion of Earth (from Mars)



[source: NASA, <https://mars.nasa.gov/all-about-mars/night-sky/retrograde/>]

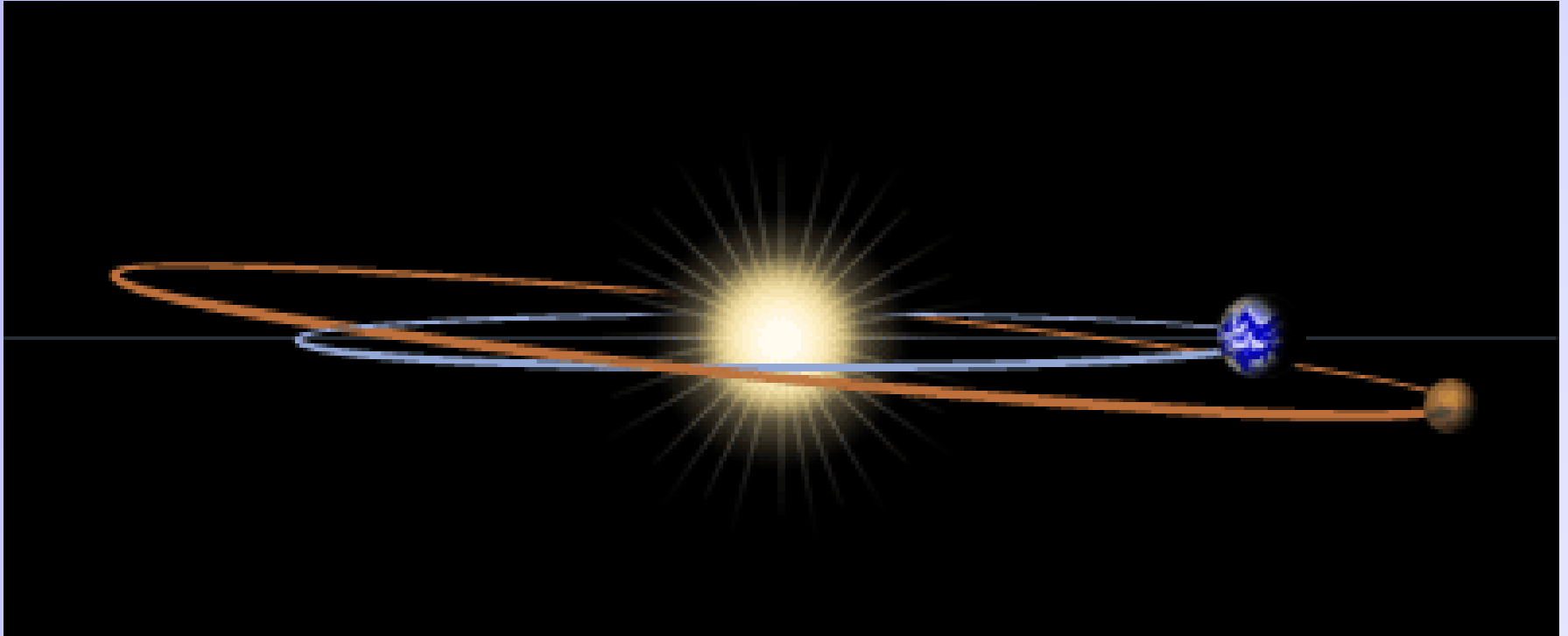
What's going on?



[OpenStax: Astronomy]

- The Earth orbits faster than Mars and passes it every 26 months.
- While Earth is passing Mars, then Mars appears to go backwards.
- “Loop-de-loop” behavior is because Earth and Mars orbital planes are angled.

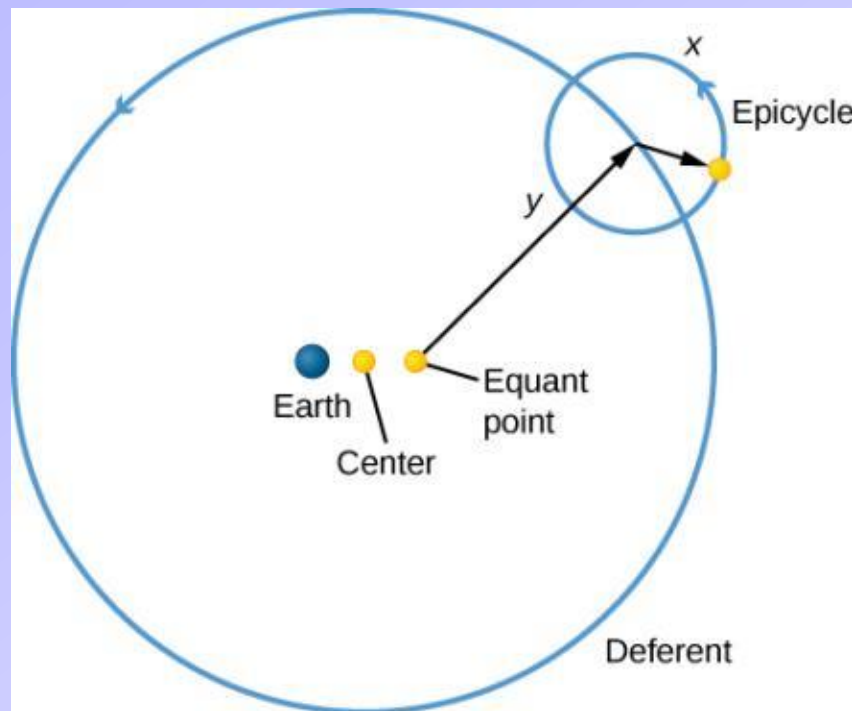
Inclination of Mars Orbit



[source: NASA, <https://mars.nasa.gov/all-about-mars/night-sky/retrograde/>]

