Geology Seminar Today
(Sept. 13, 2019)

12:05pm - 12:50pm
McGlothlin-Street Hall, Room 230

The 3 km High Subjovian Megadome on Ganymede

Simulation of Stability via Pratt Isostasy

Dr. Jonathan Kay
Geology and the Center for Geospatial Analysis
William & Mary

Today's Topics

Friday, September 13, 2019 (Week 3, lecture 7) – Chapter 3, 4.6.

1. Circular motion
2. Angular momentum
3. Escape velocity
4. Tides

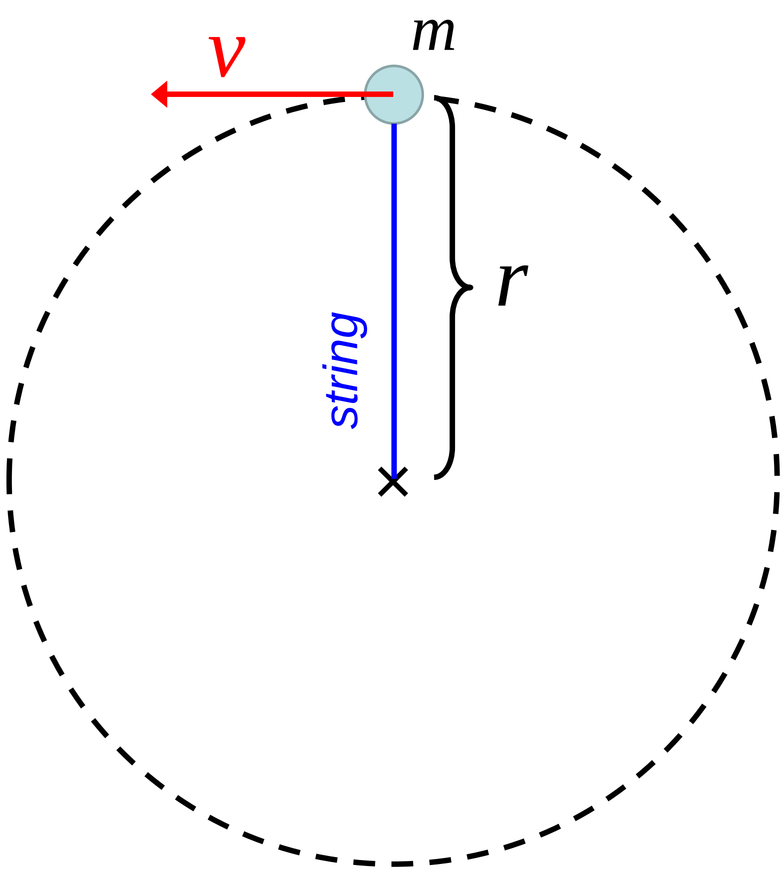
Circular Motion

<u>Recall</u>

acceleration = **change** in velocity over time

speed & direction

Rotation is a type of <u>acceleration</u> where the velocity **direction changes**, but speed is constant.



"ball on a string"

Circular Motion

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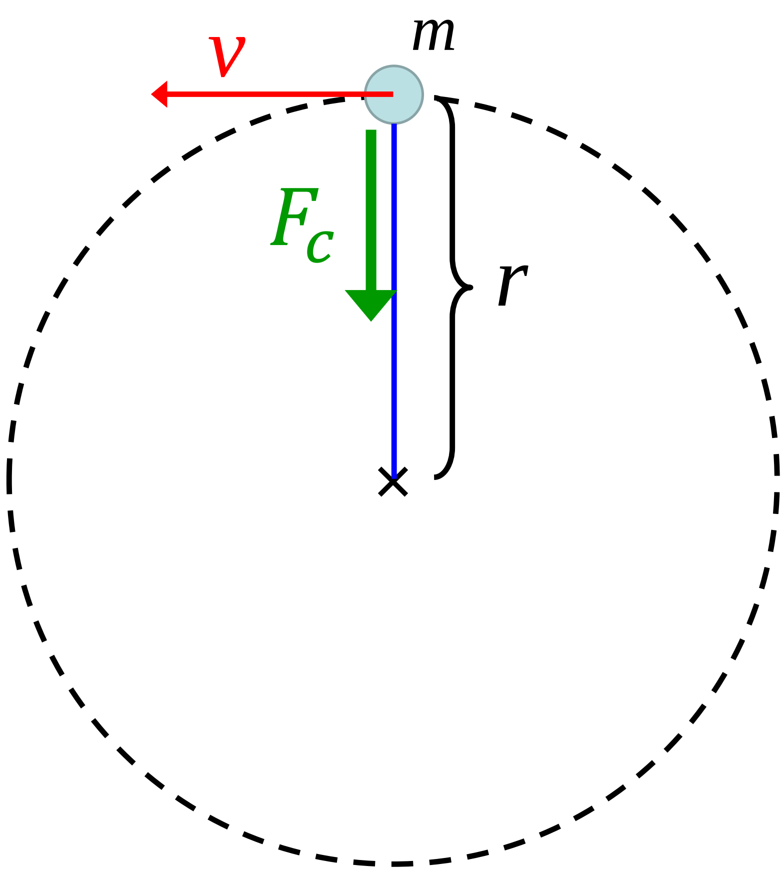
Acceleration: $a_c = \frac{v^2}{r}$

Centripetal Force: $F_c = \frac{mv^2}{r}$



"ball on a string"