Inclination of orbit (relative to ecliptic) = 1.85°

Ancient Geocentric View: Epicycles

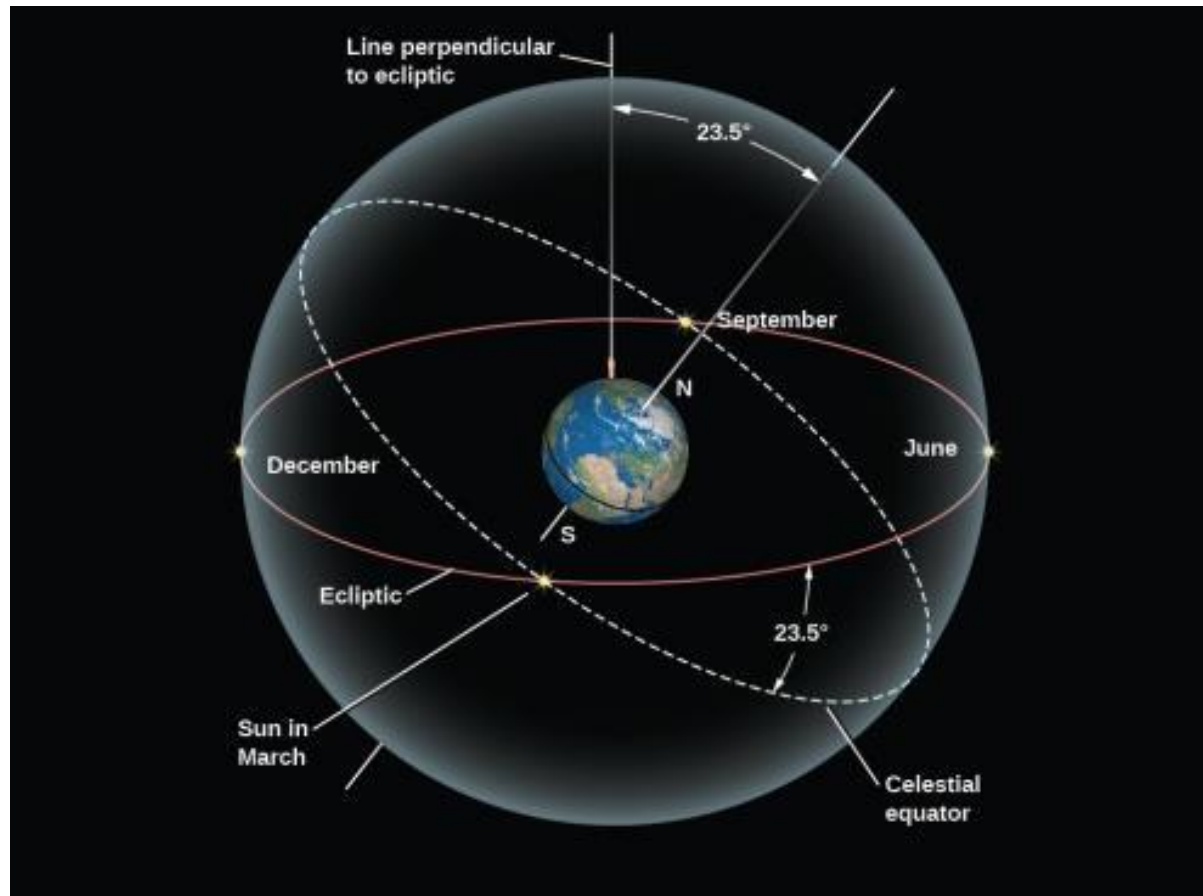


[OpenStax: Astronomy]

Ptolemy (Alexandria, c.“100-168” AD) explained retrograde motion using **epicycles**.

- Planets travel on an **epicycle circle** attached to their main orbit circle (**deferent**).
- The **deferent circle also moves** around the equant point.
- Epicycle model could **predict** apparent motion of planets.
- Replaced by Copernicus’s (Poland, 1473-1543 AD) heliocentric model.

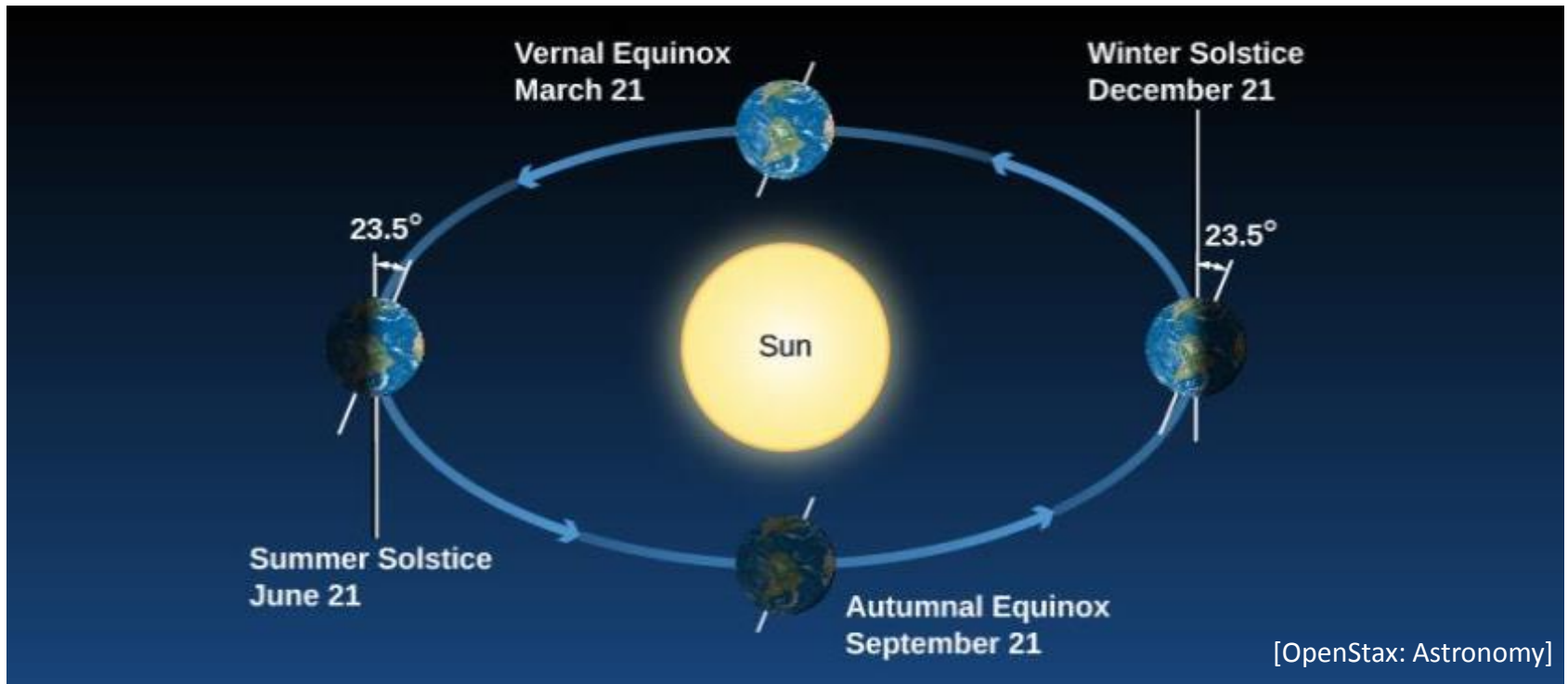
Tilt of Earth's Rotation Axis



[OpenStax: Astronomy]

- The **Ecliptic plane** is the plane in which the Earth orbits the Sun.
- The **orbital axis** is perpendicular to the Ecliptic plane.
- The **Earth rotation axis is inclined by $\theta = 23.5^\circ$** from the orbital axis.

Earth's tilt direction is constant



Earth's rotation axis always points in the same direction with respect to Sun and celestial sphere

Earth's tilt direction is constant

The celestial sphere always “rotates” around the star **Polaris**.



[Source: <https://epod.usra.edu/blog/2013/05/earths-rotation-and-polaris.html>]