Circular Motion Example: Earth's orbit of Sun

Newton's version of Kepler's 3rd Law

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

This formula is in SI units

T = orbital period in seconds

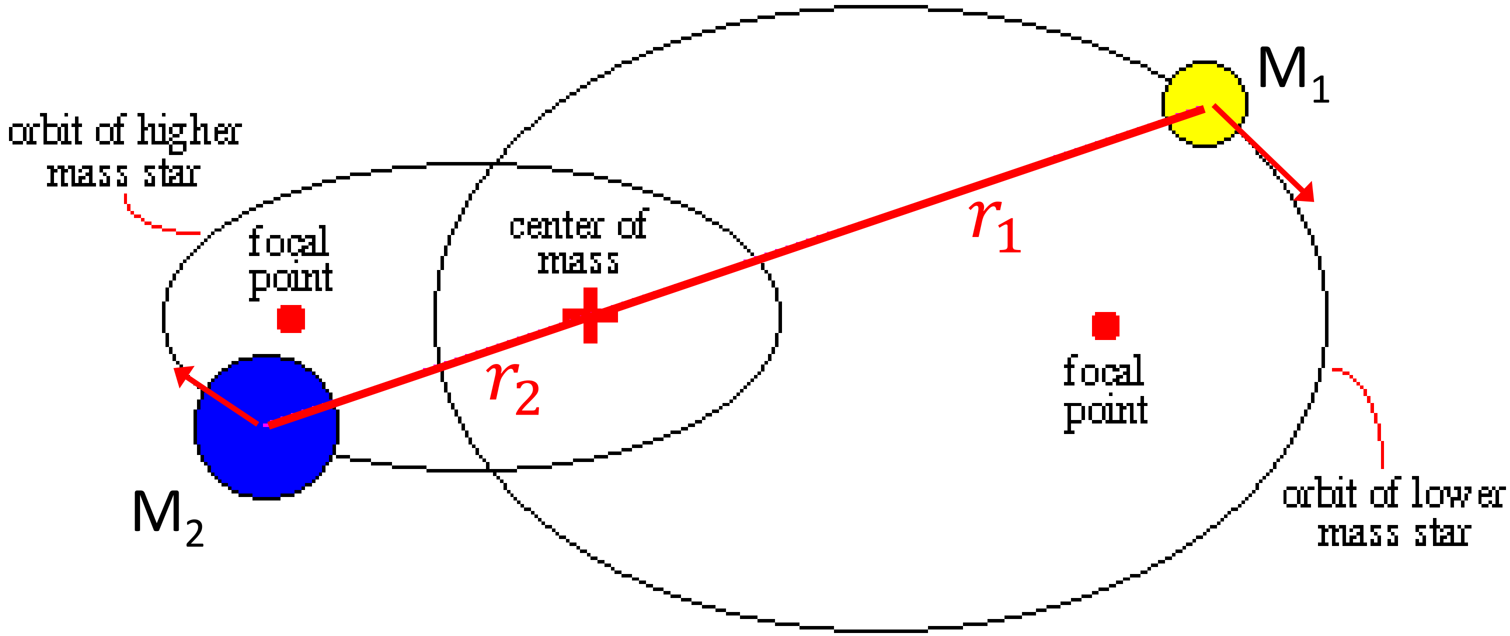
a = semimajor axis in meters

$M_{1,2}$ =Mass of orbiting objects in Kg

G = 6.6743×10^{-11} $m^3/Kg.s^2$

What happens when $M_1 \simeq M_2$?

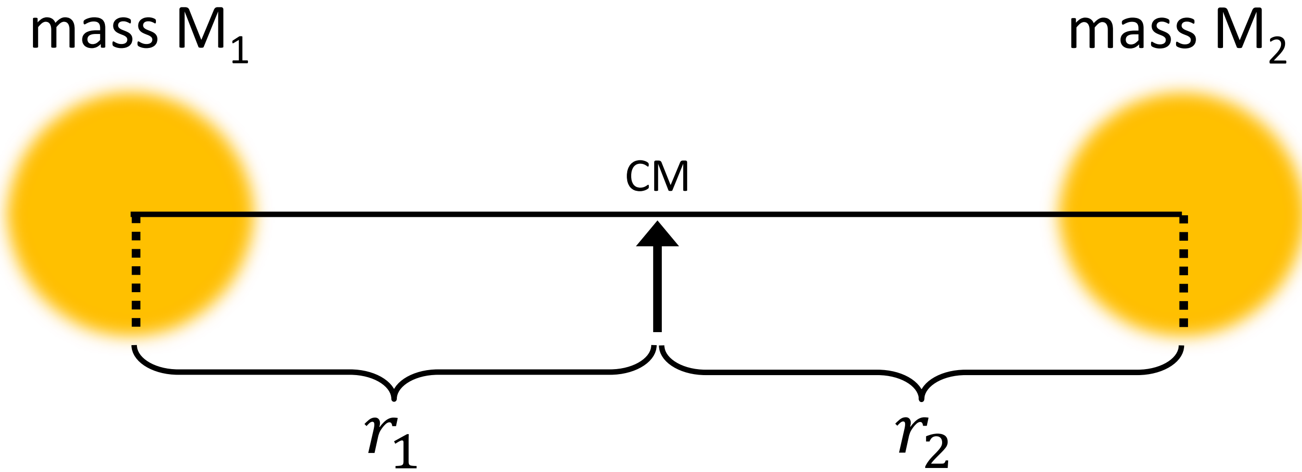
The **center of mass** of M_1 and M_2 serves as the orbiting ellipse focus.



[adapted from http://abyss.uoregon.edu]

Semimajor axis "a":
The coordinate "$r = r_1 + r_2$" is the distance between the two masses. It also describes an ellipse (not shown), whose semimajor axis "a" is used in Newton's version of Kepler's 3rd law.

Center of Mass



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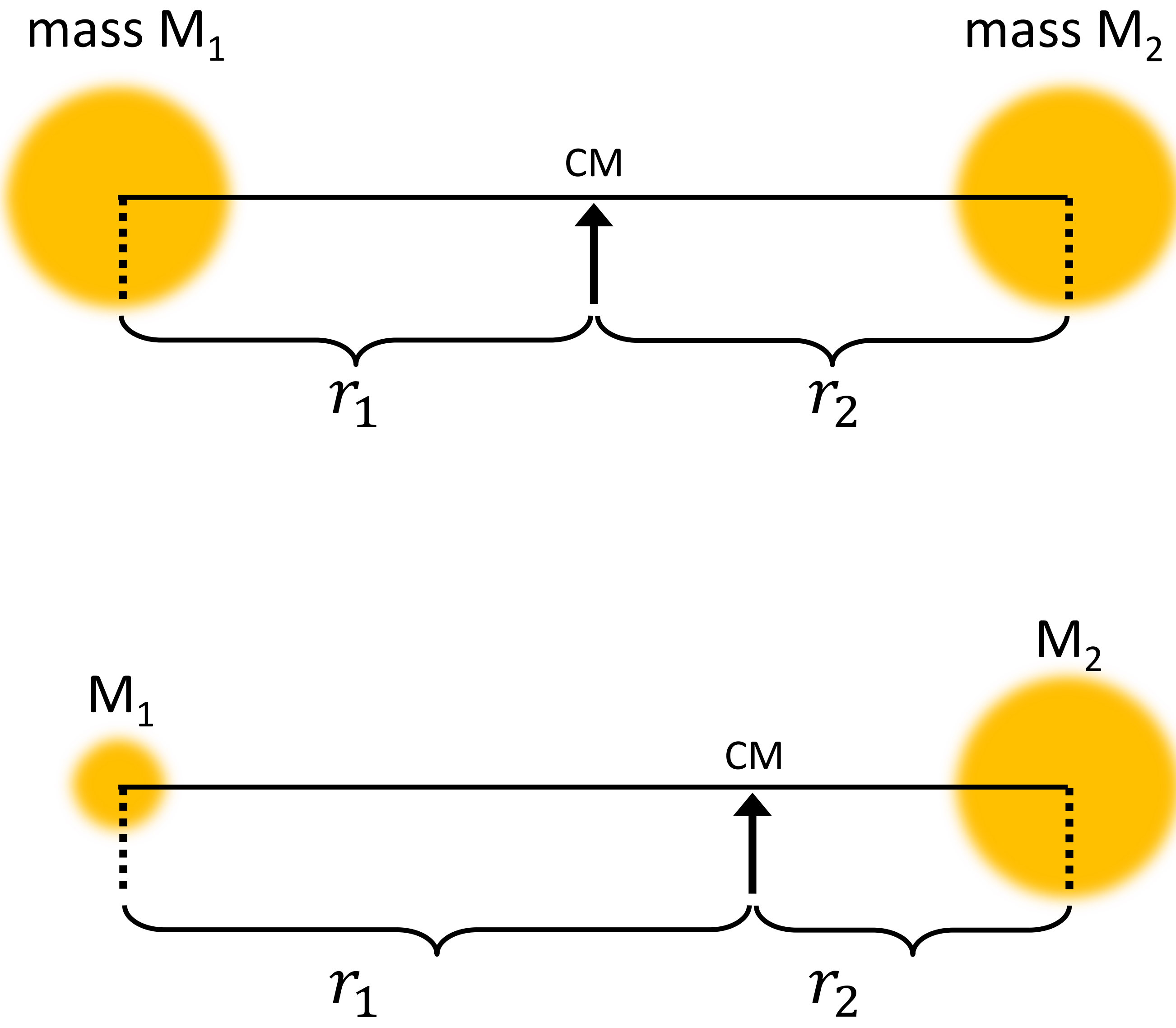

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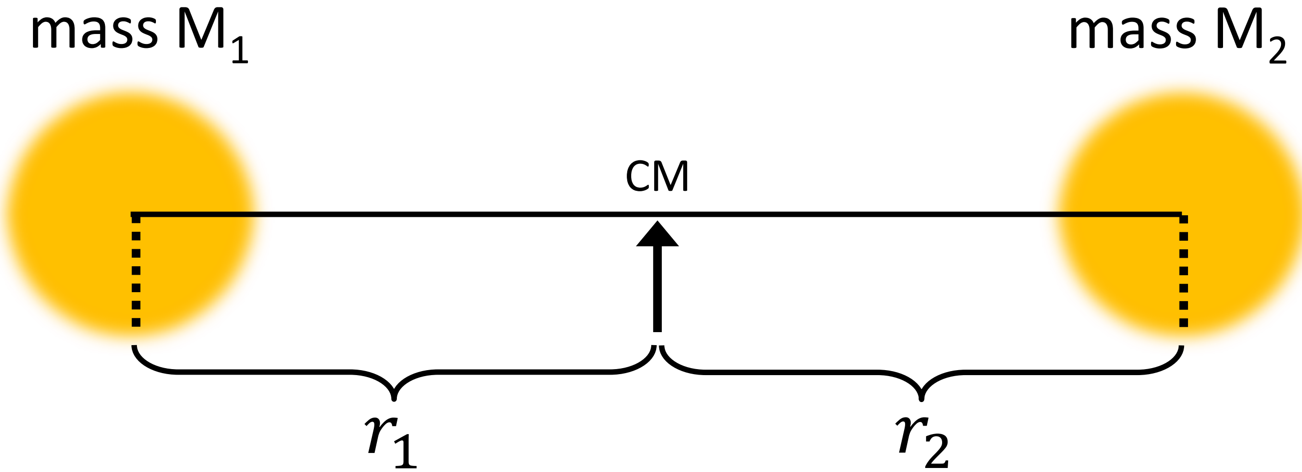
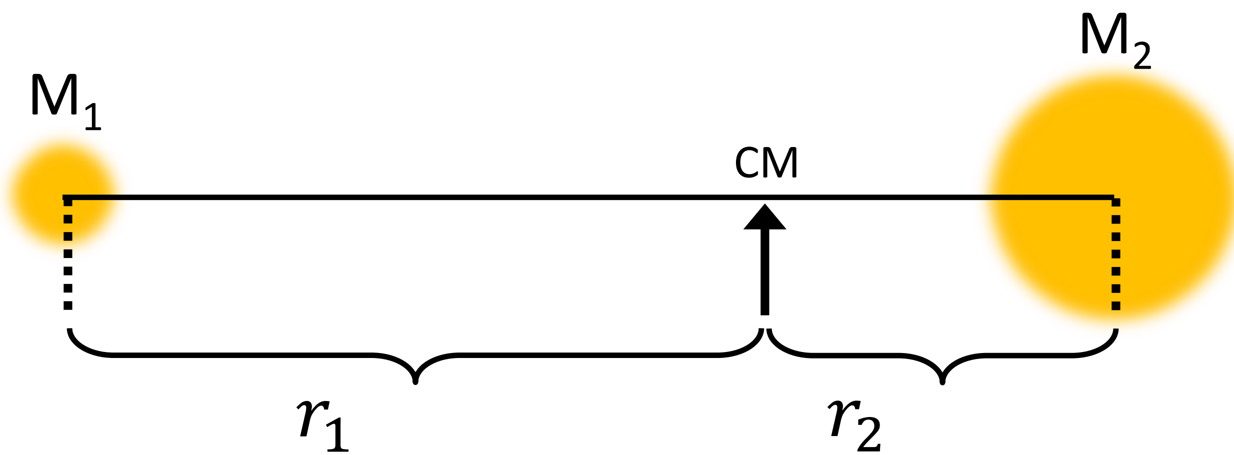


Center of mass is located such that $M_1 r_1 = M_2 r_2$

(or "barycenter")



Some Barycenters

M_2 - M_1: $r_2 = a\frac{M_1}{M_1+M_2}$ = distance from CM to M_2

Sun-Earth: $r_2 = 448\ km = 3.0 \times 10^{-6}$ AU

Earth-Moon: $r_2 = 4{,}670$ km with $a = 384{,}000$ km

= 73% of Earth's radius

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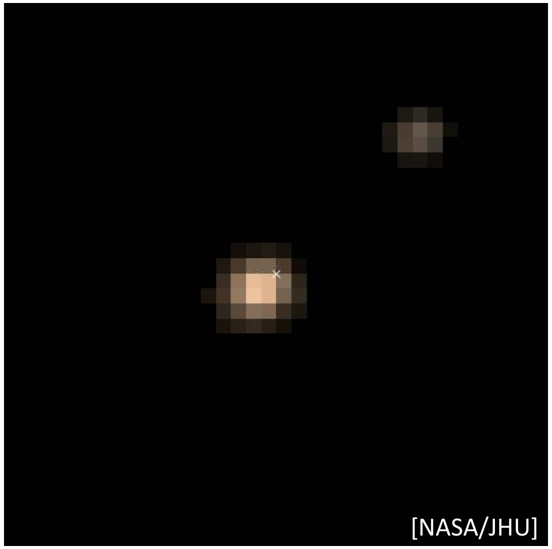
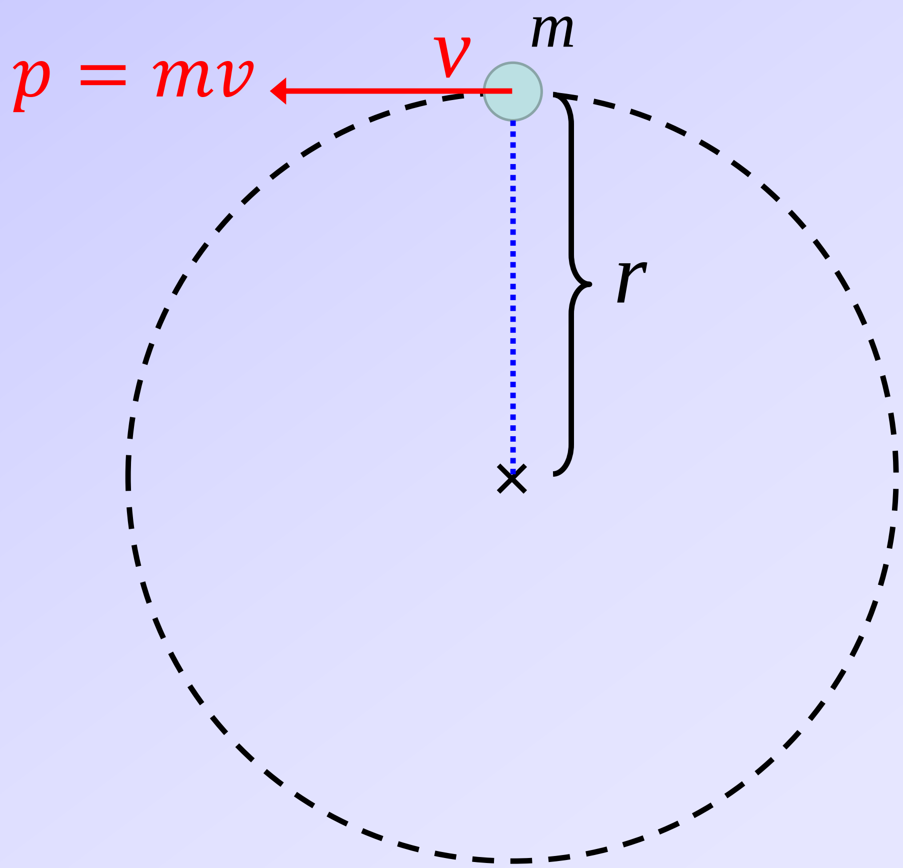
Pluto – Charon:



Conservation of Angular Momentum (1)

angular momentum $= L =$ momentum $\times$ radius

$= p \times r$ ... $= mvr$ for circular motion



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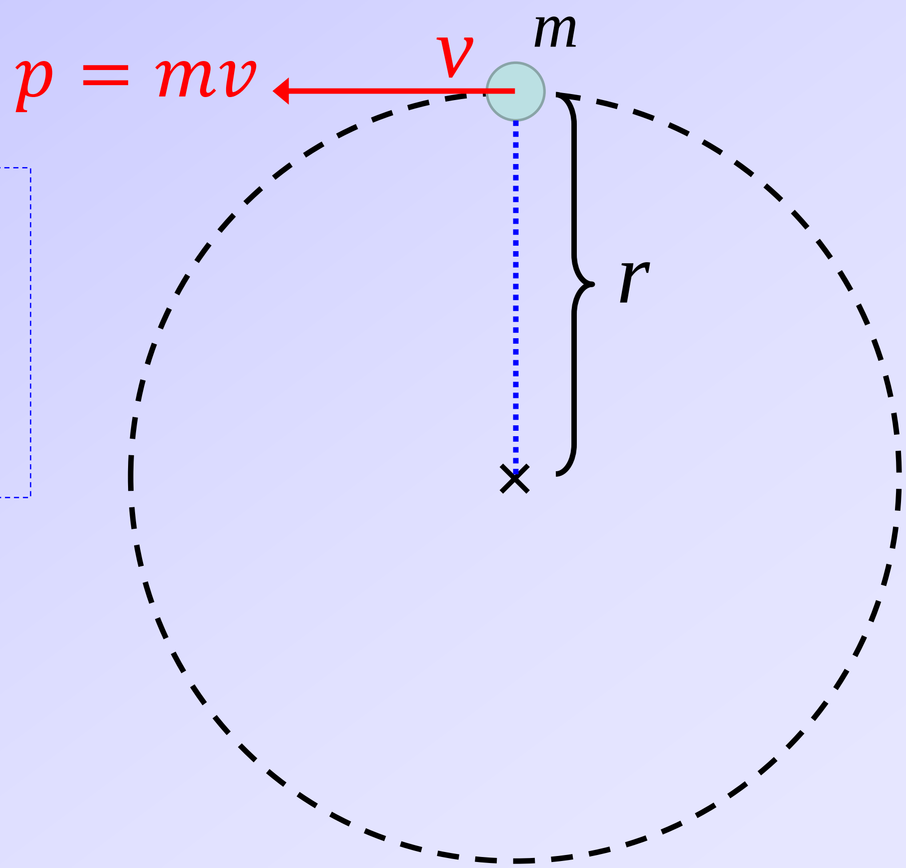

total angular momentum

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sum of the angular momenta of all the sub-parts of a system

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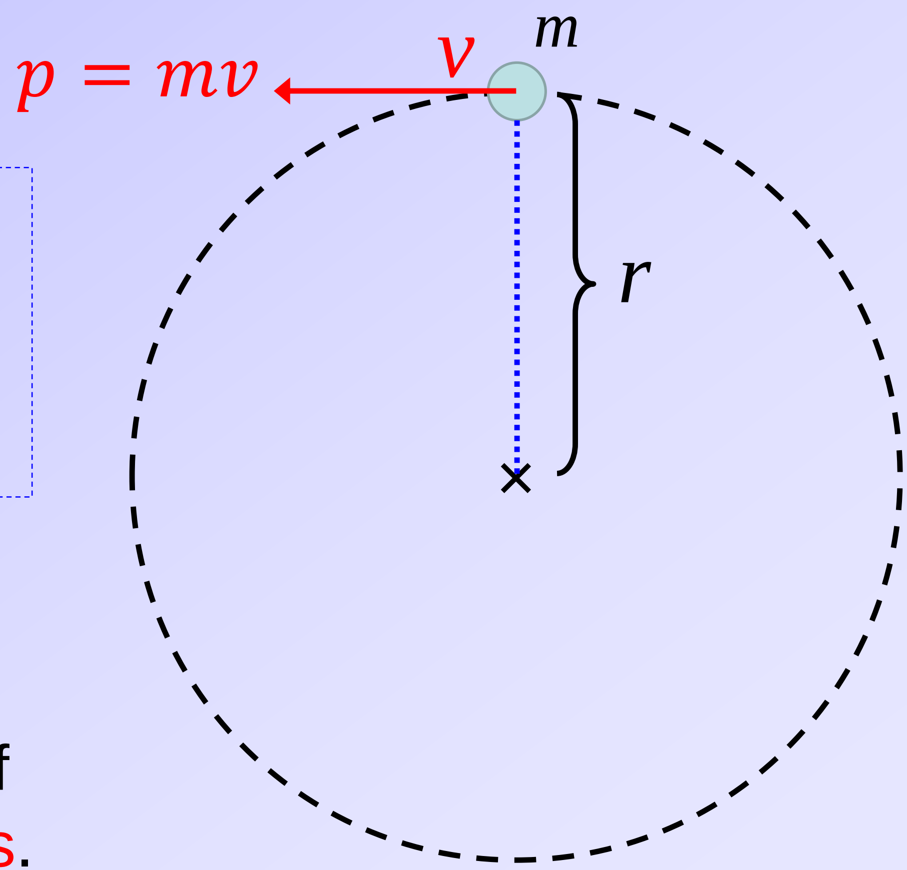

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Conservation Law

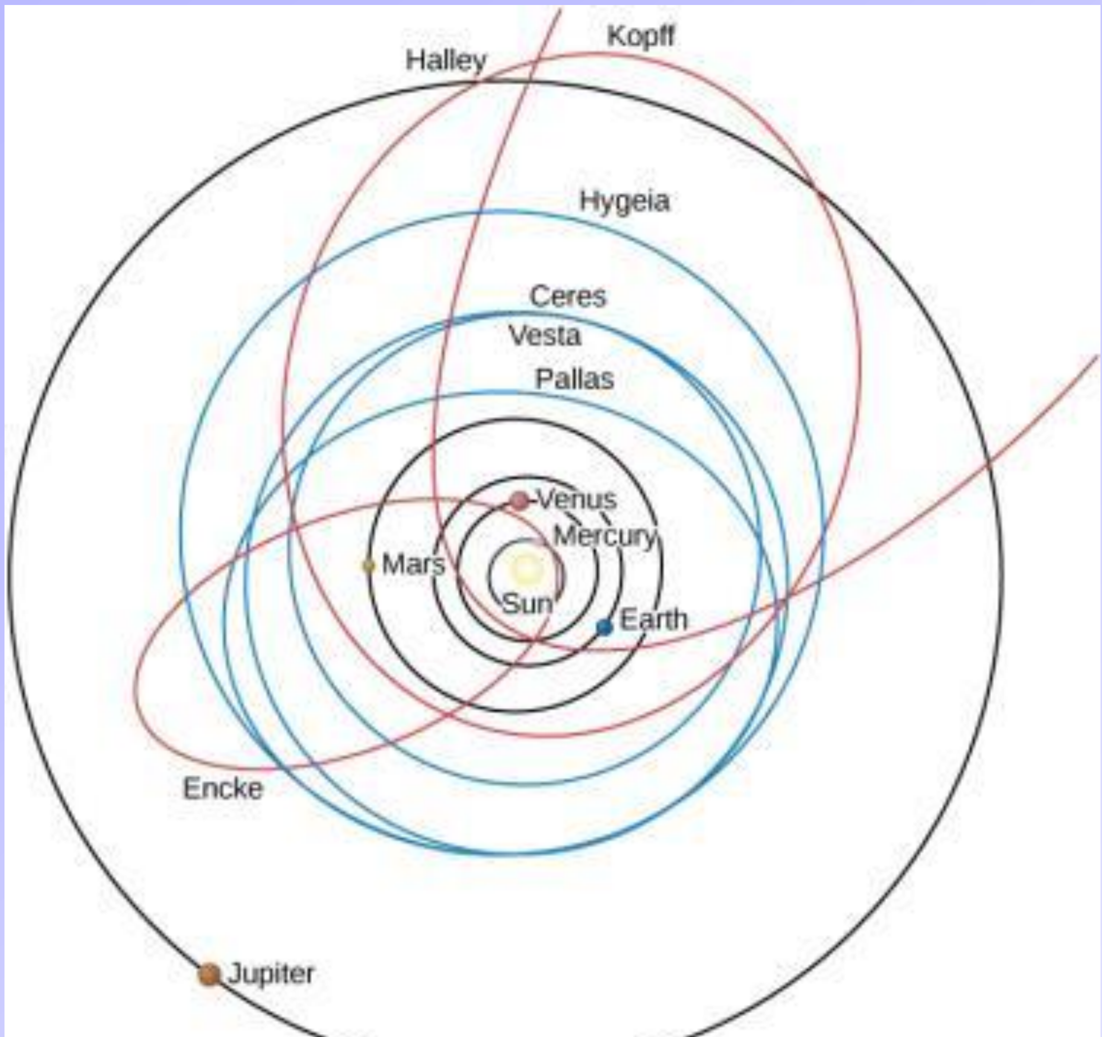
The total angular momentum of a closed system never changes.