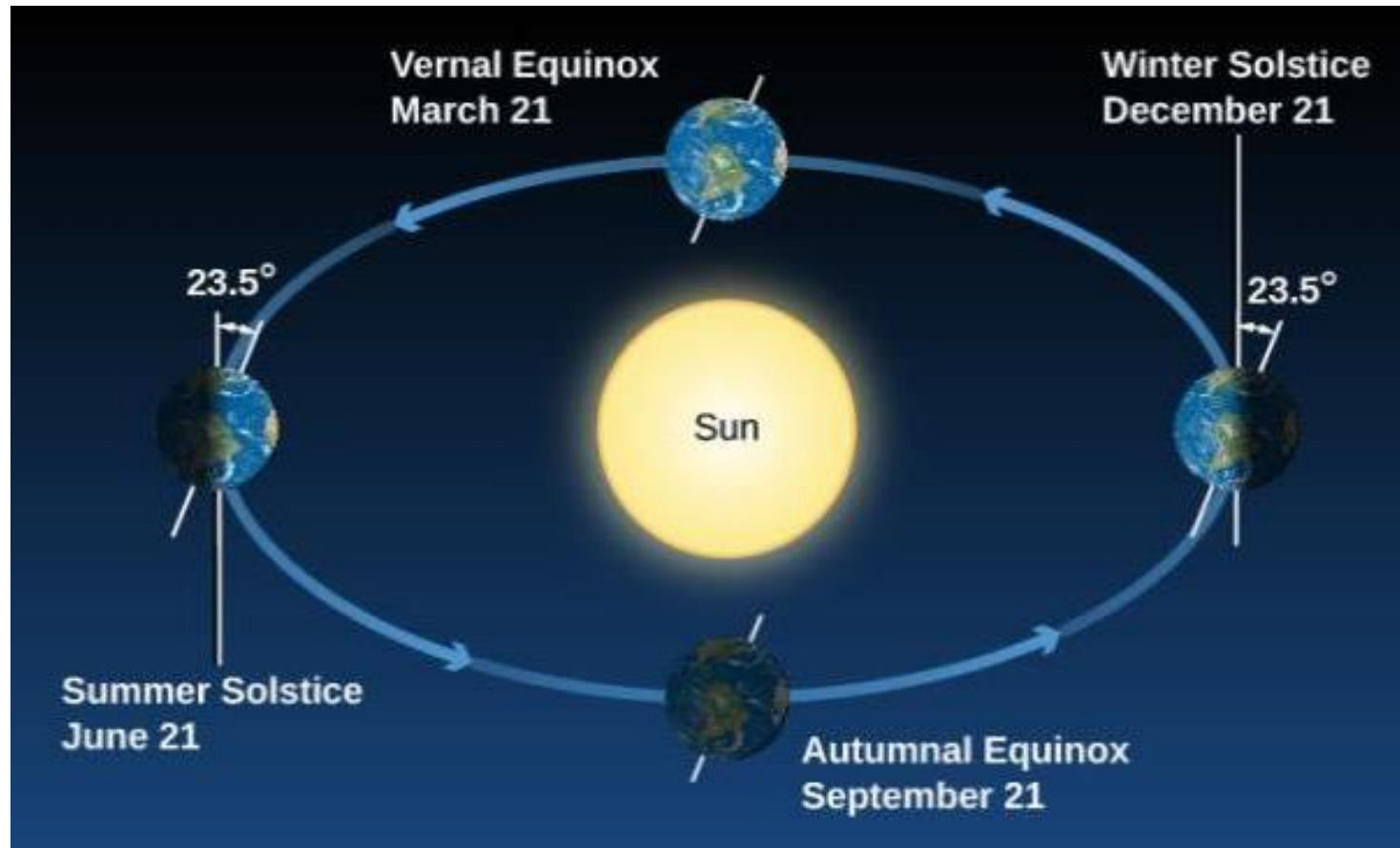
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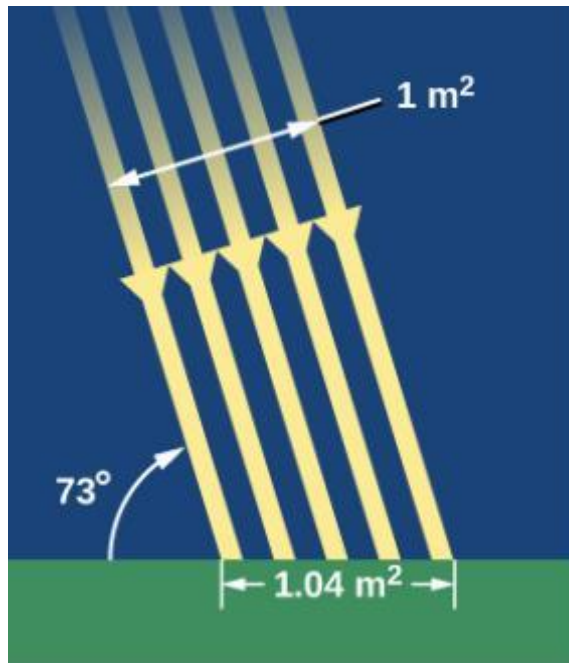
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Earth's tilt & the Seasons

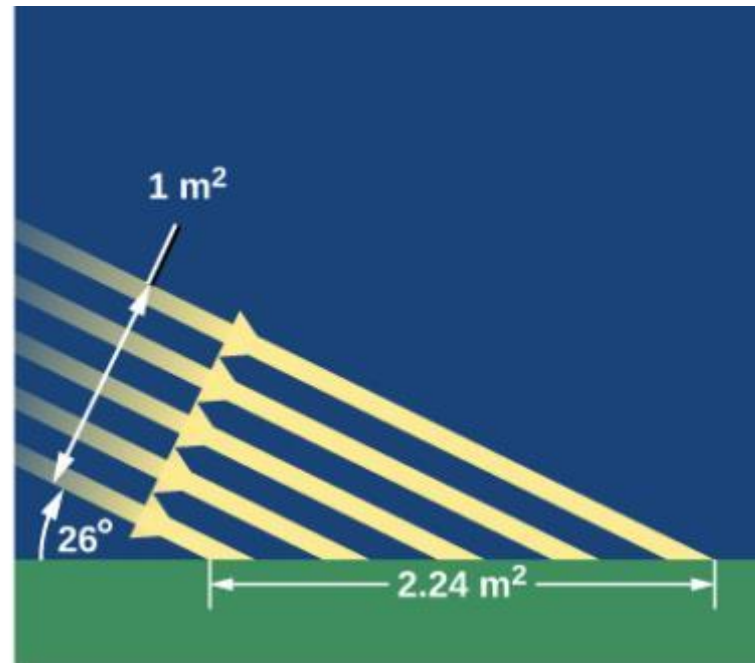


- The summer and winter seasons are determined by the **amount of sunlight** that fall in a given location on Earth.
- Amount of sunlight = light power per unit area
Recall: power = energy per time