Conservation of Angular Momentum (2)

[OpenStax: Astronomy]

- When a spinning figure skater **brings in her arms**, their distance from her spin center is **smaller**, so her speed increases.
- When her **arms are out**, their distance from the spin center is **greater**, so she **slows down**.

Conservation of Angular Momentum



[OpenStax: Astronomy]

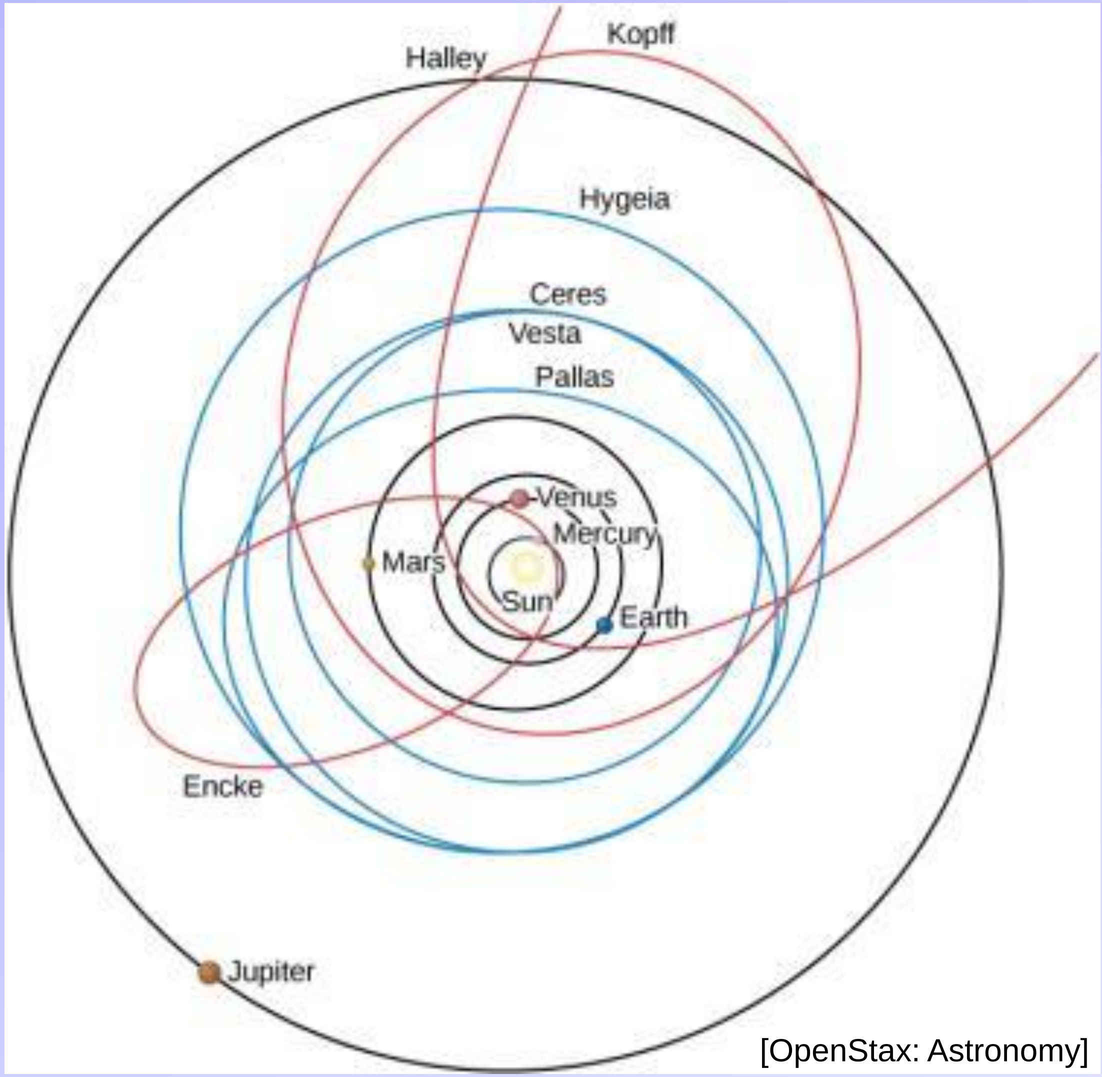
Planets (black), asteroids (blue), comets (red)

The multiple planets, asteroids, and comets all interact and modify each others orbits.

→ **Individual angular momenta change.**

→ **Total angular momentum of Solar System is constant.**

Conservation of Angular Momentum



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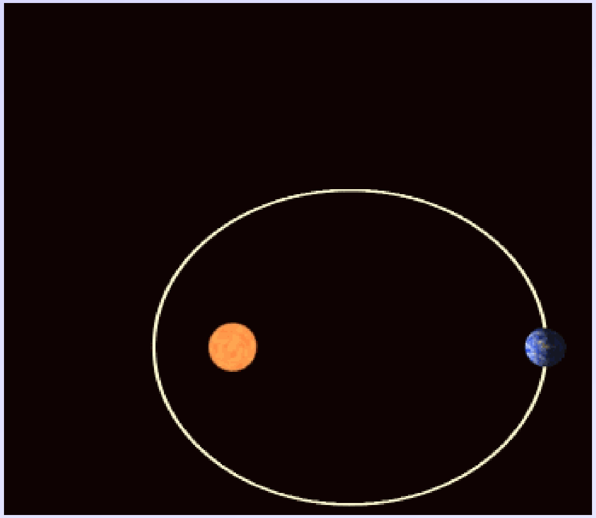
Example: Apsidal Precession