Earth's tilt & the Seasons



(a) Summer

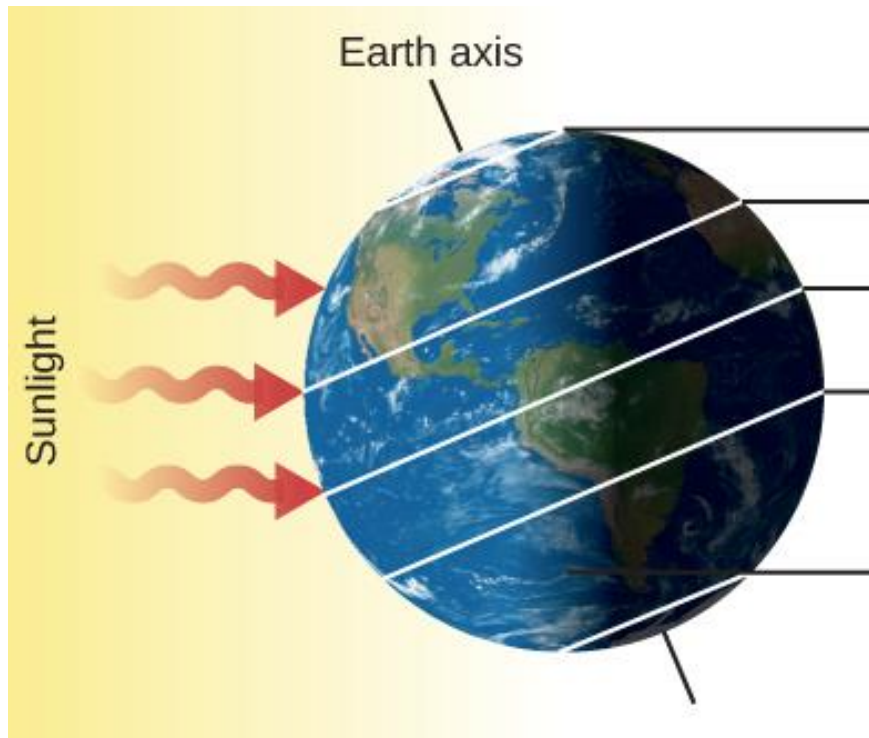


(b) Winter

- (a) In **summer**, the Sun appears high in the sky and its rays hit Earth more directly, spreading out less.
- (b) In **winter**, the Sun is low in the sky and its rays spread out over a much wider area, becoming less effective at heating the ground.

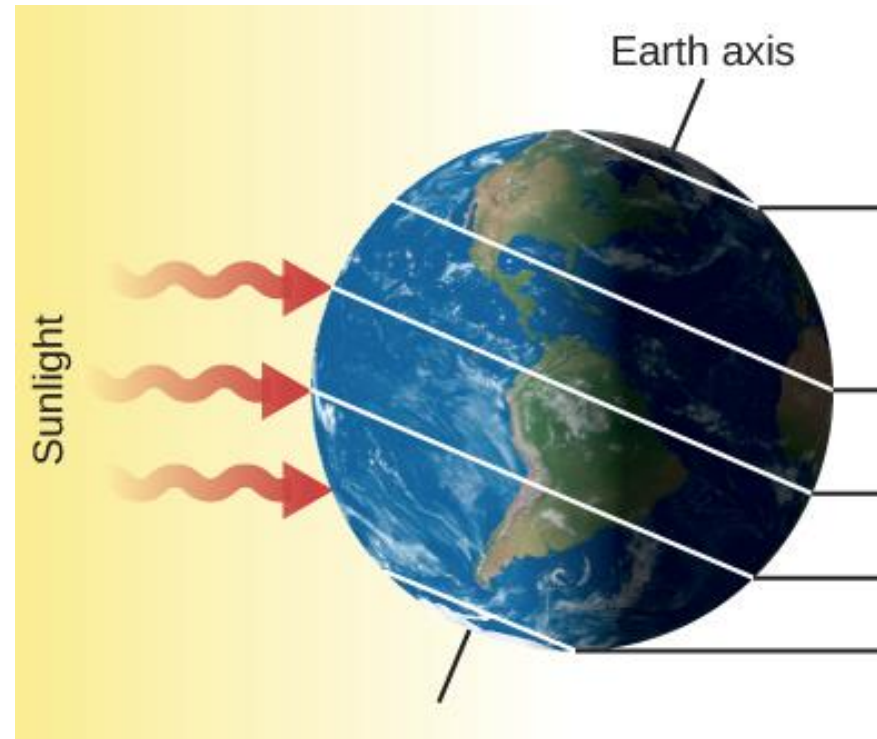
Sun's light intensity on Earth \approx 1 KiloWatt per square meter = 1 kW/m^2

Participation Question



Orientation #1

Winter ?
Summer ?



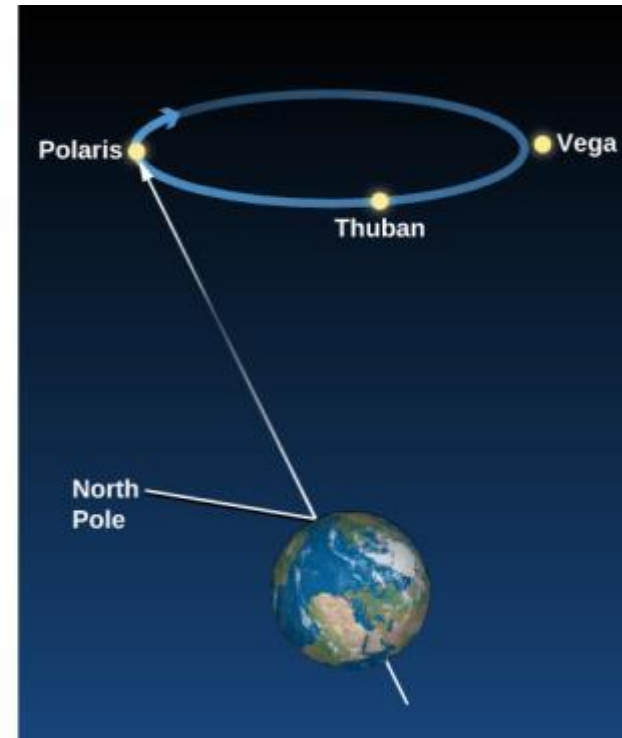
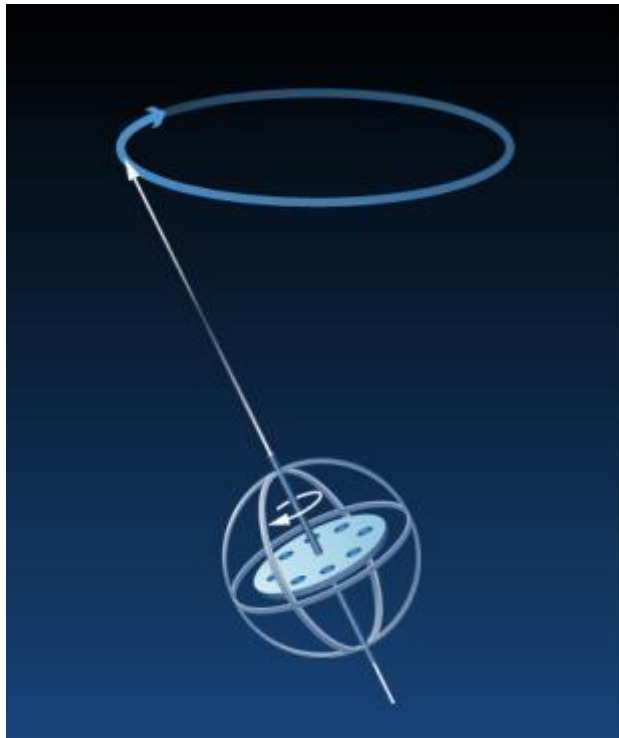
Orientation #2

Winter ?
Summer ?

Classify diagrams by season for North America

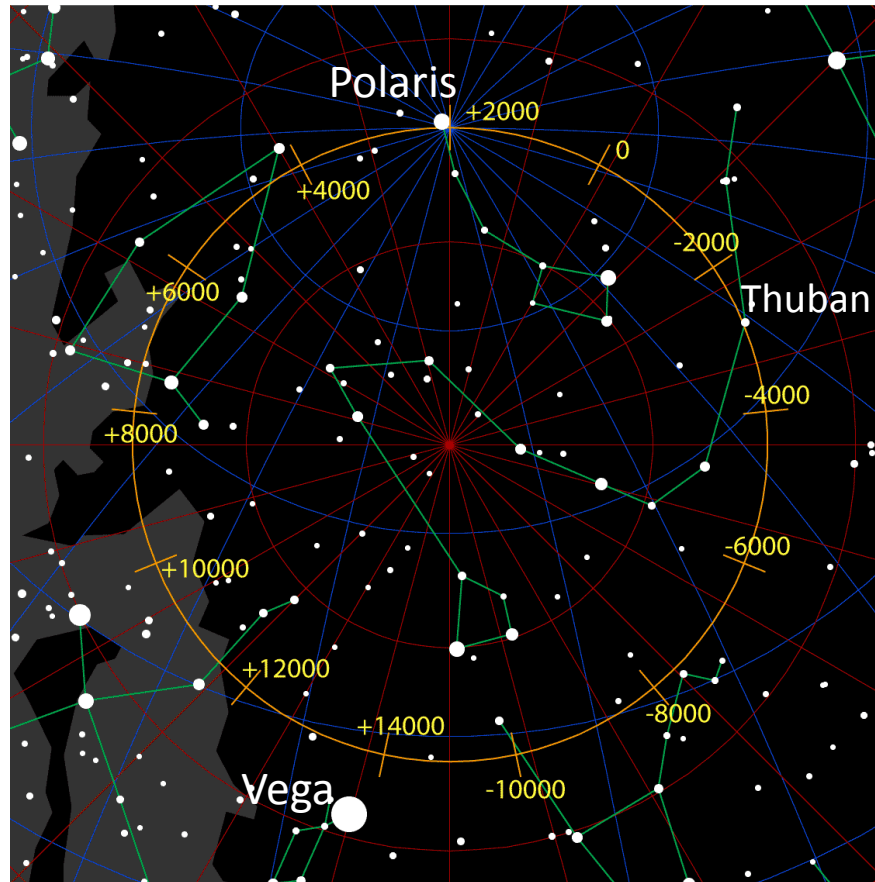
Precession of Earth's Axis

The direction of Earth's rotation axis is slowly changing.
→ The axis is precessing over a 26,000 year period.



- Today the north celestial pole is near the star Polaris
- About 5000 years ago it was close to a star called Thuban
- In 14,000 years it will be closest to the star Vega.

Precession of Earth's Axis



By Tau'olunga - self, 4 bit GIF, CC BY-SA 2.5,
<https://commons.wikimedia.org/w/index.php?curid=891838>

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Kepler's Laws of Planetary Motion