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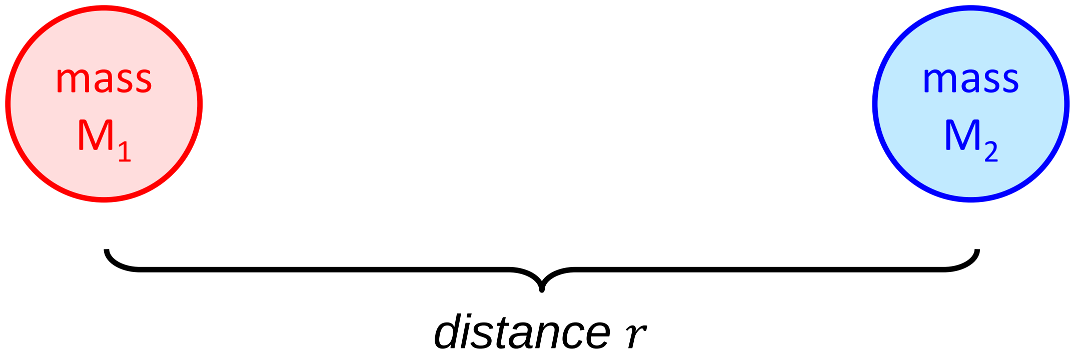
Gravitational Potential Energy



Stored gravitational energy $= E_{potential} = -G\frac{M_1 M_2}{r}$

Total Energy $= E_{total} = E_{potential} + E_{kinetic}$

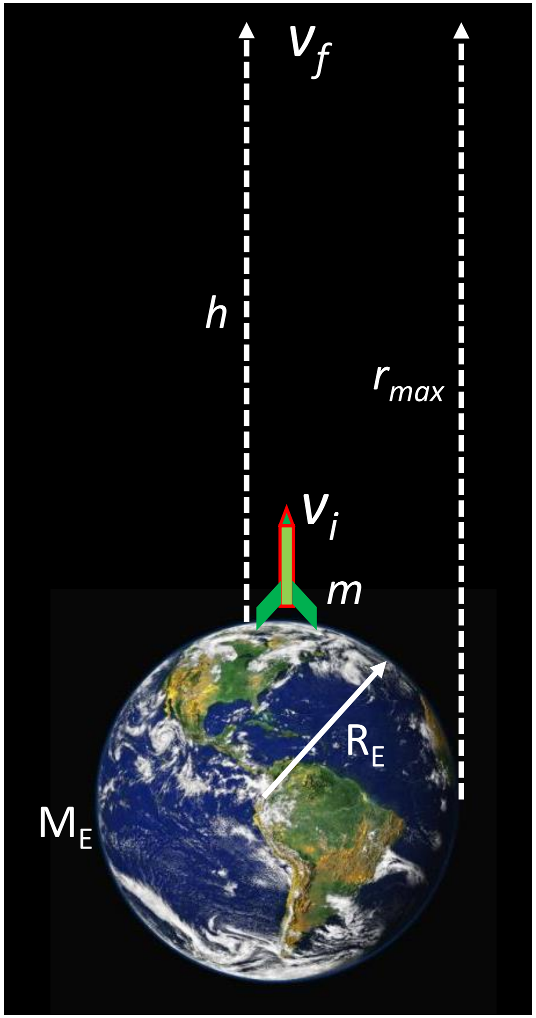
For 2 orbiting bodies (e.g. Sun + Earth): $E_{total} < 0$

For 2 unbound bodies (Earth + Mars rocket): $E_{total} > 0$

Escape Velocity

Question

What is the minimum velocity needed to escape Earth's gravity?



$$v_{escape} = \sqrt{\frac{2GM_E}{R_E}}$$

= 11.2 km/s on Earth

Note 1: escape velocity depends on your starting point.

Note 2: Since the Earth spins, objects at "rest" close to the equator already have a significant velocity.

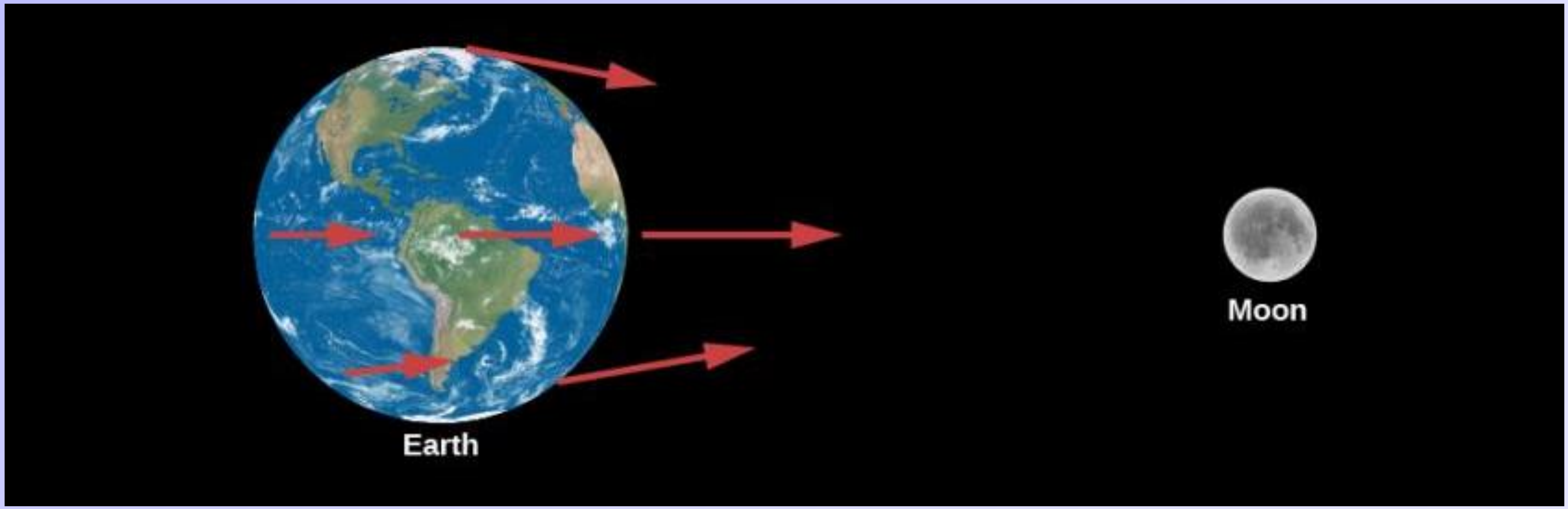
→ Rockets are typically launched close to the equator (or in Florida)

Ocean Tides

The force of **gravity** from the Moon is **not uniform** over the Earth.

→ gravity from Moon falls off as $1/r^2$.

→ Near face of Earth feels a stronger force than far face.