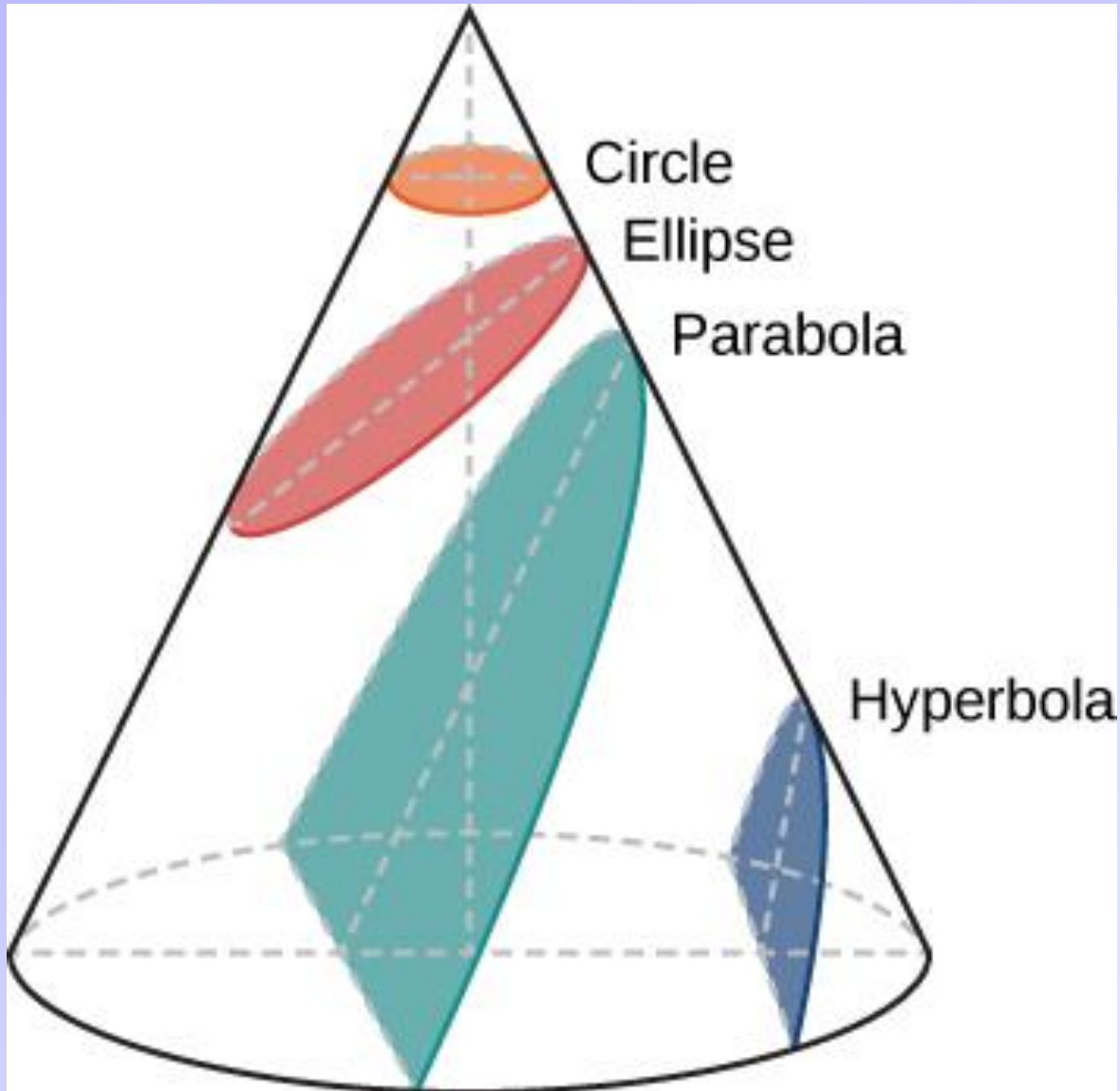
1st Law: The orbits of all planets are **ellipses**.

2nd Law: Law of **equal areas**.

3rd Law: **(orbital period)² = (semimajor axis)³**

[fine print: the “=” depends on units used]