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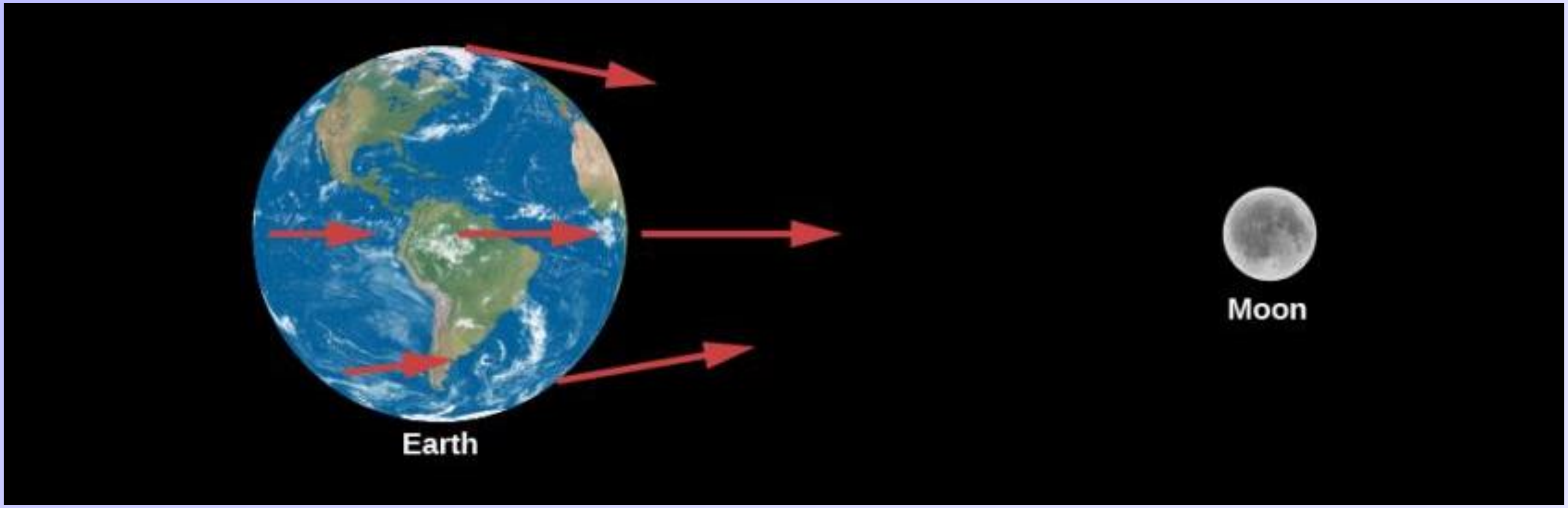


Result

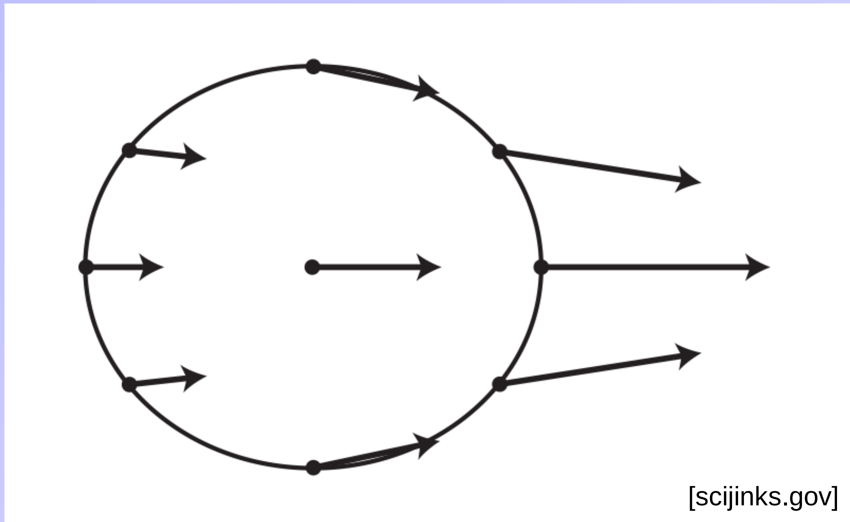
Water on **near side is pulled** towards Moon **more** than average Earth.

Water on **far side is pulled** towards Moon **less** than average Earth.

Ocean Tides: Effective Moon Gravity

Recall:

- Moon is in "free fall" orbit around Earth.
- Earth is in "free fall" orbit around Moon (albeit small orbit).



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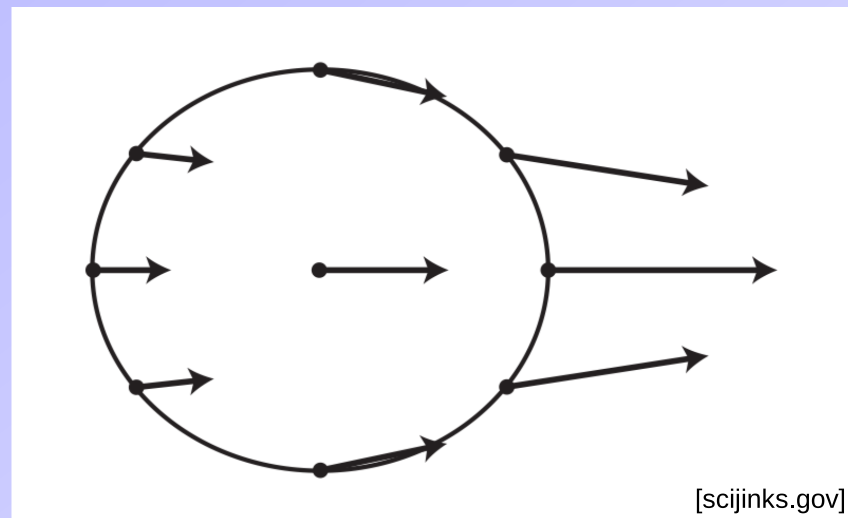
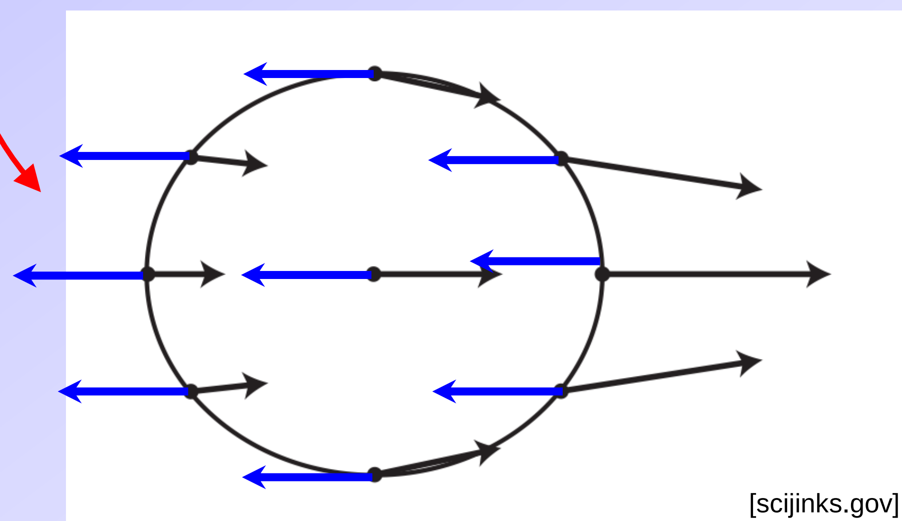


Subtract average gravitational force of Moon.

[since Earth is in "free fall" around Moon.]



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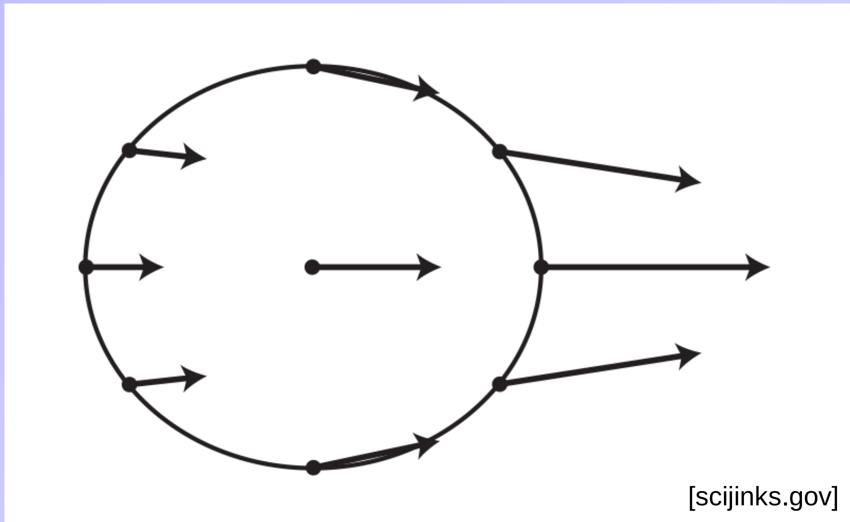

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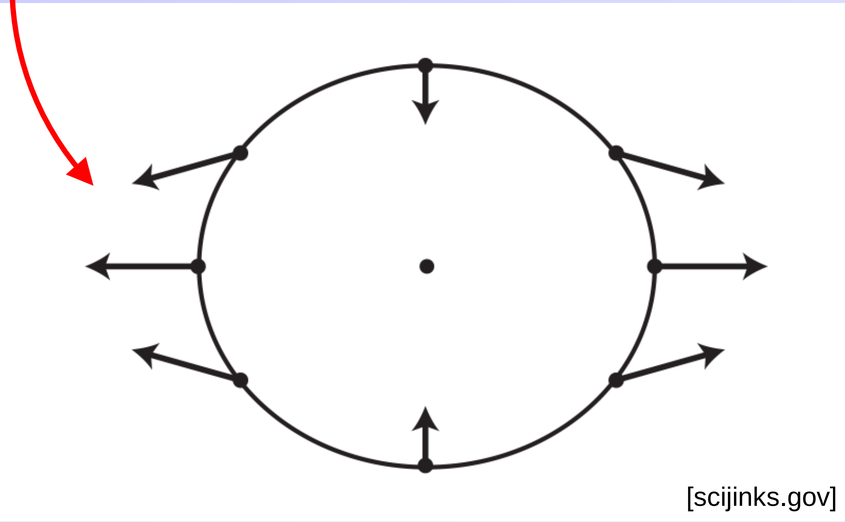

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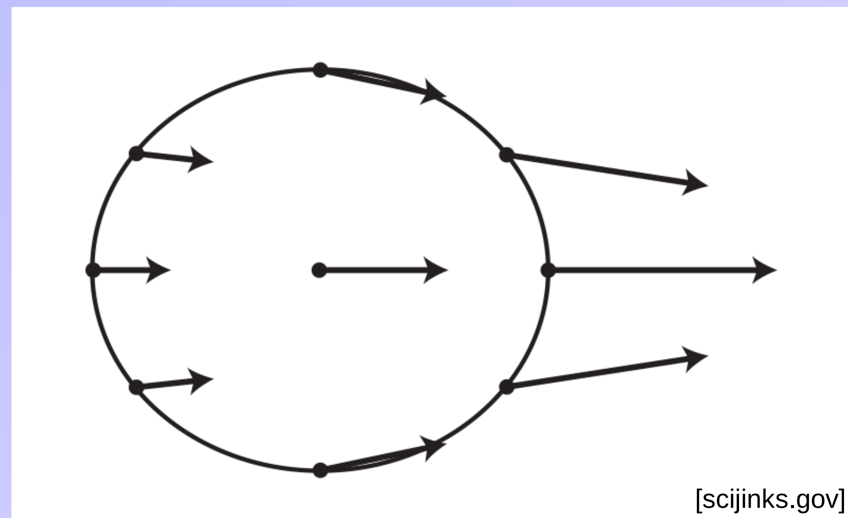
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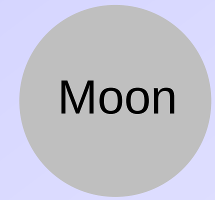
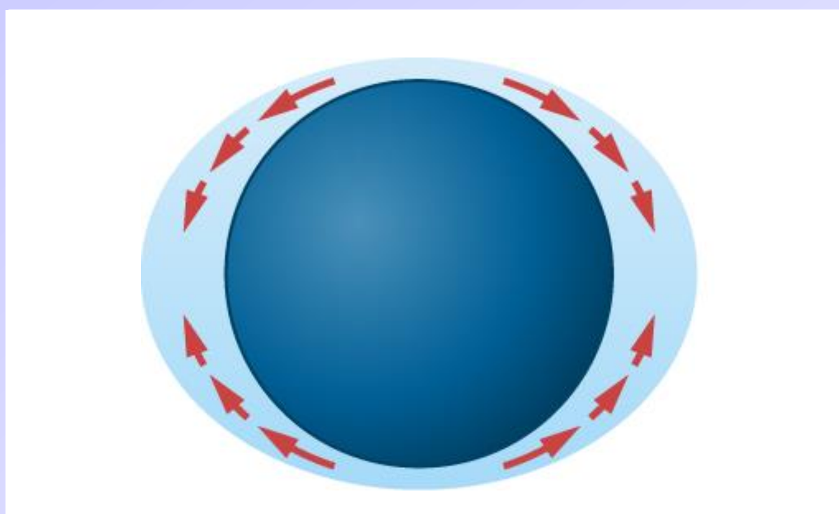


Ocean water is pulled by the effective force



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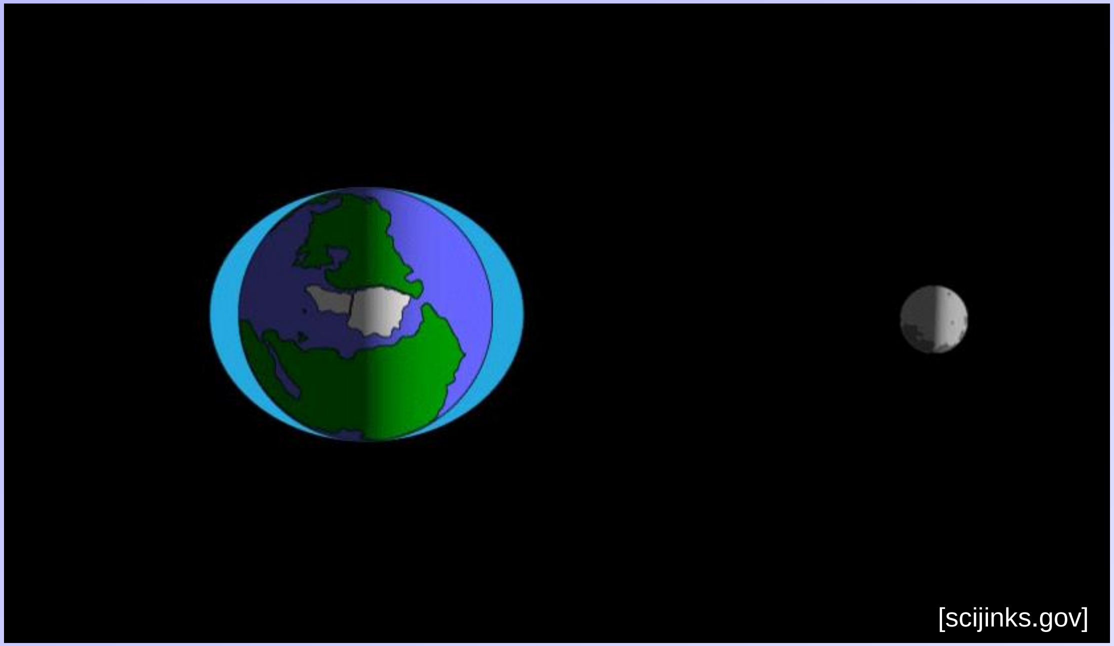

Animation of Earth and Oceans as seen from above North Pole.

Sun's gravity gradient affects tides as well: 46% of Moon's contribution.

- Tides are largest when Sun-Moon-Earth are aligned.
- Tides are weakest when Sun & Moon are at 90° to each other.
- Shape of ocean basins & winds also affect the strength of tides.
- The atmosphere also experiences tides.