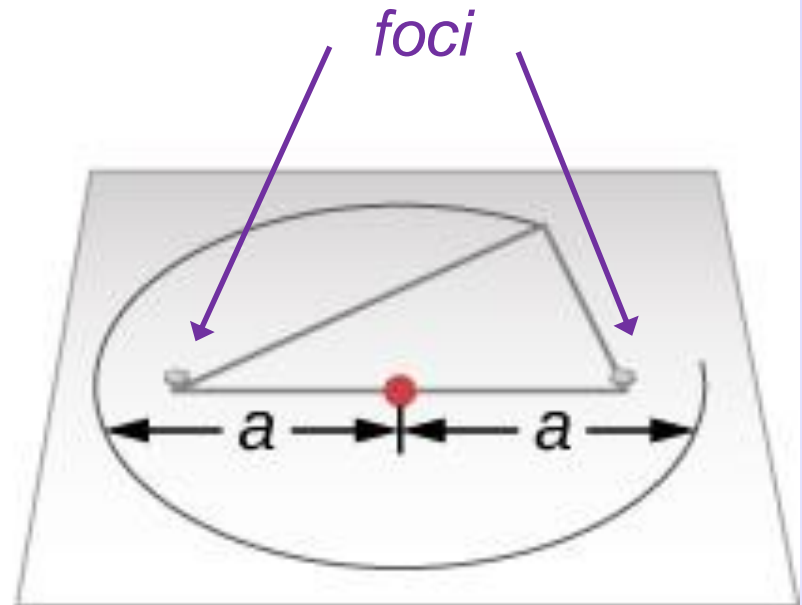
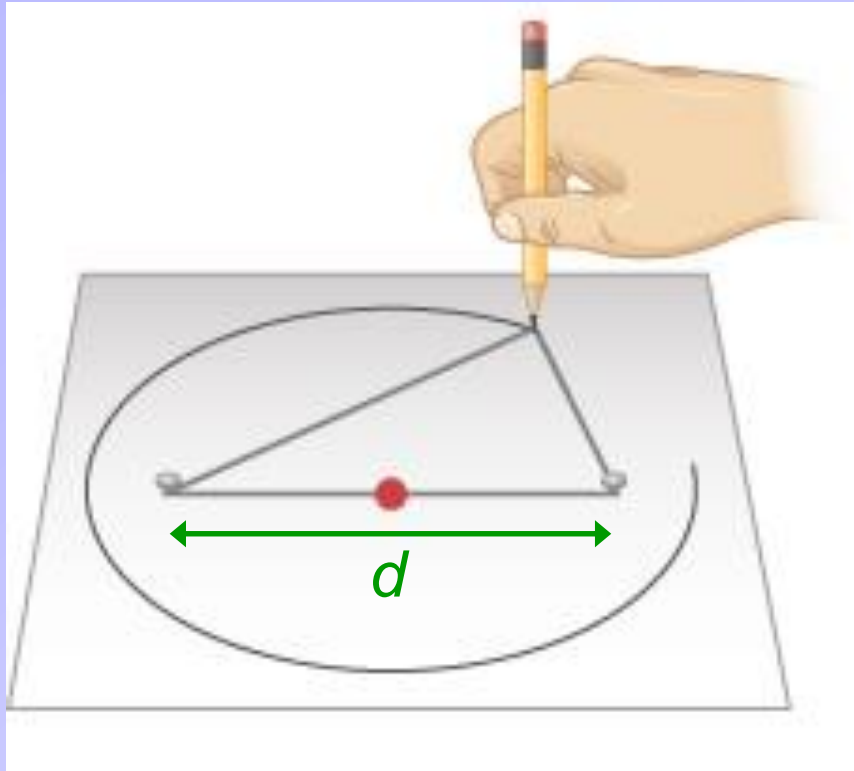
Kepler's 1st Law – Conic Sections



The **circle**, **ellipse**, **parabola**, and **hyperbola** are all formed by the intersection of a plane with a cone.

Note: Unbound orbits can be parabolic or hyperbolic.

Kepler's 1st Law -- Ellipses

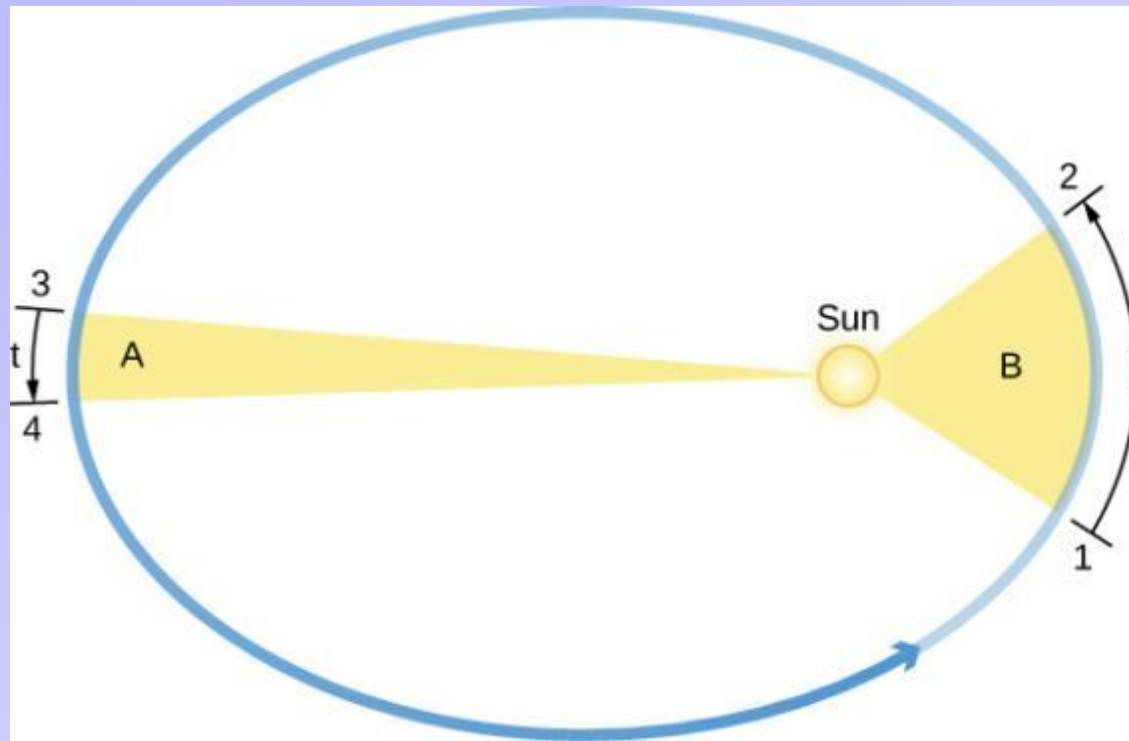


- Sun sits at one of the foci.
- Other focus is empty.

a = semimajor axis

$$\text{Eccentricity} = \varepsilon = \frac{d}{2a}$$

Kepler's 2nd Law



The Law of Equal Areas. The orbital speed of a planet traveling around the Sun varies such that in equal intervals of time t , a line between the Sun and a planet sweeps out equal areas (area A = area B).

Kepler's 3rd Law

T = orbital period in units of Earth years

a = semimajor axis in AU

$$T^2 = a^3$$